PAPER 3C4 - 2003 - CRIB D.J. COLE

1 a)

Algebraic method (recommended)

from data sheet $\omega_s = (1+R)\omega_c - R\omega_a$

For first epicyclic:

$$\omega_i = (1+R_1)\omega_{c1} - R_1\omega_N \quad \text{—} (1)$$

For second epicyclic

$$\omega_i = (1+R_2)\omega_o - R_2\omega_{a2} \quad \text{—} (2)$$

But $\omega_{c1} = \omega_{a2}$

so (1) becomes $\omega_i = (1+R_1)\omega_{a2} - R_1\omega_N$ — (3)

Eliminate ω_{a2} from equations (2) and (3):

$$\omega_i = (1+R_2)\omega_o - R_2 \frac{\omega_i + R_1\omega_N}{(1+R_1)}$$

$$\omega_i(1+R_1) = (1+R_1)(1+R_2)\omega_o - R_2\omega_i - R_1R_2\omega_N$$

$$\frac{\omega_o}{\omega_i} = \frac{1+R_1+R_2+R_1R_2(\omega_N/\omega_i)}{(1+R_1)(1+R_2)}$$

when $\omega_N = 0$, $\dfrac{\omega_o}{\omega_i} = \dfrac{1+R_1+R_2}{(1+R_1)(1+R_2)}$

when $\dfrac{\omega_N}{\omega_i} = -0.2$, $\dfrac{\omega_o}{\omega_i} = \dfrac{1+R_1+R_2-0.2R_1R_2}{(1+R_1)(1+R_2)}$

[50%]

1 a) Tabular method (alternative to algebraic method)

	S_1	C_1	A_1
fix C_1	X	0	$-\frac{X}{R_1}$
$+N$	N	N	N
	$N+X$	N	$N-\frac{X}{R_1}$

	S_2	C_2	A_2
fix C_2	Y	0	$-\frac{Y}{R_2}$
$+M$	M	M	M
	$M+Y$	M	$M-\frac{Y}{R_2}$

- $$\frac{\omega_N}{\omega_i} = \frac{A_1}{S_1} = W = \frac{N-\frac{X}{R_1}}{X+N}$$

  $$\therefore N = \frac{X\left(W+\frac{1}{R_1}\right)}{(1-W)}$$

- $S_1 = S_2$

  $$X+N = Y+M$$

  $$\therefore Y = X + \frac{X\left(W+\frac{1}{R_1}\right)}{(1-W)} - M$$

- $C_1 = A_2$

  $$N = M - \frac{Y}{R_2}$$

  $$\therefore M = \frac{X\left(\frac{\left(W+\frac{1}{R_1}\right)}{(1-W)} + \frac{1}{R_2} + \frac{1}{R_2}\frac{\left(W+\frac{1}{R_1}\right)}{(1-W)}\right)}{\left(1+\frac{1}{R_2}\right)}$$

- $$\frac{\omega_o}{\omega_i} = \frac{C_2}{S_1} = \frac{M}{X+N}$$

$$= \frac{X\left(\frac{\left(W+\frac{1}{R_1}\right)}{(1-W)} + \frac{1}{R_2} + \frac{1}{R_2}\,\frac{\left(W+\frac{1}{R_1}\right)}{(1-W)}\right)}{\left(X + X\frac{\left(W+\frac{1}{R_1}\right)}{(1-W)}\right)\left(1+\frac{1}{R_2}\right)}$$

$$= \frac{\left(R_1R_2W + R_2\right) + R_1\left(W + \frac{1}{R_1}\right) + R_1(1-W)}{(1+R_1)(1+R_2)}$$

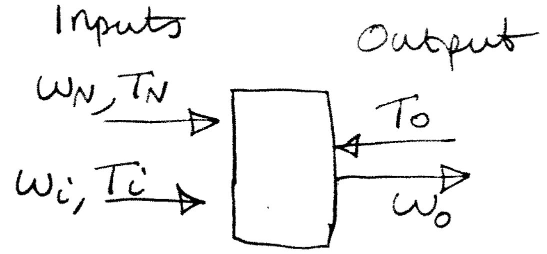
$$= \frac{1 + R_2 + R_2 + W R_1 R_2}{(1+R_1)(1+R_2)}$$

when $\omega_N = 0$, $W = 0$ $\therefore$ $$\underline{\underline{\frac{\omega_o}{\omega_i} = \frac{1+R_1+R_2}{(1+R_1)(1+R_2)}}}$$

when $\frac{\omega_N}{\omega_i} = -0.2$, $W = -0.2$ $\therefore$ $$\underline{\underline{\frac{\omega_o}{\omega_i} = \frac{1+R_1+R_2-0.2R_1R_2}{(1+R_1)(1+R_2)}}}$$

[50%]

b) i)



Sign conventions.

Virtual work

$$T_i\,\omega_i' + T_N\,\omega_N' = T_o\,\omega_o'$$

ω' are virtual speeds.

Set $\omega_N' = 0$ to get T_o/T_i :

$$\frac{T_o}{T_i} = \left.\frac{\omega_i'}{\omega_o'}\right|_{\omega_N'=0} = \frac{(1+R_1)(1+R_2)}{1+R_1+R_2}$$

from part (a)

where $R_1 = 3$, $R_2 = 4$, $T_i = 100$ Nm

so $$T_o = \frac{100\,(1+3)(1+4)}{1+3+4}$$

$$\underline{\underline{T_o = 250\text{ Nm}}}$$

[15%]

ii) Torque ratio is independent of speeds, so output torque is same for $\frac{\omega_N}{\omega_i} = 0$ and $\frac{\omega_N}{\omega_i} = -0.2$.

[10%]

iii) $\eta = \dfrac{\sum \text{power out}}{\sum \text{power in}}$

Need to determine whether ω_N is an input or an output.

Assuming no losses, $\dfrac{T_o}{T_i} = 2.5$

$$\frac{\omega_o}{\omega_i} = \frac{1 + R_1 + R_2 - 0.2R_1R_2}{(1+R_1)(1+R_2)}$$

$$= \frac{1+3+4-0.2.3.4}{(1+3)(1+4)}$$

$$= 0.28$$

hence $\dfrac{P_o}{P_i} = \dfrac{\omega_o T_o}{\omega_i T_i} = 0.28 \times 2.5 = 0.7$

hence power <u>output</u> from ω_N

Power:

$\eta T_i \omega_i = T_o \omega_o - T_N \omega_N$ (noting sign convention)

Torque equilibrium: $T_i + T_N = T_o$

Eliminating T_N:

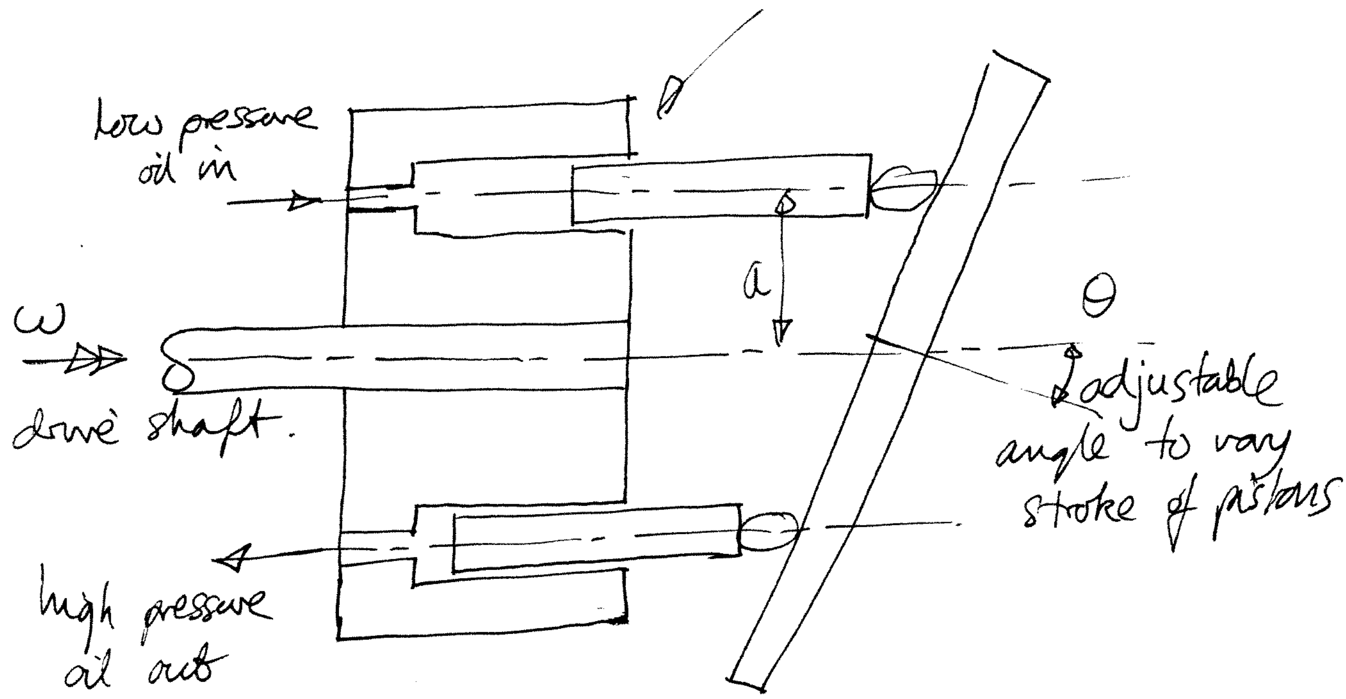
$$\eta T_i \omega_i = T_o \omega_o - (T_o - T_i)\omega_N$$

$$\frac{T_o}{T_i} = \frac{\eta - \frac{\omega_N}{\omega_i}}{\frac{\omega_o}{\omega_i} - \frac{\omega_N}{\omega_i}} = \frac{0.95 + 0.2}{0.28 + 0.2} = 2.4$$

$\therefore T_o = 2.4 \times 100 = \underline{\underline{240\ \text{Nm}}}$

[25%]

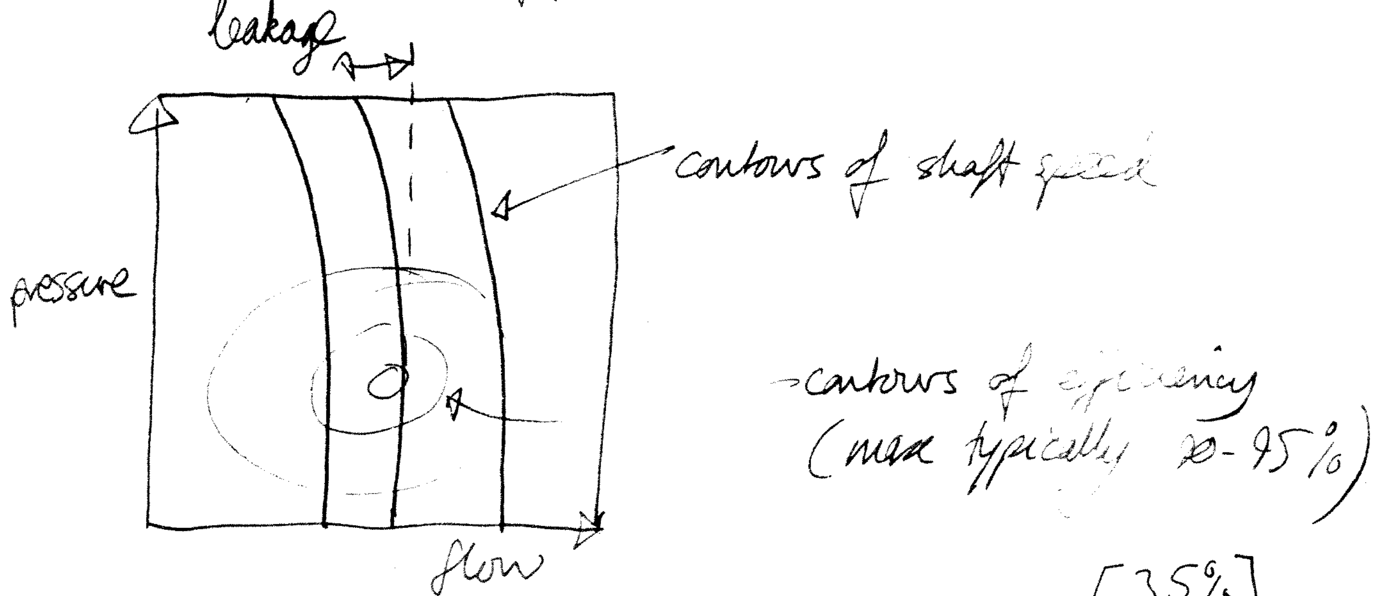
2 a)



series of pistons run inside cylinders

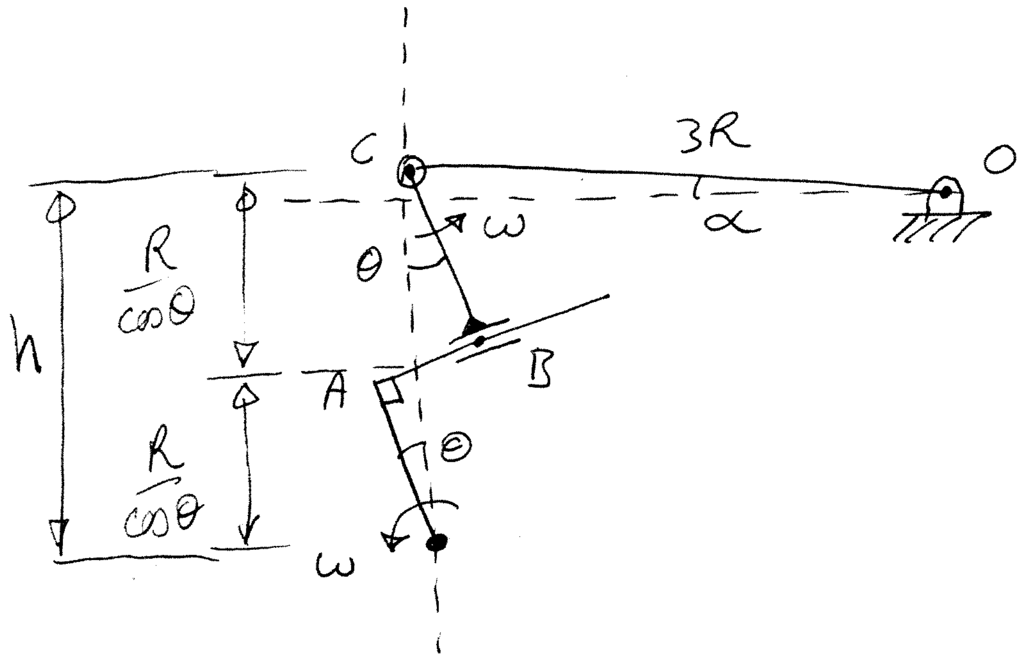
Inlet and outlet ports receive low pressure oil at top of stroke and generate high pressure at bottom of stroke.

$$\text{Ideal flow rate} = \underbrace{n}_{\text{no. of pistons}} \cdot \overbrace{A}^{\text{area of piston}} \cdot \underbrace{2a\tan\theta}_{\text{stroke}} \cdot \overbrace{\frac{\omega}{2\pi}}^{\text{speed of shaft revs/sec}}$$



[35%]

b) i) equivalent mechanism



[20%]

ii) $\alpha = \dfrac{h - 2R}{3R}$ (small angles)

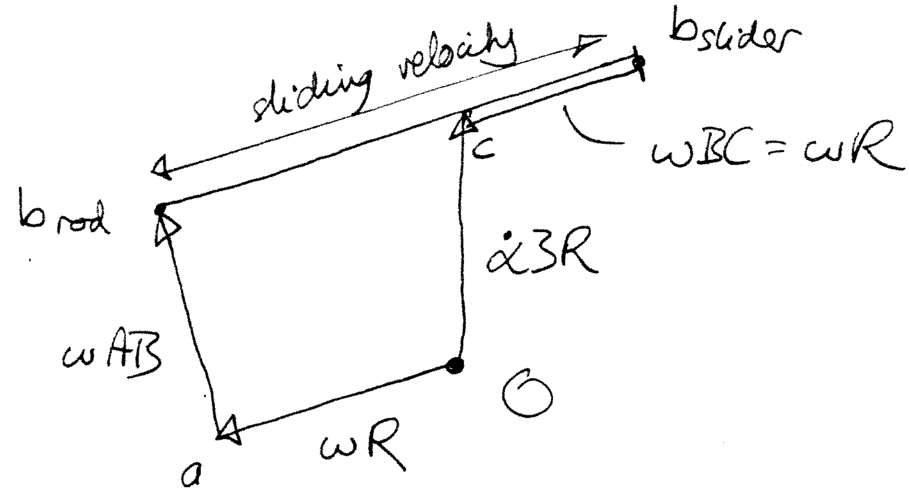
$$\alpha = \frac{2R\left(\frac{1}{\cos\theta} - 1\right)}{3R}$$

$$= \frac{2}{3}\left(\frac{1}{\cos\theta} - 1\right)$$

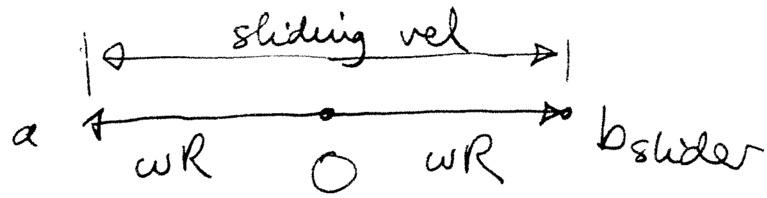
$$\dot{\alpha} = \frac{2}{3}\,\frac{\sin\theta}{\cos^2\theta}\,.\,\dot{\theta}$$

so when $\theta = 0$, $\dot{\alpha} = 0$

alternatively, using velocity diagram:



when θ = 0, AB = 0, so diagram becomes:



hence $\dot{\alpha} = 0$

Now for $\ddot{\alpha}$, differentiate expression for $\dot{\alpha}$ to give:

$$\frac{3}{2}\ddot{\alpha} = \frac{\cos^2(\sin\theta.\ddot{\theta} + \dot{\theta}^2\cos\theta) + \sin\theta.\dot{\theta}.2\cos\theta\sin\theta.\dot{\theta}}{\cos^4\theta}$$

$$= \frac{\sin\theta}{\cos^2\theta}\ddot{\theta} + \frac{\dot{\theta}^2}{\cos\theta} + \frac{2\sin^2\theta\,\dot{\theta}^2}{\cos^3\theta}$$

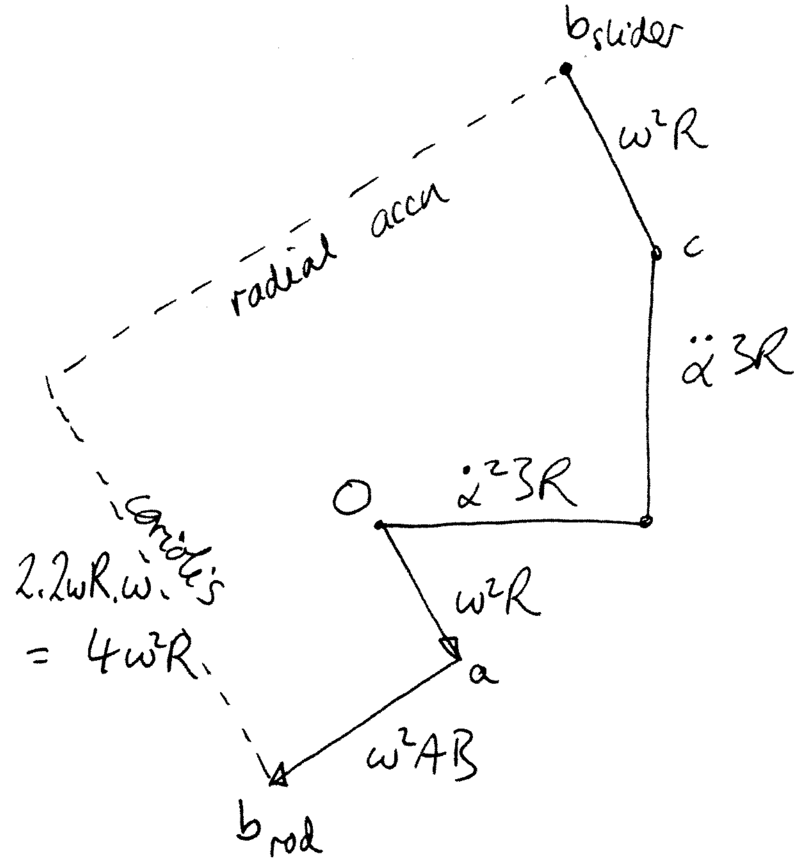
but $\ddot{\theta} = 0$ and $\sin\theta = 0$

so $$\frac{3}{2}\ddot{\alpha} = \frac{\dot{\theta}^2}{\cos\theta}$$

$$\ddot{\alpha} = \frac{2}{3}\dot{\theta}^2 \qquad (\cos\theta = 1)$$

$$\ddot{\alpha} = \frac{2}{3}\omega^2$$

alternatively, using acceleration diagram:



which becomes as shown below when $\theta = 0$:

$\therefore \ddot{\alpha} 3R + 2\omega^2 R = 4\omega^2 R$

$\therefore \ddot{\alpha} = \dfrac{2}{3}\omega^2$

[45%]

3 a)

	Cost	Speed	Load	Accuracy
Dry/rubbing	L	L	L	L/M
Rolling element	M	M	M	H
Hydrodynamic	M	H (not L)	H	M

(L = Low, M = medium, H = high)

Applications:

Dry/rubbing – photocopier

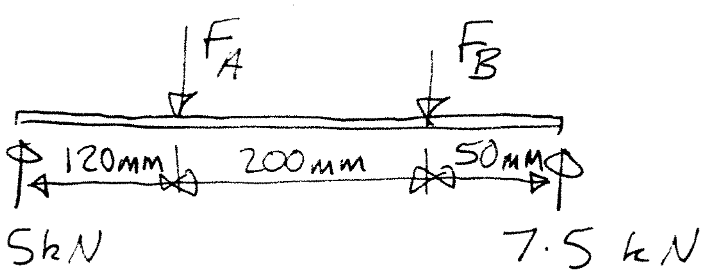
Rolling element – wheel bearing of car

Hydrodynamic – crankshaft of i.c. engine

[25%]

b) i)

calculate radial forces on bearings.



$F_A + F_B = 7.5 + 7.5 = 15\,kN$

Moments about B

$$7.5 \cdot 320 = F_A \cdot 200 + 7.5 \cdot 50$$

$$F_A = 10.125 \text{ kN}$$

$$\therefore \quad F_B = 4.875 \text{ kN}.$$

Axial force F_3 can be reacted at bearing A or B. Calculate equivalent radial forces P at each bearing for each case:

axial force at A : $P_A = 10.125 + 2.5 = 12.625 \text{ kN}$

$P_B = 4.875 \text{ kN}$

axial force at B : $P_A = 10.125 \text{ kN}$

$P_B = 4.875 + 2.5 = 7.375 \text{ kN}.$

So axial force at B gives smallest maximum equivalent radial force. Select bearing for this force of 10.125 kN :

Life equation : $L = a_1 a_{23} \left(\frac{C}{P}\right)^p$

- $L = \dfrac{1000 \text{ hours} \times 60 \text{ minutes/hour} \times 3000 \text{ revs/min}}{10^6}$

$L = 180$

- for 95% reliability, $a_1 = 0.62$ (data sheet)
- $a_{23} = 1$ (question).

hence
$$180 = 0.62\left(\frac{C}{10.125}\right)^{p=3}$$
$$\underline{\underline{C = 67.04 \text{ kN}}}$$

so smallest deep groove ball bearing has <u>O.D. 120mm</u>
(<u>bearing 6409</u>)

[40%]

ii) Use a ball bearing to react the axial force, and a roller bearing at the other location. The ball bearing is likely to be the larger of the two, so put it where the equivalent radial force is least, at B, $P_B = 7.375$ kN

$$180 = 0.62\left(\frac{C}{7.375}\right)^{p=3}$$
$$\underline{\underline{C = 48.82 \text{ kN}}}$$

so select <u>bearing 6309, O.D. 100mm</u>

Now select roller bearing, $P_A = 10.125$ kN
$$180 = 0.62\left(\frac{C}{10.125}\right)^{p=\frac{10}{3}}$$
$$\underline{\underline{C = 55.50 \text{ kN}}}$$

so select <u>roller bearing 209, O.D. 85mm</u>

<u>Hence max diameter reduced from 120mm to 100mm</u>

[35%]

4\. a) Main benefit is improved efficiency over a wide range of speed ratios.

A split transmission can also provide: (i) zero output speed without the need for a clutch to disconnect the input power; and (ii) forward and reverse ratios. [10%]

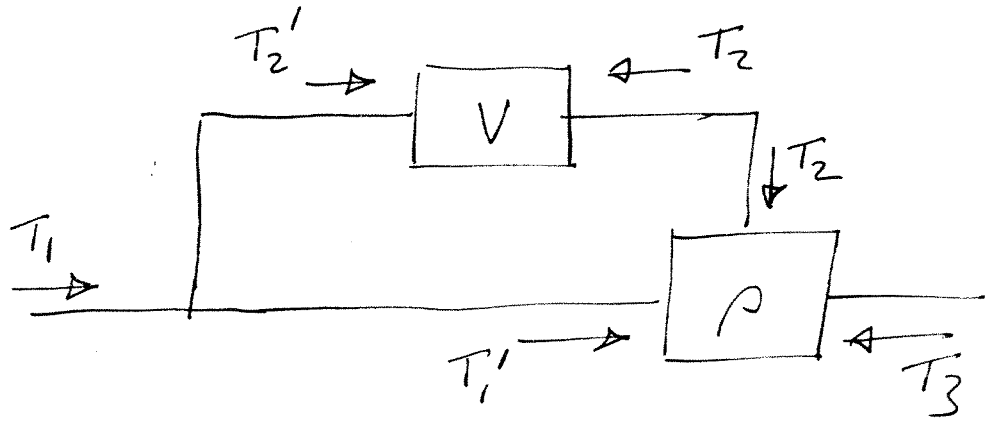
b)
$$\omega_3 = \rho\omega_1 + (1-\rho)\omega_2 \qquad (1)$$

when $\omega_3 = 0$, $\quad \rho\omega_1 = (\rho-1)\omega_2$

$$\text{so } V = \frac{\omega_2}{\omega_1} = \frac{\rho}{\rho-1}$$

[5%]

c) consider torques:



for the variable speed unit

$$\eta = \frac{T_2 \omega_2}{T_2' \omega_1} \quad \therefore \quad \frac{T_2'}{T_2} = \frac{V}{\eta} \qquad \text{— (2)}$$

for the epicyclic, consider virtual speeds and power balance to get torque ratios:

$$T_1' \Omega_1 + T_2 \Omega_2 - T_3 \Omega_3 = 0$$

set $\Omega_3 = 0$ $\quad \therefore \quad \frac{T_2}{T_1'} = -\left(\frac{\Omega_1}{\Omega_2}\right)_{\Omega_3 = 0} = \frac{1-\rho}{\rho} = \alpha$ — (3)

so power split

$$\frac{P_V}{P_1} = \frac{T_2' \omega_1}{T_1' \omega_1 + T_2' \omega_1} = \frac{T_2'/T_1'}{1 + T_2'/T_1'}$$

where $\frac{T_2'}{T_1'} = \frac{T_2'}{T_2} \cdot \frac{T_2}{T_1'} = \frac{V}{\eta} \alpha$

$$\therefore \quad \frac{P_V}{P_1} = \frac{V\alpha/\eta}{1 + V\alpha/\eta}$$

$$\frac{P_V}{P_1} = \frac{V\alpha}{\eta + V\alpha} \qquad \text{— (4)}$$

[40%]

$$\text{Efficiency } \frac{P_o}{P_1} = \frac{\eta P_V + (P_1 - P_V)}{P_1}$$

$$= \frac{P_V}{P_1}(\eta - 1) + 1$$

$$= \frac{V\alpha(\eta-1)}{\eta + V\alpha} + 1$$

$$\frac{P_o}{P_1} = \frac{\eta(V\alpha + 1)}{\eta + V\alpha} \qquad [15\%]$$

d) For power flow right to left $\eta \Rightarrow \frac{1}{\eta}$

so (4) becomes

$$\frac{P_V}{P_1} = \frac{V\alpha}{\frac{1}{\eta} + V\alpha}$$

for P_V negative and P_1 positive, $\frac{P_V}{P_1} < 0$

so $$-\frac{1}{\eta} < V\alpha < 0$$

from (1) $$\frac{\omega_3}{\omega_1} = (1-\rho)\frac{\omega_2}{\omega_1} + \rho$$

$$\frac{\omega_3}{\omega_1 \rho} = \alpha . V + 1$$

thus $$1 - \frac{1}{\eta} < \frac{\omega_3}{\omega_1 \rho} < 1$$

when $\eta = 0.7$ and $\rho = -0.25$

$$-0.107 < -\frac{\omega_3}{\omega_1} < 0.25$$

$$0.107 > \frac{\omega_3}{\omega_1} > -0.25 \qquad [30\%]$$