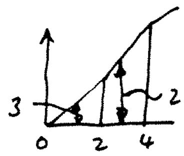
Solutions. 2003

JFA SLEATH

1

a) %'s : 3, 18, 35, 27, 12, 5 with 2 hr unit time.
Cumulants : 3, 21, 56, 83, 95, 100

Piecewise linear approx



etc.

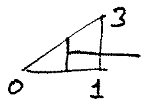
Plot at "½ time" intervals
i.e. at 1, 3, 5, 7, ... hours.

→ see graph paper → S-curve.
lag by 1 hour and subtract
→ curve proportional to 1 hour hydrograph

∴ Instantaneous flows at hourly intervals proportional to

0, 3, 7, 11, 16, 19, 16, 11, 7, 5, 3, 2, 0

Using piecewise linear approx, distribution %'s are therefore

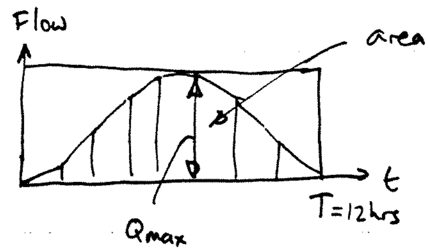


etc

20%

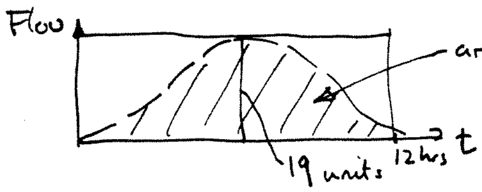
→ 1.5, 5, 9, 13.5, 17.5, 17.5, 13.5, 9, 6, 4, 2.5, 1 , $\Sigma = 100\%$ ✓

ii) 1 cm XS rainfall over 60 km² = $10^{-2}\,m \times 60 \times 10^6\,m^2$ = $6 \times 10^5\,m^3$
total runoff above baseflow



area under graph = $6 \times 10^5\,m^3$

area of rectangle = $Q_{max} \times T$



area = 100 units x hrs.

area of rectangle = 12 hrs x 19 units.

$$\therefore \text{area under curve} = \frac{100}{12 \times 19} \times \text{area of rectangle}$$

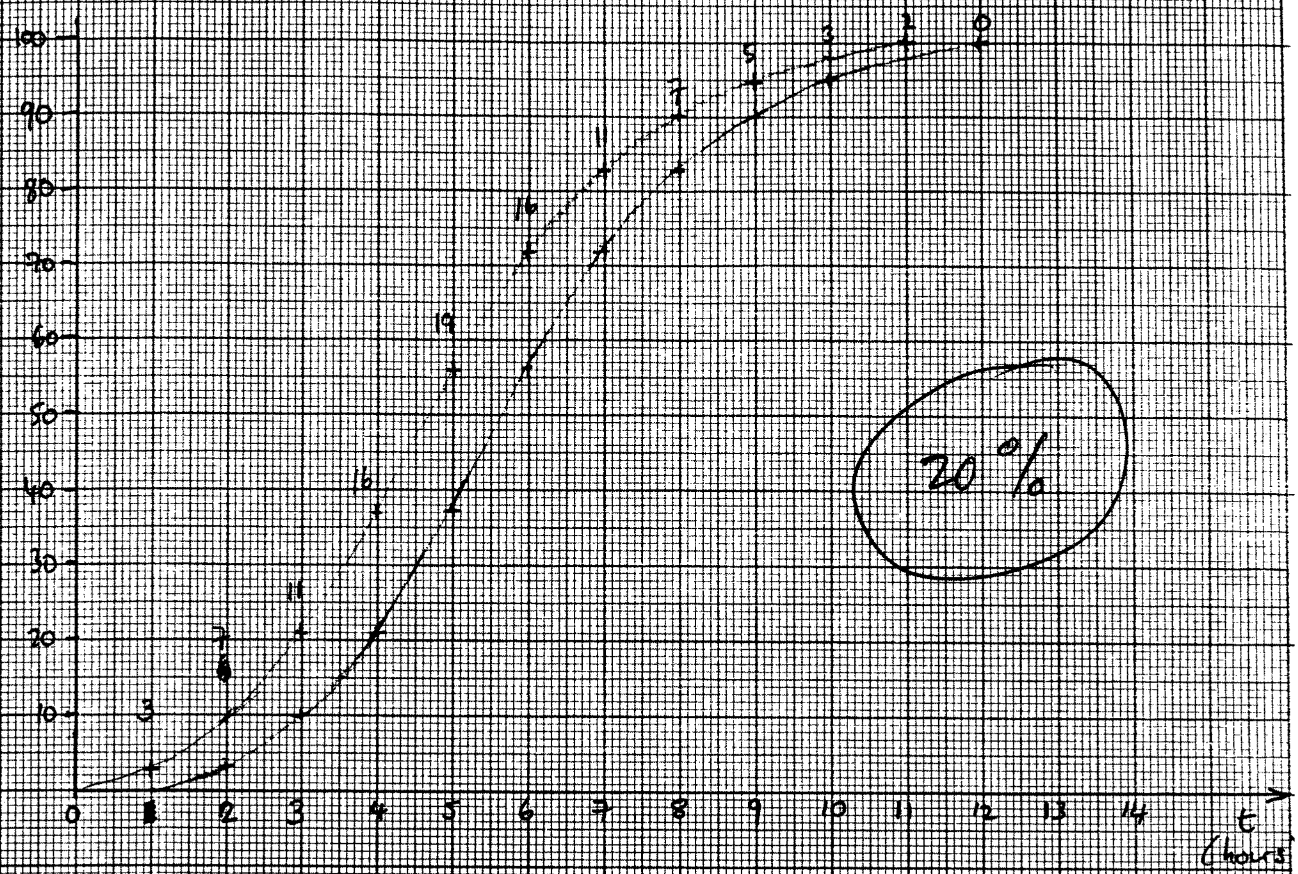
$$= 0.44 \times \text{area of rectangle}$$

∴ From top graph, $6 \times 10^5\,m^3 = 0.44 \times Q_{max} \times 12$ hours.

$$\therefore \quad Q_{max} = \frac{6 \times 10^5\,m^3}{0.44 \times 12 \times 3600\,s} = \underline{32\ m^3/s}$$

30%

1 (Contd)


100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
t (hours)
3
7
11
16
19
16
11
7
5
3
2
0

20%

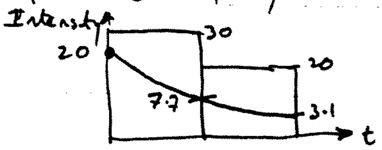
1

(b) .
$f_o = 20$ mm/h
$f_c = 0{\cdot}5$ mm/h
$K_f = 1\ h^{-1}$

$$f = f_c + (f_o - f_c)e^{-kt}$$

$\therefore$ At $t=0$, $f = 20$ mm/hr
$t = 1$hr, $f = 0{\cdot}5 + 19{\cdot}5\,e^{-1} = 7{\cdot}7$ mm/hr
$t = 2$hr, $f = 0{\cdot}5 + 19{\cdot}5\,e^{-2} = 3{\cdot}1$ mm/hr.

Rainfall = 30 mm/hr, 20 mm/hr.



$\therefore$ Infiltration curve always under rainfall curve.

$\therefore$ Infiltration in 1st hour $= \int_0^1 f\,dt = \int_0^1 f_c + (f_o - f_c)e^{-kt}\,dt$

$$= \left[f_c t + \frac{(f_o - f_c)}{-k} e^{-kt} \right]_0^1$$

$$= f_c - \frac{(f_o - f_c)}{k}\left[e^{-1} - e^{0}\right]$$

$$= 0{\cdot}5 + \frac{19{\cdot}5}{1}(0{\cdot}6321) = 12{\cdot}8\text{ mm}.$$

In 2nd hour,

$$\text{Infiltration} = 0{\cdot}5 + \frac{19{\cdot}5}{1}\left[e^{-1} - e^{-2}\right]$$

$$= 0{\cdot}5 + 19{\cdot}5\,(0{\cdot}2325) = 5{\cdot}0\text{ mm}.$$

$\therefore$ XS rainfall $= \begin{cases} 30 - 12{\cdot}8 = 17{\cdot}2\text{mm} & \text{1st hour} \\ 20 - 5{\cdot}0 = 15\text{mm} & \text{2nd hour.} \end{cases}$

25%

c). Assumptions:
e.g. rainfall uniform in space and time over catchment,

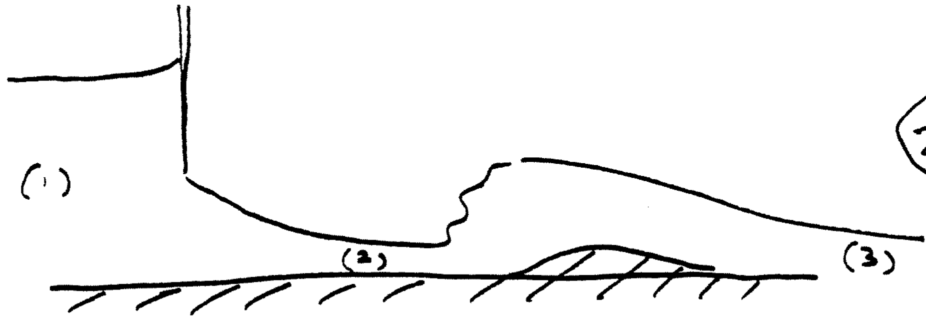
Main problematic assumption is linearity. – ie. that doubling the excess rainfall doubles the instantaneous runoff, but does not change the shape of the hydrograph.

In reality, if there is more run-off, it may travel faster and so peak may arrive sooner.

Alternatively, if there is much more run-off, the flow may spill onto a floodplain, thereby attenuating the peak.

25%

2(a)



20%

Bernoulli $5 + \frac{v_1^2}{2g} = 1 + \frac{v_2^2}{2g}$

Continuity $5v_1 = v_2$

$\therefore \quad 4 = \frac{1}{2g}\left[v_2^2 - v_1^2\right] \qquad v_1^2 = \frac{4 \times 2 \times 9.81}{24}$

$v_1 = 1.808$ m/s

$v_2 = 9.04$ m/s

$F_1 = \frac{v_1}{\sqrt{gh_1}} = 0.258, \quad F_2 = \frac{v_2}{\sqrt{gh_2}} = 2.886$

when depth is 3m $\quad v_2 = 3v_3, \quad v_3 = 3.013$ m/s

$F_3 = \frac{v_3}{\sqrt{gh_3}} = 0.555$

and hydraulic jump 2 → 3.

(b) Force on obstruction

$$-F + \frac{\rho g 1^2}{2} - \frac{\rho g 3^2}{2} = \dot{m}(v_3 - v_1)$$

$$\therefore -F = \rho \times 5 \times 1.808(3.013 - 9.04) + \left(\tfrac{9}{2} - \tfrac{1}{2}\right)\rho g$$

$$\therefore F = 15.24 \frac{kN}{m} = \underline{\underline{304.8 \text{ kN}}} \text{ for 20m width}$$

Force on sluice

$$-F + \frac{\rho g 5^2}{2} - \frac{\rho g 1}{2} = \dot{m}(v_2 - v_1)$$

80%

$$-F = \rho 5 \times 1.808(9.04 - 1.808) - 12\rho g$$

$$= 53.37 \frac{kN}{m} = \underline{\underline{1067.4 \text{ kN}}} \text{ for 20m}$$

3 (a)(i) As flow rate increases the bed geometry passes through the following stages:

Flat bed with little or no sediment motion

Ripples } Move downstream. $F<1$

Dunes

20%

Transition – Dunes / Flat bed

Flat bed with intense sediment motion and calm surface

Flat bed with stationary waves on water surface

Antidunes – usually move upstream. F usually >1.

(ii) Relevant dimensionless groups for bed forms are

$$\frac{\tau}{(\rho_s-\rho)gD}\ ,\quad \frac{u_* D}{\nu}\ ,\quad \frac{u_*}{W}\ ,\quad \frac{u}{\sqrt{gd}}$$

15%

The group relevant to water flows is the

Froude number $\frac{u}{\sqrt{gd}}$

(b) $$\frac{\tau}{\rho} = u_*^2 = gSd = 9.81\times10^{-2}\times1 = 0.0981\ m^2/s^2$$

$$\therefore \frac{u_* D}{\nu} = \frac{0.313\times3\times10^{-3}}{10^{-6}} = 0.939\times10^{+3}$$

Hydraulically rough $\therefore$ $\bar{u} = 0.313\times2.5\ln\left(\frac{12.1\times1}{3\times10^{-3}}\right) = 6.5\ m/s$

$F = 6.5/\sqrt{9.81\times1} = 2.075$

20%

$$\frac{\tau}{(\rho_s-\rho)gD} = \frac{0.0981}{1.65\times9.81\times10^{-3}\times3} = 2.02$$

Garde + Albertson (Data Sheet) gives antidunes

Also $W = 3.3(.003\times1.65)^{\frac{1}{2}} = 0.232\ m/s$

$\therefore u_*/W = 1.35$

$\therefore$ Albertson Simons & Richardson give antidunes

3 (c) Modelling: if depth d is reduced by 100 then u^+ is reduced by 10 since slope is same

Exact geometric similitude would reduce D to $3\times10^{-3}\times10^{-2}$ ie $30\,\mu m$ which would cause coagulation.

Also $\frac{u^+D}{\nu} = \frac{0.939\times10^3}{10\times10^2} = 0.939$ smooth whereas original was rough.

Try to keep $\frac{\tau}{\rho(\frac{\rho_s}{\rho}-1)gD}$ about same

Keep $\frac{u^+D}{\nu}$ rough or at least not smooth

and $F = \frac{\bar{u}}{(gd)^{1/2}}$ about same

if D is reduced by 10 (not 100) then

$$\frac{\left(\frac{\bar{U}}{u^+}\right)_{full}}{\left(\frac{\bar{U}}{U^+}\right)_{mod}} = \frac{\ln \frac{12.1\times1}{3\times10^{-3}}}{\ln \frac{12.1\times10^{-2}}{3\times10^{-4}}} = \frac{8.3}{6}$$

45%

$$\therefore \frac{\bar{u}_{full}}{\bar{u}_{mod}} = \frac{8.3}{6}\times10 = 13.8$$

New $F = \frac{2.075\times10}{13.8} = 1.5$ so reasonable

then adjust $\frac{\tau}{\rho(\frac{\rho_s}{\rho}-1)gD}$ to be same by playing with $\frac{\rho_s}{\rho}-1$. τ reduced by 100, D reduced by 10 so above by 10 and hence $\frac{\rho_s}{\rho}-1$ goes from 1.65 to 0.165!

4 (a) Bookwork – see any standard textbook (e.g. Raudkivi "Loose Boundary Hydraulics") 30%

(b) Assumptions

Isotropic turbulence

Turbulent eddies have a recognisable mixing length

Fall velocity is unaffected by sediment concentration

Turbulence is unaffected by sediment 15%

(c) If ε is constant

$$\int \frac{dc}{c} = -\int \frac{W}{\varepsilon}\, dy$$

$$\therefore c = c_o \exp\left(-\frac{Wy}{\varepsilon}\right) \qquad \text{(A)}$$

20%

where c_o is a constant of integration

(d) Not likely to be correct:-

If $\varepsilon = v'\ell = \ell^2 \frac{\partial u}{\partial y}$ and $\ell = \kappa y$ and u is taken from log law, we obtain $\varepsilon = \kappa u_* y$ 15%

(e) From Data Sheet $W = 56 \times 10^4 \times (0.2 \times 10^{-3})^2 \times 1.65$

$= 0.037$ m/s

If river is of depth d, Equation (A) gives

$$\frac{10}{2} = \frac{\exp\left(-\frac{W(d-3)}{\varepsilon}\right)}{\exp\left(-\frac{W(d-1)}{\varepsilon}\right)} = \exp\left(\frac{2W}{\varepsilon}\right)$$

$$\therefore \varepsilon = 2W / \ln 5 = 0.046 \; (m^2/s)$$

20%