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PART IIA MODULE 3D3 2003 SOLUTIONS

Answers

Q1 (a) Bookwork. Mention definition of efficiency factor ϕ_f as bending strength of section (on elastic theory, $\sigma_f Z_e$) divided by that of a solid square bent about major axis. Then log-log charts (Data Sheet): plots of strength σ_f against density ρ — for given material move horizontally to chart of σ_f against Z_e, with sloping lines of constant required strength M: then vertically to chart of Z_e versus A, selecting max ϕ_f for that material (hence section shape): then horizontally to chart of ρ versus A, to meet vertical ρ-line, read off mass/length. Compare other materials similarly. Limitations: have to use linear-elastic theory, since not all materials can go plastic; only one cross-section of beam studied (no collapse mechanism); many designs have to limit SLS deflection as well as strength. [30%]

[10%] (b) (i) $I = 719600\ cm^4$ $A = 494\ cm^2$ $\phi_e = \frac{12I}{A^2} = \underline{35.4}$

(ii) $A = \frac{M}{L\rho} = \frac{850 \times 10^3}{146 \times 7840} = 0.743\ m^2$

deflection $\frac{5\rho g A L^4}{384 EI} \leq \frac{L}{300}$

$$\therefore I \geq \frac{1500 \times 7840 \times 9.81 \times 0.743 \times 146^3}{384 \times 210 \times 10^9}$$

$$= 3.308 \ m^4$$

$$\therefore \phi_e \geq \frac{12 \times 3.308}{(0.743)^2} = \underline{71.9}$$

Rather high – outside Ashby's chart, and twice the efficiency of a large UB. If data correct, built-up trusses can be much more efficient than I-beams (which [25%] are prone to buckling, have weight in web, etc.)

(c) Mainly bookwork. Limiting L/d helpful in initial design, and allows designer to make full use of strength, while providing adequate stiffness.
This example, plastic, hinges midspan and supports

$$\gamma_f \frac{WL}{8} = \sigma_f Z_p . 2 \quad \ldots (1)$$

deflection : $\frac{cWL^3}{384 EI} \leq \frac{L}{F} \quad \ldots (2)$

divide (2) by (1) : $\frac{cL^2}{48 \gamma_f EI} \leq \frac{L}{2F \sigma_f Z_p}$

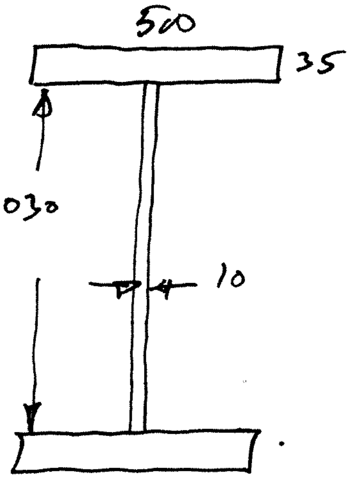
divide by d and simplify $\frac{L}{d} \leq \frac{24 \gamma_f}{cF} \cdot \frac{E}{\sigma_f} \cdot \frac{I}{Z_p d}$

First group numerical; second material (stronger material of same modulus needs <u>deeper</u> beam); third group depends on section geometry.

[35%] Could <u>not</u> use for brittle material – not plastic theory.

Q2

(a)



$$Z_p = 515 \times 10 \times 515 + 500 \times 35 \times 1065 = 21{,}290 \text{ cm}^3$$

$$M_{resist} = \frac{355}{1.1} \times 21{,}290 \times 10^{-3} \text{ kNm} = 6{,}870 \text{ kNm}$$

[20%] $\therefore$ allowable $W = \frac{6{,}870 \times 8}{26} = \underline{2{,}113 \text{ kN}}$

(b) Firstly, shear stress on web plate near supports

$$\tau \approx \frac{V}{dt} = \frac{1050 \times 10^3}{10 \times 1030} = 102 \text{ MPa}$$

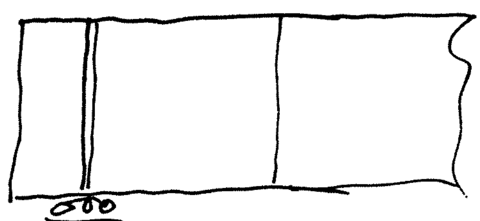
from Data Sheet, plastic $q_w = 0.6 \times 355 = 213$ MPa. So no danger of yield

But buckling? If no stiffeners, $a/d \to \infty$, Data Sheet

$$q_{cr} \approx 0.75 \times \left(\frac{1000 \times 10}{1030}\right)^2 = 71 \text{ MPa} < \tau$$

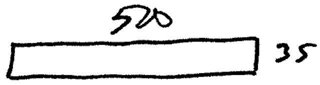
so stiffeners required. Try $a/d = 1$, $q_{cr} \approx 165$ MPa, probably ok even allowing for imperfections, interaction etc.

Second bearing: force 1050 kN, at yield requires 300 mm length of 10 mm plate, even without crippling or buckling. So probably need one stiffener at support:—



Midspan — check local buckling, b/t ratios for plastic

[30%] behaviour; welds take longitudinal shear, shear at supports.

(c) 

$P_{cr} = \dfrac{\pi^2 EI}{l^2}$ etc.

For full yield of compression member, Data Sheet, $\bar{\lambda} \leq 0.2$

$$\lambda_o = \pi\sqrt{E/\sigma_y} = 76.4 \qquad \therefore \lambda = 0.2 \times 76.4 = 15.3$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 144 \text{ mm} \quad \text{(for lateral buckling)}$$

$$\therefore \text{max. } l = 15.3 \times 144 = \underline{\underline{2.2\,m}}$$

This is quite close together! Could be awkward to

[30%] provide, unless a deck or floor slab helps.

(d) Could prevent lateral-torsional buckling — but web plates even thinner than before so would need more stiffening near supports. And how do you practically get such stiffeners inside the box?

C_w negligible : so $M_{cr} = \dfrac{\pi}{L}\sqrt{EI_{yy}.GJ}$ for uniform moment

$$I_{yy} = \frac{7 \times 50^3}{12} + 2 \times 103 \times 0.5 \times 24.7^2 = 136,000 \text{ cm}^4$$

$$\therefore M_{cr} = \frac{\pi}{26}\sqrt{\begin{array}{c}210 \times 10^9 \times 136,000 \times 10^{-8} \\ 81 \times 10^9 \times 245,000 \times 10^{-8}\end{array}} = 28,800 \text{ kNm}$$

so max. M at midspan say $\dfrac{1}{0.88} \times 28,800 = 32,800$ kNm

$$\therefore \bar{\lambda} = \sqrt{\frac{6,870}{32,800}} = 0.46$$

[Detailed calculation not really needed]

[20%] From charts not much reduction in strength at midspan due to LT buckling — but problems at supports!

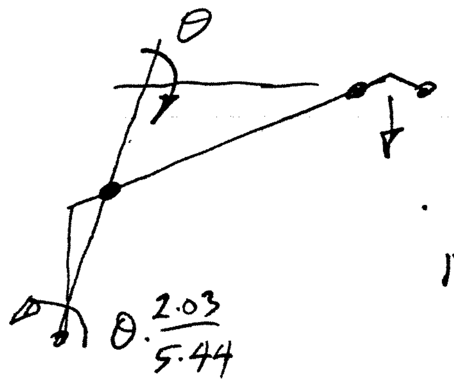
Q3. (a) Mainly bookwork. Lower bound theorem relies on ductile behaviour, convex yield criteria, perfect plasticity, flow-rule normal to yield surface, small deflections during "collapse", no buckling. Load factor λ_s, find any system of stress, moments etc in equilibrium with $\lambda_s W$ and everywhere within yield surface — then $\lambda_s \leq \lambda_c$ at collapse. In design, invent any load path (i.e. equil. system) for the loads to be taken at ULS, clothe with material such that stresses within yield, structure designed will not collapse under specified loading.

Very powerful freedom for designer. No need to consider complex real stress patterns, only simple equilibrium ones.

Limitations: steel — ductile, but buckles if thin; must use "compact" sections: Concrete — OK for low steel percentages (slabs) otherwise need rules limiting departure from elastic pattern: timber — no, not a plastic material)

[30%]

(b) (i) Example of plasticity theory — any equilibrium system will do, no need to consider elasticity, if buckling prevented.



equal moments of hinges (like plastic theory)

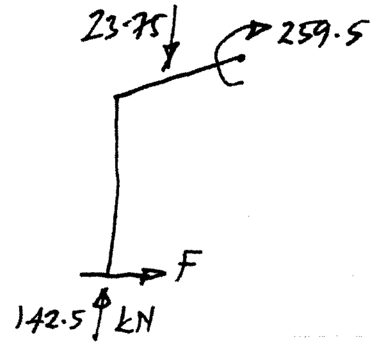
$$M\theta\left[1+1+\frac{2.03}{5.44}\right] = \theta \cdot 9.5\left[\begin{array}{l}1.0\times10.57\\ +11.5\times4.82\\ -2.5\times\frac{2.03}{5.44}\times1.25\end{array}\right]$$

$$\Rightarrow \quad M = 259.5 \text{ kNm}$$

∴ required $Z_p = \frac{259.5\times10^3}{275}\times1.1 = 1{,}038 \text{ cm}^3$

try **406×178×54 UB** $[Z_p = 1055 \text{ cm}^3]$

stanchion :



$$F\times5.44 + 23.75\times1.25$$
$$= 259.5 + 142.5\times2.5$$

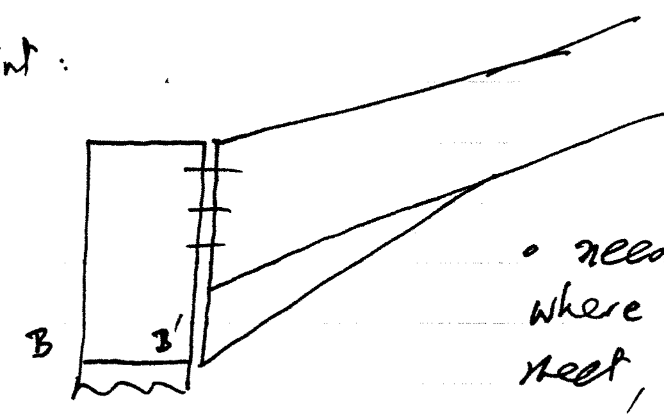
whence $F = 107.7$ kN

moment $4.2\times107.7 = 452.5$ kNm

so $Z_{p,req}$ (as above calc) is 1810 cm^3
try **457×191×82 UB** (some margin)

[50%] Further checks : max. moment away from haunch at midspan : interaction axial compression : LT buckling

(ii) joint :

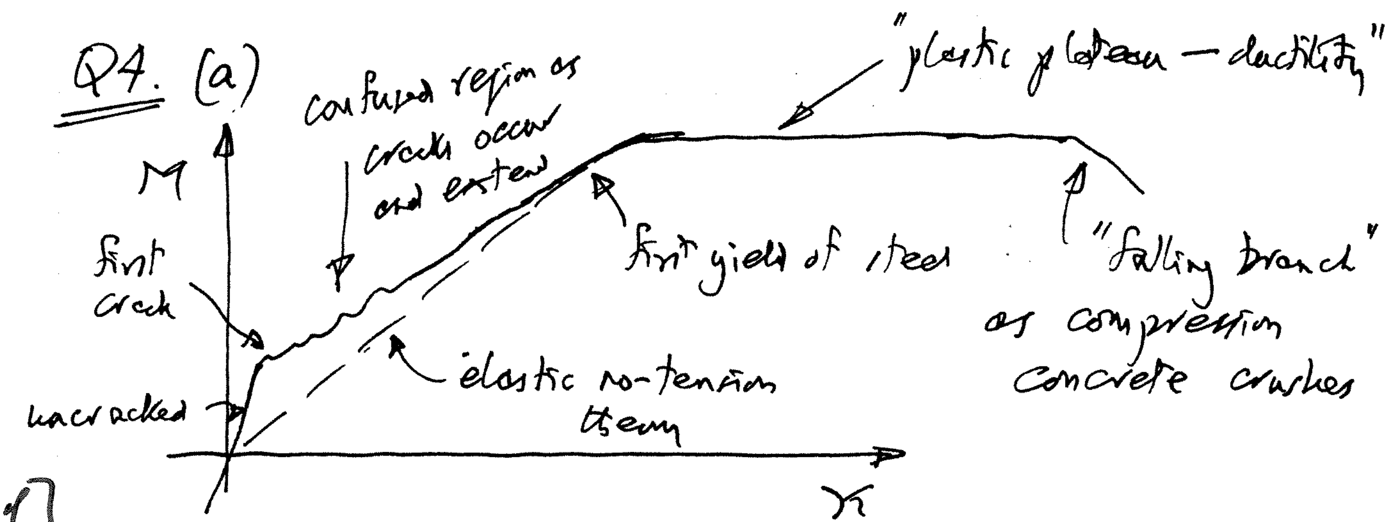


- need stiffener BB' where haunch foot is in compression
- need lateral restraint for B' where 3 compressive forces meet, could buckle out-of-plane

- bolt design by equilibrium, moment transferred, tension in bolts

[20%] • end plates thick enough to take bolt forces without bending.

Q4. (a)



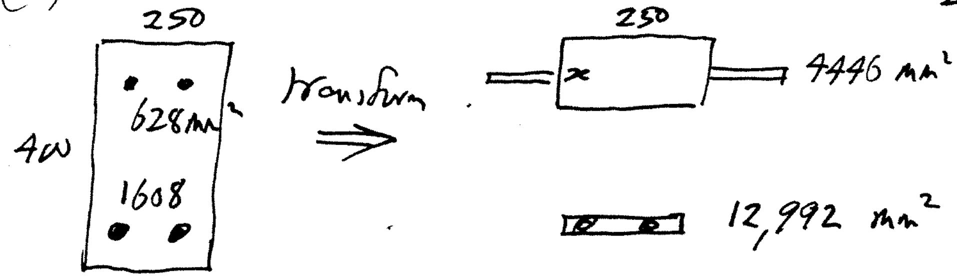
[20%]

(b) (i) $\sigma = \frac{My}{I}$ neglecting steel $\therefore M_{cr} = \sigma . bh^2/6$

[10%]

$$= \frac{3 \times 250 \times 400^2}{6} = \underline{20 \text{ kNm}}$$

(ii) cracked-elastic. modular ratio $m = \frac{210}{26} = 8.08$



zero first moment of area, about depth x.

$$12,992\,(344 - x) = 4446\,(x - 50) + 250\,x^2/2$$

quadratic $\quad x^2 + 139.5x - 4.691 \times 10^6 = 0$

$$\Rightarrow \underline{x = 136 \text{ mm}}$$

$$I_{cr} = \tfrac{1}{3} \times 250 \times (136)^3 + 4,446 \times 86^2 + 12,992 \times 208^2$$
$$= 804.6 \times 10^6 \text{ mm}^4$$

concrete stress at $\varepsilon_c = 0.0005$ is 13 MPa

$$\therefore M = \frac{13 \times 805 \times 10^6}{136} \text{ Nmm} = \underline{76.9 \text{ kNm}}$$

[30%]

Actually, more because of tension stiffening.

Crack widths limited (i) for appearance and public confidence, and (ii) to limit entry of deleterious materials (oxygen, water, chlorides etc) which might corrode the steel or attack concrete.

[10%]

(iii) $A_s f_y = \frac{450}{1.1} \times 1608 = 657.8$ kN

$A_s' f_y = \frac{450}{1.1} \times 628 = \underline{256.9}$

400.9

depth x of concrete in compression

$0.6 \times \frac{30}{1.5} \times 250x = 400.9 \times 10^3 \Rightarrow x = 134$ mm

(similar to cracked-elastic range)

Then $M = 657.8 \times (344 - 67) + 256.9 \times 17$

$= \underline{\underline{186.6 \text{ kNm.}}}$

Check strains in steel: $\varepsilon_{tension} = 0.0035 \times \frac{210}{134} = .0055$, yielded well

$\varepsilon_{compression} = 0.0035 \times \frac{84}{134} = 0.0022$, just yielded

[30%]

5. (a) $\Delta = \frac{5}{384} \cdot \frac{wL^4}{EI}$ $\qquad I = \frac{bh^3}{12}$

at buckling $\quad 0.88 \cdot \frac{wL^2}{8} = \frac{\pi E b^3 h}{24 L}$

$\therefore \quad wL^3 = \frac{\pi}{2.64} \cdot E b^3 h$

at deflection limit. $\quad \frac{L}{200} = \frac{5}{384} \cdot \frac{wL^4}{E b h^3} \cdot 12$

$\therefore \quad wL^3 = \frac{32}{1000} \cdot E b h^3$

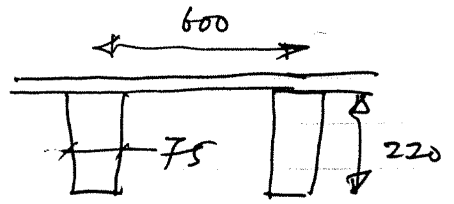
for buckling first $\quad \frac{\pi}{2.64} E b^3 h \leq \frac{32}{1000} E b h^3$

$\therefore \quad \frac{b^2}{h^2} \leq \frac{0.032 \times 2.64}{\pi}$

$\underline{\frac{b}{h} \leq 0.164}$

[30%]

(b) (i)



$k_{mod} = 0.7$ from Data Sheet

$f_{m,d} = k_{mod}\, k_h\, k_{crit}\, k_{ls}\, f_{m,k} / \gamma_m$

$= 0.7 \times 1.0 \times k_{crit} \times 1.0 \times 24/1.3 = 12.9\, k_{crit}$

Load on section $= 3.5 \times 0.6 = 2.1$ kN/m

$\sigma_{max} = \frac{wL^2}{8} \cdot \frac{6}{bh^2} = \frac{3 \times 2.1}{4 \times 75 \times 220^2} \cdot L^2 = 4.34 \times 10^{-7} L^2$

MPa if L in mm

if $k_{crit} = 1$, $\quad f_{m,d} = 12.9$

when equal to σ_{max}, $\quad L = \sqrt{\frac{12.9}{4.34 \times 10^{-7}}} = 5{,}453$ mm

But we must check for LT buckling.

$$\sigma_{m.crit} = \frac{M_{crit} \cdot y}{I} = \frac{\pi}{L} \cdot \frac{Eb^3h}{24} \cdot \frac{1}{0.88} \cdot \frac{6}{bh^2}$$

$$= \frac{\pi}{L} \cdot \frac{Eb^2}{3.52h} = \frac{\pi \times 7.4 \times 10^3 \times 75^2}{3.52 \times 220 . L}$$ MPa if L in mm

$$= \frac{169,000}{L}$$

Guess $L = 5,400$: $\sigma_{m,cr} = 31.27$

$$\lambda_{rel,m} = \sqrt{\frac{24}{31.27}} = 0.88$$

$$k_{cr} = 1.56 - 0.75 \times 0.88 = 0.902$$

$$f_{m,d} = 11.64 \text{ MPa} \qquad L_{max} = \underline{5,178 \text{ mm}}$$

guess $L = 5,200$ $\quad \sigma_{m,cr} = 32.5 \quad \lambda_{rel,m} = 0.859$

$$k_{cr} = 1.56 - 0.75 \times 0.859 = 0.916$$

[50%] $\quad f_{m,d} = 11.81$ MPa $\qquad L_{max} = \underline{5,217 \text{ mm}}$

take $\underline{\underline{5.2m}}$

(ii) <u>Bookwork</u> on $E_{0.05}$, k_h etc.

On deflection limit : $\frac{b}{h}$ is in fact $\frac{75}{220} = 0.34$ which is well above the limit calculated in (a). So Δ limit will occur first. But it still might be OK — much will depend on γ_f at ULS. Assume 2, deflection under 1.05 kN/m

$$\frac{5}{384} \cdot \frac{1,050 \cdot (5,200)^4}{7.4 \times 10^3} \cdot \frac{\times 12}{75 \times 220^3} = 20.1 \text{ mm deflection}$$

$$\frac{L}{200} = \frac{5200}{200} = 26 \text{ mm deflection, So OK.}$$

But if $\gamma_f \approx 1.5$, <u>not</u> OK.

[20%]