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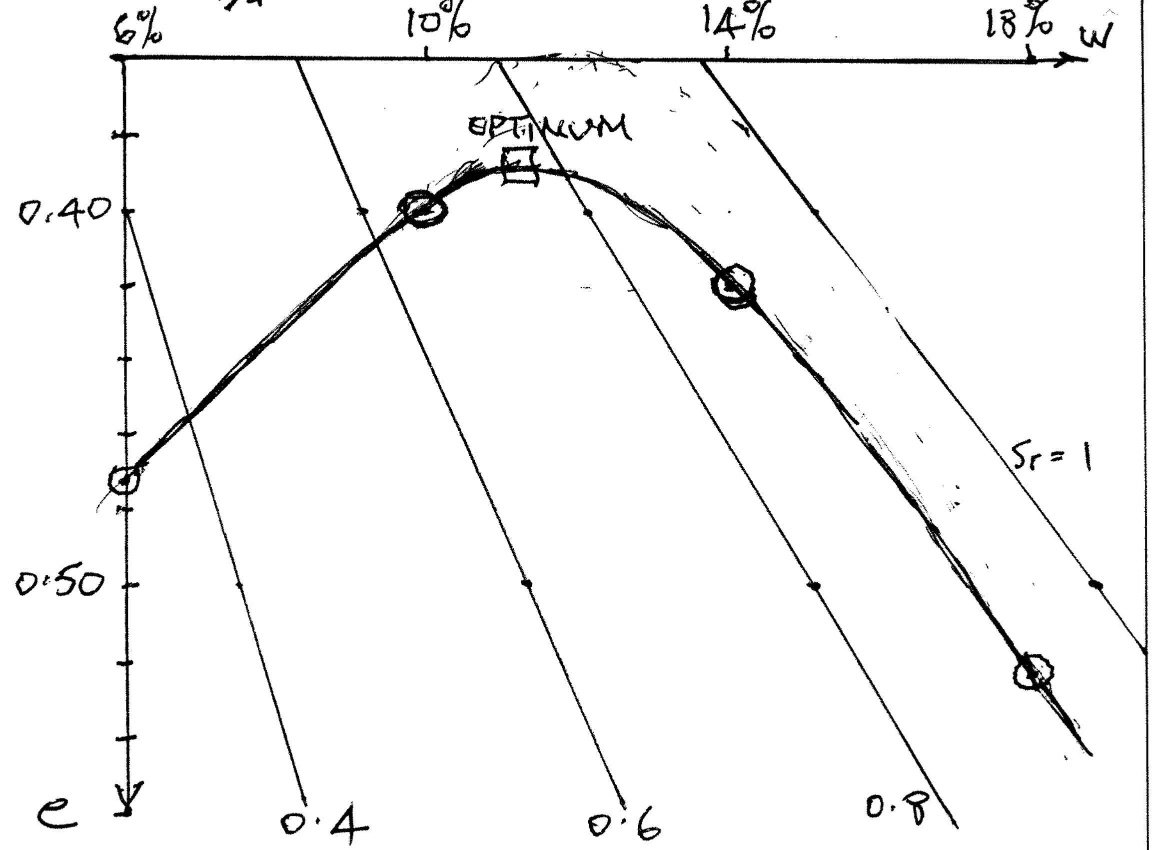
Engineering Tripos Part IIA 2003
Module 3D1 Soil Mechanics
Answers

1. a)

	W%	6	10	14	18
	ρ kgm^{-3}	1908	2082	2127	2053
$\rho_d = \rho/(1+w)$	ρ_d kgm^{-3}	1800	1893	1866	1740
$e = G_s \rho_w/\rho_d - 1$	e	0.472	0.400	0.420	0.523



$w = \dfrac{S_r e}{G_s}$

	S_r	1	0.8	0.6	0.4
e = 0.4,	W =	15.1%	12.1%	9.1%	6.0%
e = 0.5;	W =	18.9%	15.1%	11.3%	7.5%

Proctor optimum is $e = 0.39$, $\rho_d = 1906$ kg m^{-3}

Engineers prefer ρ_d because they do not need to measure G_s, and otherwise ρ_d and e are equally valid.

The fill in the field may contain large particles which have been removed for the laboratory test. So field water content may differ. Compactive effort of field plant may be different than Proctor standard. So curves may shift up (heavier compaction). 8

1 b) Procter optimum water content ~ 11%.

Wet of optimum, water is trapped in the voids. The more water, the more voids,

Dry of optimum, clumps of fines held in capillary suction trap air in voids which become interconnected below $S_r \sim 0.6$. 4

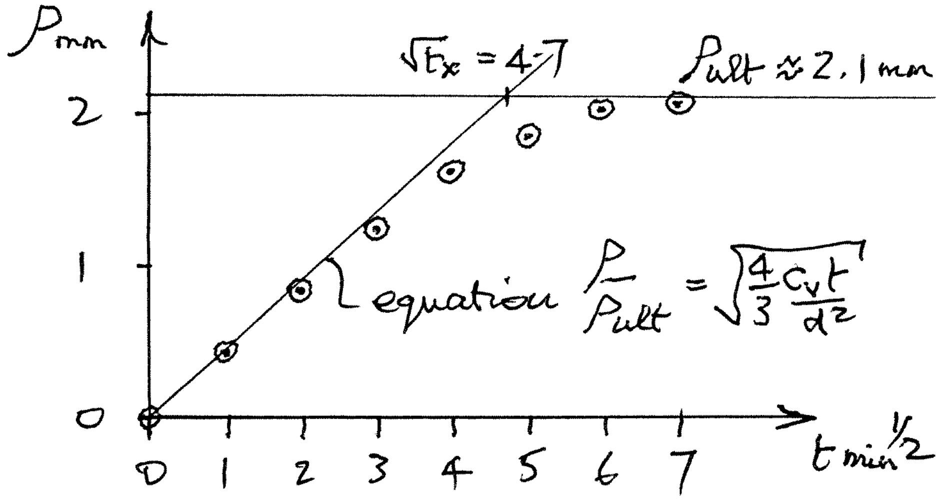
c) Pore suction on the dry side does create larger effective stress, and hence larger stiffness & strength.

The problem will be collapse of unstable air voids due to wetting when the reservoir is impounded. The till compacted at $w = 6\%$, for example, would not swell on wetting – it would settle. The clumps of fines would swell and soften, permitting the cavernous structure to collapse and settle.

The engineer should demand $w \not< 10\%$, and should rehydrate any desiccated clay at compaction. 8

2.

a)



$E_o \simeq \dfrac{50}{(2.1/20)} \simeq 476$ kPa

$C_v \simeq \dfrac{3}{4}\dfrac{(10\,mm)^2}{4.7^2\,min} \simeq 3.4\ mm^2/min \simeq 5.7\times10^{-8}\ m^2/s$

But $C_v = \dfrac{E_o k}{\gamma_w}$ so $k = \dfrac{5.7\times10^{-8}\times 10}{476} \simeq 1.2\times10^{-9}$ m/s 8

b) The median element in the clay is at the centre, starting at 3m depth. Check the stress levels used in the lab test:

$\rho = 1600\ kg/m^3$

$\therefore \gamma = 15.7\ kN/m^3$

$\therefore \gamma' = 15.7 - 9.8 = 5.9\ kN/m^3$

$\therefore \sigma'_{v,0} = 3\times5.9 = 17.7\ kN/m^2$

For the sand fill

$\rho = 1800\ kg/m^3$

$\therefore \gamma = 17.7\ kN/m^3$

$\therefore \Delta\sigma_v = 2\times17.7 = 35.4\ kN/m^2$

So there is a small mismatch between 20 kPa and 18 kPa for the initial effective stress, and a larger mismatch between 70 kPa and 53 kPa for the final effective stress

2 b cont.)

The compressibility of normally consolidated clay changes with stress change.

$$\Delta V = \lambda \ln(\sigma_f'/\sigma_i')$$

So $\Delta V_{lab} \propto \ln(70/20) \propto 1.25$

$\Delta V_{field} \propto \ln(53/18) \propto 1.08$

$\therefore \varepsilon_{field} = \varepsilon_{lab} \times \frac{1.08}{1.25} = \frac{2.1}{20} \times 0.864 = 0.0907$

and $\Delta\sigma'_{field} = 35.4$ kPa for a 2m surcharge of fill

$\therefore E_o$ field $= 35.4/0.0907 = 390$ kPa

k field $= 1.2\times10^{-9}$ m/s as before

C_v field $= \frac{E_o k}{\gamma_w} = 4.8\times10^{-8}$ m²/s | 4

c) $\rho_{ult} \simeq \varepsilon_{field} \times 6m \simeq 0.54$ m

At 12 months, $T_v = \frac{C_v t}{d^2} \simeq \frac{4.8\times10^{-8}\times3600\times24\times365}{1.5^2}$

$T_v \simeq 0.67$

$\therefore R_v \simeq 0.85$

$\therefore \rho_{1\,year} \simeq 0.46$ m

We have assumed the middepth sand layer is a drain, bringing $d = 3/2 = 1.5$m. | 4

d) We apparently need 2.54m of sand, so the total mass will be $2.54\times10^4\times1.8$ Tonnes or 45700 Tonnes.

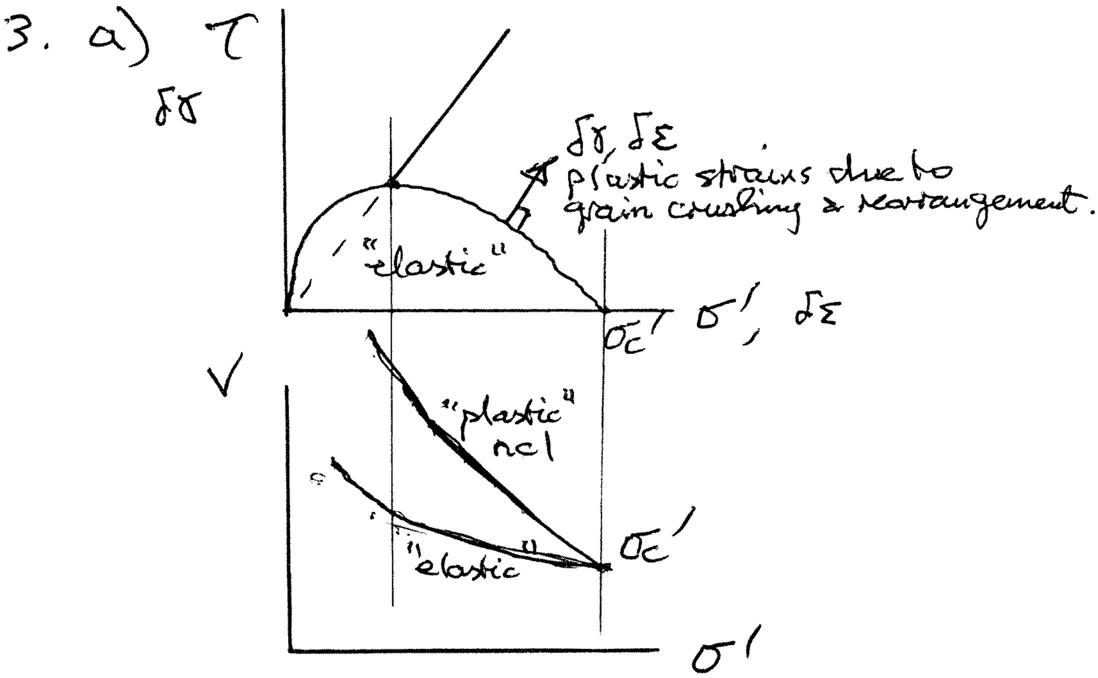
The extra 0.58m of sand, which will be pushed below the water table, will creat an extra ultimate effective stress increment of

$$0.54\times(17.7-9.8) \simeq 4.3 \text{ kPa}$$

So the corrected value $\Delta\sigma_v' \simeq 40$ kPa

This should be reflected in the calculations. An instrumented section of trial fill would be desirable to confirm results. | 4

3. a)



The normal compression line (ncl) involves crushing and rearrangement of grains. This is irrecoverable, i.e plastic.

Unloading from σ_c' follows a κ-line involving grain contact elasticity with a small amount of rearrangement (hysteresis, cyclic compaction). 4

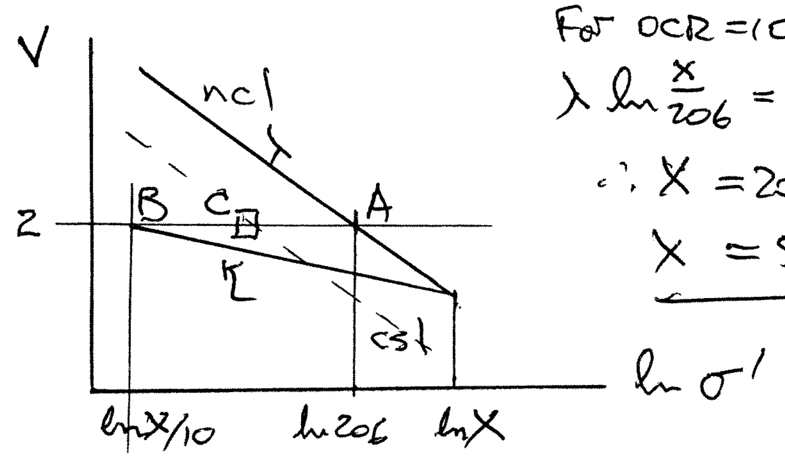
b) For London Clay, ncl is, for A:

$$V = (\Gamma + \lambda - \kappa) - \lambda \ln \sigma' = 2.759 + 0.161 - 0.062 - 0.161 \ln \sigma'$$

i.e. $V = 2.858 - 0.161 \ln \sigma'$

If $e = 1$, $V = 2$, so $\ln \sigma' = 0.858/0.161$

$\therefore \sigma'_{OCR=1} = \underline{206 \text{ kPa}}$.



For OCR = 10, At B

$$\lambda \ln \frac{X}{206} = \kappa \ln 10$$

$$\therefore X = 206\ 10^{\kappa/\lambda}$$

$$\underline{X = 500 \text{ kPa}}$$

So the clay is preloaded to 500 kPa and unloaded to 50 kPa to achieve V = 2 again.

3 b cont.)

Both samples A & B seek a critical state C at $V = 2.000$

The csl is $V = 2.759 - 0.161 \ln \sigma'$

$\therefore \ln \sigma'_c = 0.759 / 0.161 = 4.714$

$\sigma'_c = 111$ kPa

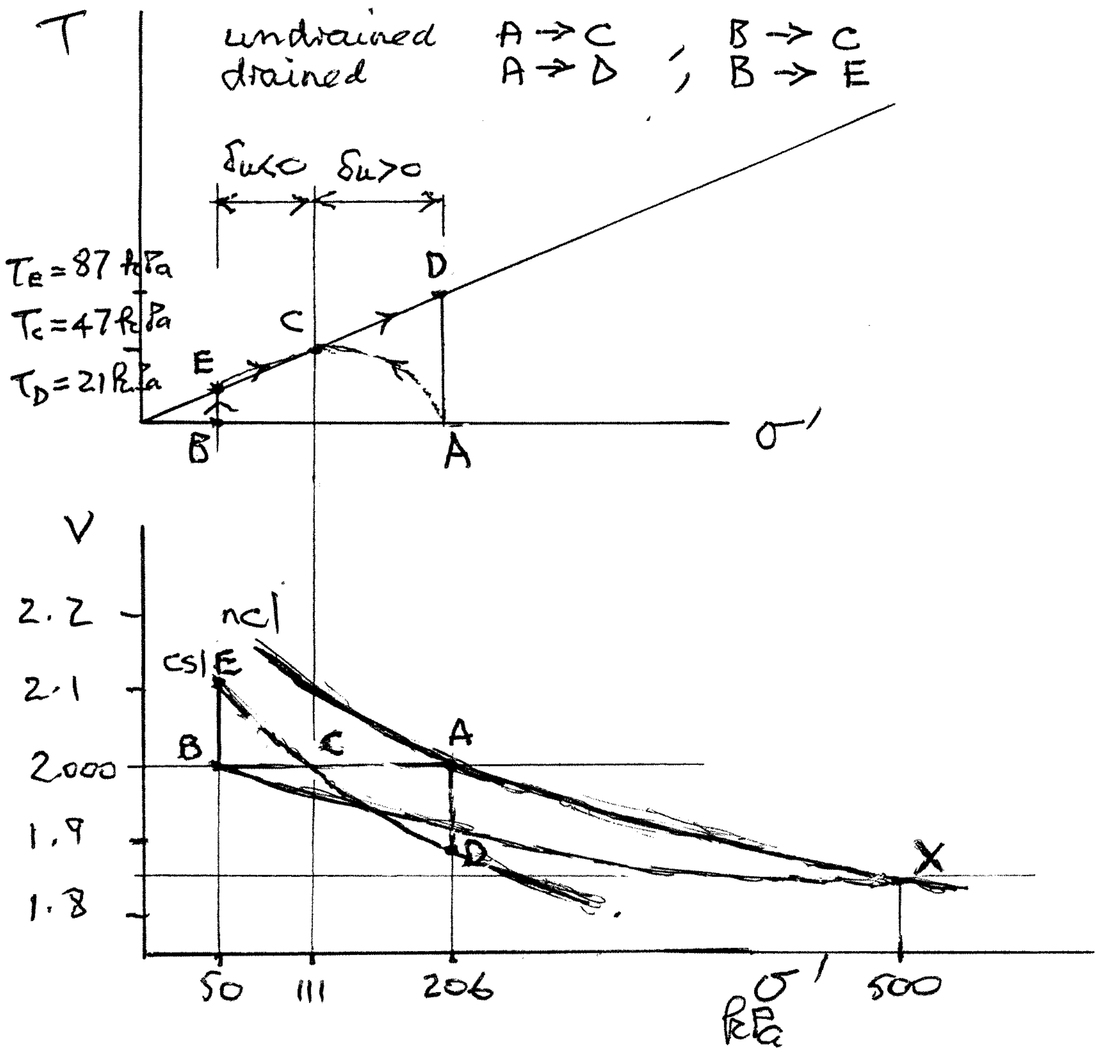
$\therefore \tau_c = 111 \tan 23^\circ = 47$ kPa

So their undrained shear strength is 47 kPa

Test $A \to C$ has σ' falling from 206 to 111 kPa
Test $B \to C$ has σ' rising from 50 to 111 kPa

$\therefore \delta u_{A\to C} = +95$ kPa

$\delta u_{B\to C} = -61$ kPa.



10

3 c) Apparent cohesion is an old-fashioned word for undrained shear strength, which was τ_c = 47kPa in the example above. This is used when there has been relatively little time for drainage between c_u being measured and it being mobilised, e.g. in the side walls of a trench which is quickly excavated, or beneath a spread foundation that is suddenly loaded.

Two stress histories are covered here. Soil A is normally consolidated. If it permitted to drain there will be large settlements as indicated along path A → D. Soil B is overconsolidated. If it is permitted to drain, it will soften as indicated along path B → E where the strength may reduce to 21 kPa.

Normally consolidated soil is too compliant.
Overconsolidated soil is brittle. 6

4 a) Because grain crushing reduces over-riding and dilatancy which otherwise enhance the shear strength of dense granular materials,

We need to define relative crushability

$$I_C = \ln \sigma_c / \sigma'$$

where σ_c is the crushing stress of grains.

Then $I_R = I_D I_C - 1$ correlates with dilatancy giving an extra component $\Delta\phi$ in addition to ϕ_{crit}. 5

b) Take $\sigma_c = 15000$ kPa from P.10

Constant volume shear demands $I_R = 0$

$$\therefore I_C = 1/I_D = 1.33$$

$$\therefore \sigma'_{crit} = \sigma_c / 3.79 = 3954 \text{ kPa}$$

$$\therefore \tau_{crit} = \sigma'_{crit} \tan 32° = 2471 \text{ kPa}$$

This would apply to constant volume shearing of sand in the field. The sand would have to be saturated and sheared very quickly, e.g. by a very fast CPT or explosive. 5

c) $\phi_{max} - \phi_{crit} = 0.8\,\psi_{max} = 5 I_R$

$$I_R = 0.75 \times \ln \frac{15000}{\sigma'} - 1$$

$$= 1.727 \quad \text{and} \quad 3.454$$

$$\Delta\phi = 8.6° \qquad 17.3°$$

$$\phi_{max} = 40.6° \qquad 49.3°$$

$$\psi_{max} = 10.8° \qquad 21.6°$$

5

d) At $\sigma' = 395$ kPa, $\tau_{max} = 339$ kPa

At $\sigma' = 39.5$ kPa, $\tau_{max} = 46$ kPa

4 d cont.)

Suppose you make a linear fit over the last two points. Fit with

$$\tau = c' + \sigma' \tan \phi'$$

Then
$$c' + 395 \tan\phi' = 339$$
$$c' + 39.5 \tan\phi' = 46$$
$$\therefore \tan \phi' = 293/355.5$$
So $\tan \phi' = 0.82$
$$\phi' = 39.5^\circ$$
And
$$c' = 13 \text{ kPa}$$

The danger is that engineers may come to believe that $\tau_{max} = 13$ kPa at $\sigma' = 0$ for which there is no evidence.
At $\sigma' = 3954$ kPa it gives $\tau_{max} \doteq 3273$ kPa which is much larger than the critical state shear strength we know to be 2471 kPa.

Actually, the envelope is close to a power curve

$$\frac{\tau}{\tau_{crit}} = \left(\frac{\sigma}{\sigma_{crit}}\right)^{\beta} \quad \text{say}$$

Let us calculate two independent values of β from the two estimates:

$$0.137 = 0.1^{\beta} \qquad \beta = 0.86$$
$$0.0186 = 0.01^{\beta} \qquad \beta = 0.86$$

and of course it fits exactly at the critical state. Good! 5