1\. (a) $N_{235} = N_{0235}\exp(-\lambda_{235}t)$

$N_{238} = N_{0238}\exp(-\lambda_{238}t)$

(20%)

$$\therefore \frac{N_{0235}}{N_{0238}} = \frac{N_{235}\exp(\lambda_{235}t)}{N_{238}\exp(\lambda_{238}t)}$$

$$\lambda_{235} = \frac{\ln 2}{T_{1/2\,235}} = \frac{0.693}{7.1\times10^{8}} = 9.763\times10^{-10}\ y^{-1}$$

$$\lambda_{238} = \frac{0.693}{4.5\times10^{9}} = 1.540\times10^{-10}\ y^{-1}$$

At present day $\dfrac{N_{235}}{N_{235}} = \dfrac{0.0072}{0.9928} = 7.252\times10^{-3}$

So 2×10^{9} years ago

$$\frac{N_{0235}}{N_{0238}} = 7.252\times10^{-3}\ \frac{\exp(9.763\times10^{-10}\times2\times10^{9})}{\exp(1.540\times10^{-10}\times2\times10^{9})}$$

$$= 0.0376$$

If N is isotopic abundance then

$$N_{0235} + N_{0238} = 1$$

$$\therefore N_{0235}\left(1+\frac{1}{0.0376}\right) = 1 \Rightarrow \underline{\underline{N_{0235} = 0.0362}}$$

and $\underline{\underline{N_{0238} = 0.9638}}$

(b) For thermal fission to be maintained neutrons produced in fission will need to be moderated to thermal energies, so a moderator is required. This is likely to have been water at Oklo.

(15%)

The fuel/moderator and other materials (in a designed reactor control materials, fuel cladding etc.; in a natural reactor just extraneous soil, minerals etc.) will need to form a critical mass.

An initial neutron (e.g. produced by radioactive decay) will be required to initiate the fission chain reaction.

1\. (c) $\Sigma_x = \sum_i \Sigma_{xi}$ for interaction x

(30%) $\Sigma_{xi} = N_i \sigma_{xi} = \dfrac{f_i M_i N_A}{m_i} \sigma_{xi}$

f_i = isotopic abundance M_i = mass per unit volume

m_i = molar mass N_A = Avogadro's number

$\sigma_a = \sigma_c + \sigma_f$

$$\therefore \Sigma_a = \frac{0.0362 \times U \times 6.022\times10^{23} \times (107+580)\times10^{-28}}{0.235} \quad \text{U-235}$$

$$+ \frac{0.9638 \times U \times 6.022\times10^{23} \times (2.75)\times10^{-28}}{0.238} \quad \text{U-238}$$

$$+ 10 \quad \text{everything else}$$

$$= 7.044\times10^{-3}\, U + 10 \ \ m^{-1}$$

Only U-235 is fissile

$$\therefore \Sigma_f = \frac{0.0362 \times U \times 6.022\times10^{23} \times 580 \times 10^{-28}}{0.235} = 5.380\times10^{-3}\, U \ \ m^{-1}$$

$$\eta = \nu \frac{\Sigma_f}{\Sigma_a} = \frac{2.43 \times 5.380\times10^{-3}\, U}{10 + 7.044\times10^{-3}\, U}$$

$$\text{If } \eta > 1 \Rightarrow 2.43 \times 5.380\times10^{-3}\, U > 10 + 7.044\times10^{-3}\, U$$

$$\therefore U\left(13.073\times10^{-3} - 7.044\times10^{-3}\right) > 10$$

$$\therefore \underline{\underline{U > 1659 \ \ kg/m^3}}$$

(d) If $U = 2000 \ kg/m^3$

$\Sigma_a = 24.088 \ m^{-1}$

(20%) $\Sigma_f = 10.760 \ m^{-1}$

Hence $\eta = 2.43 \times \dfrac{10.760}{24.088} = 1.0855$

Hence $B_m^2 = (\eta - 1)\dfrac{\Sigma_a}{D} = \dfrac{0.0855 \times 24.088}{0.01} = 205.95 \ m^{-2}$

$\therefore B_m = 14.35 \ m^{-1}$

[Continued on next page]

1. (d) continued

In a critical spherical reactor with negligible extrapolation distance the flux falls to zero when

$\sin(B_m R) = 0 \qquad (R \neq 0)$

$\therefore \quad B_m R = \pi$

$\therefore \quad R = \pi / B_m = \pi / 14.35 = \underline{\underline{0.219\ m}}$

(e) If a natural fission reactor had operated then evidence would be:

(15%) (i) A lower isotopic abundance of U-235 (due to some of it having been fissioned)

(ii) Higher than nomal isotopic abundances of common fission products and their daughter products (remembering that most fission products are radioactive)

(iii) Higher than nomal isotopic abundances of the isotopes created by neutron irradiation of the local soil (and their daughter products)

2 (a) Many fission products are unstable. Some decay by neutron emission. Unlike the neutrons emitted promptly in fission these neutrons are emitted some time after the fission reaction that produced the relevant fission products (at a time determined by the decay constant of the fission products in question). These neutrons are in consequence known as "delayed neutrons".

(15%)

Delayed neutrons have a very significant (beneficial) effect, increasing the average neutron lifetime and hence lengthening the dominant time constant governing the dynamic behaviour of the neutron population.

(b) $\frac{dn}{dt}$ = the rate of change of the neutron population

(15%) $\frac{\rho-\beta}{\Lambda}n$ = the rate of production of prompt neutrons

λc = the rate of production of delayed neutrons and also the rate of decay of precursors

s = the independent neutron source rate

$\frac{dc}{dt}$ = the rate of change of the precursor population

$\frac{\beta n}{\Lambda}$ = the rate of production of precursors through fission

(c) In steady-state operation $\frac{dc}{dt} = 0$

(10%)

$$\therefore \frac{\beta}{\Lambda} n_0 = \lambda c_0$$

$$\therefore \frac{c_0}{n_0} = \frac{\beta}{\lambda\Lambda} = \frac{0.007}{0.1 \times 10^{-3}} = \underline{\underline{70}}$$

2(d) For a source-free system the governing equations are:

(50%)

$$\frac{dn}{dt} = \frac{\rho-\beta}{\Lambda} n + \lambda c \quad \text{and} \quad \frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

Taking Laplace transforms (with p as the transform variable):

$$p\bar{n} - n_0 = \frac{\rho-\beta}{\Lambda}\bar{n} + \lambda\bar{c} \qquad (1)$$

$$p\bar{c} - c_0 = \frac{\beta}{\Lambda}\bar{n} - \lambda\bar{c} \qquad (2)$$

Now, from (c), $c_0 = \frac{\beta}{\lambda\Lambda} n_0$

$$(2) \Rightarrow \therefore \quad \bar{c}(p+\lambda) = \frac{\beta}{\lambda\Lambda}(\lambda\bar{n} + n_0)$$

$$(1) \Rightarrow \therefore \quad \bar{n}\left(p + \frac{\beta-\rho}{\Lambda}\right) = n_0 + \frac{\beta}{\Lambda}\frac{(\lambda\bar{n}+n_0)}{(p+\lambda)}$$

$$\therefore \bar{n}\left(p+\frac{\beta-\rho}{\Lambda}\right)(p+\lambda) = n_0\left(p+\lambda+\frac{\beta}{\Lambda}\right) + \frac{\beta\lambda\bar{n}}{\Lambda}$$

$$\therefore \bar{n}\left(p^2 + \left[\lambda + \frac{\beta-\rho}{\Lambda}\right]p + \cancel{\frac{\beta\lambda}{\Lambda}} - \frac{\rho\lambda}{\Lambda} - \cancel{\frac{\beta\lambda}{\Lambda}}\right) = n_0\left(p+\lambda+\frac{\beta}{\Lambda}\right)$$

To find the system time constants solve

$$p^2 + \left[\lambda + \frac{\beta-\rho}{\Lambda}\right]p - \frac{\rho\lambda}{\Lambda} = 0$$

$$\therefore p^2 + \left[0.1 + \frac{0.007-0.003}{10^{-3}}\right]p - \frac{0.003\times 0.1}{10^{-3}} = 0$$

$$\therefore p^2 + 4.1p - 0.3 = 0$$

$$\therefore p = 0.0719 \text{ or } -4.1719 \text{ s}^{-1}$$

$\therefore$ The dominant time constant (the +ve one) is

$$T_+ = \frac{1}{p_+} = \frac{1}{0.0719} = \underline{\underline{13.91 \text{ s}}}$$

(e) Without precursors the equation is $\frac{dn}{dt} = \frac{\rho n}{\Lambda}$

(10%) By inspection, the +ve time constant $T_+ = \frac{\Lambda}{\rho} = \frac{10^{-3}}{0.003} = \underline{\underline{0.333 \text{ s}}}$

3 (a) At EOC $\sum_{i=1}^{M} \frac{1}{M}\rho_0\left(1-\frac{i\mu}{T}\right) = 0$

(20%) $\therefore \frac{\rho_0}{M}\left\{\sum_{i=1}^{M} 1 - \frac{\mu}{T}\sum_{i=1}^{M} i\right\} = 0$

$\therefore \quad M - \frac{\mu}{T}\frac{1}{2}M(M+1) = 0$

$$\therefore \quad \underline{\underline{\mu = \frac{2T}{M+1}}} \qquad (1)$$

(b) Including the refueling outage the total cycle length

$\tau = \mu + \Delta$

(35%) $= \frac{2T}{M+1} + \alpha + \frac{\beta}{M}$

The availability $A = \frac{\mu}{\tau} = \frac{\mu}{\mu+\Delta} = \frac{1}{1+\Delta/\mu}$

This is maximised when $\frac{\Delta}{\mu}$ is minimised, i.e.
when $\frac{d}{dM}\left(\frac{\Delta}{\mu}\right) = 0$

$$\frac{\Delta}{\mu} = \left(\alpha+\frac{\beta}{M}\right)\left(\frac{M+1}{2T}\right)$$

$$\therefore \frac{d}{dM}\left(\frac{\Delta}{\mu}\right) = -\frac{\beta}{M^2}\left(\frac{M+1}{2T}\right) + \left(\alpha+\frac{\beta}{M}\right)\frac{1}{2T}$$

$$\therefore \frac{d}{dM}\left(\frac{\Delta}{\mu}\right) = 0 \text{ when } \left(\alpha+\frac{\beta}{M}\right)\frac{1}{2T} = \frac{\beta}{M^2}\frac{(M+1)}{2T}$$

$$\therefore \alpha + \cancel{\frac{\beta}{M}} = \cancel{\frac{\beta}{M}} + \frac{\beta}{M^2}$$

$$\therefore M = \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{18}{2}} = \underline{\underline{3}}$$

(c) For $M = 3$ $\quad \mu = \frac{1}{2}T$ from (1)

(25%) For immediate equilibrium operation the initial reactivity inventory must be the same as the equilibrium reactivity inventory at start of cycle.

3 (c) continued

In equilibrium at the start of cycle

$\frac{1}{3}$ of the fuel is fresh ($\rho = \rho_0$)

$\frac{1}{3}$ of the fuel has been in the reactor 1 cycle

$$\rho = \rho_0\left(1 - \frac{\mu}{T}\right) = \rho_0\left(1 - \frac{1}{2}\right) = \frac{1}{2}\rho_0$$

$\frac{1}{3}$ of the fuel has been in the reactor 2 cycles

$$\rho = \rho_0\left(1 - \frac{2\mu}{T}\right) = \rho_0(1-1) = 0$$

$\therefore$ Initial inventory is $\underline{\underline{\frac{1}{3} \text{ with } \rho_0,\ \frac{1}{3} \text{ with } \frac{\rho_0}{2},\ \frac{1}{3} \text{ with } 0}}$

(d) $$\tau = \frac{2T}{M+1} + \alpha + \frac{\beta}{M}$$

(20%) $$= \frac{2 \times 120}{3+1} + 2 + \frac{18}{3} = \underline{\underline{68 \text{ weeks}}}$$

From an operational point of view it is better to have an annual cycle, i.e. 52 weeks, so that the outage can be scheduled in a period where electricity demand is lowest (in the summer in the UK).

The above analysis shows that optimal value of M is independent of T, so this could be adjusted to give $\mu = 44$ weeks ($\Delta = 8$ weeks for $M = 3$), i.e.

$$\frac{2T}{M+1} = 44$$

$$\Rightarrow T = 88 \text{ weeks for } M = 3$$

T can be reduced by operating the reactor at higher power (probably not possible) or by reducing the enrichment of the fuel (certainly possible and also makes the fuel cheaper).

4(a) The key factor in any fission power reactor is the neutron balance. On average about 2.5 neutrons are produced per fission but some are absorbed or otherwise lost in the moderator, fuel cladding, reactor structure and physically lost through the walls of the reactor. Others are absorbed in the non-fissile U-238 and in non-fissile (capture) reactions in U-235. In a PWR the moderator is light water and the fuel cladding is a zirconium alloy, both of which have relatively high neutron absorption cross-sections. Hence it would be impossible to maintain a critical neutron balance using natural uranium in a PWR

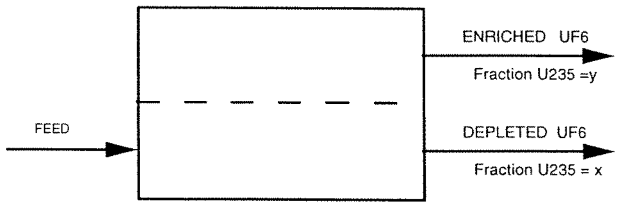
(30%)

If enriched fuel were not available it would be necessary to change both the fuel cladding and the moderator. The only two commercial moderators used for natural uranium fuel are graphite and heavy water (D_2O). Possible fuel claddings are aluminium and magnesium alloys.

(b) Because an isotopic separation is required it must be by physical rather than by chemical means. All established techniques make use of the difference in density between U-235 and U-238. The only compound of U that is suitable for density separation is Uranium Hexafloride (usually know as HEX). HEX is a gas at just above ambient temperature so advantage can be taken in the difference in gas density. The original process was based on gaseous diffusion using a counter-current cascade as shown in Figs 1.

ENRICHMENT PROCESSES

GASEOUS DIFFUSION



(30%)

$$\text{SEPARATION FACTOR } \alpha = \frac{y(1-x)}{x(1-y)}$$

The separation Factor is the ratio of the diffusion rate which equals the ratio of the square root of the molecular weight

$$\alpha = \sqrt{\frac{\text{MW U238 hex}}{\text{MW U235 hex}}} = \sqrt{\frac{352}{349}}$$

$$= 1.00429$$

Thus many stage are needed to give a reasonable degree of enrichment. In order to produce a typical PWR fuel at 3% U-235 from natural Uranium at 0.7% and a tails composition of 0.2% would require 1272 stages. As each stage needs a compressor and a cooler the energy costs of this process are enormous.

The alternative process uses high-speed gas centrifuges, again relying on density difference. Because it is possible to get a much greater degree of separation per stage only 25 centrifuge stages are needed per 1000 diffusion stages and the power consumption is about one tenth. The main problems are engineering, the centrifugal forces are several thousand g needing very expensive materials and very accurate design.

The centrifuge process has not totally replaced the diffusion process partly because of various complex cross subsidies that keeps it going in the USA, the Russian Federation and France.

Other techniques have been tried and one based on laser technology using Uranium vapour has shown some promise. The Uranium vapour is excited by a tuned laser and the U-235 is selectively ionised allowing separation by means of strong electrical fields. The engineering would be very complex and expensive so it is still at the development stage.

4 (c) Thermal power = $1100 \div 0.3$ = 3667 MW

(40%)

Reactor power is given by: $N \times \phi_{ave} \times \sigma_f \times \omega$

Where N is no of atoms of fissile material
ϕ_{ave} is average neutron flux = 4.0E17 n/m^2s
σ_f is fission cross section = 580 barn = 580E-28 m^2
ω is energy per fission = 200 MeV = 200E6 x 1.6E-19 J

N = 3667E6 ÷ (4E17 x 580E-28 x 200 x 1.6E-13)

= 4.939E27 atoms U235

= 4.939E27 x 235 ÷ 6.022E26 = 1927.4 kg U235

Enrichment is 3.5% thus mass of fuel (as U) is given by:

1927.4 ÷ 0.035 = 55068 kg U

Mass balance across enrichment plant:

$F = P + W$

Component balance across enrichment plant:

$Fx_f = Px_p + Wx_w$

Thus $F = 55068 + W$

$0.007F = 1927.4 + 0.003W$

Hence $F = 440549$ kg U

$W = 385481$ kg U

Separation work is given by:

$P(-\ln 0.035) + W(-\ln 0.003) - F(-\ln 0.007)$

$= 184610 + 2239314 - 2185936 = 237988$ kg SWU