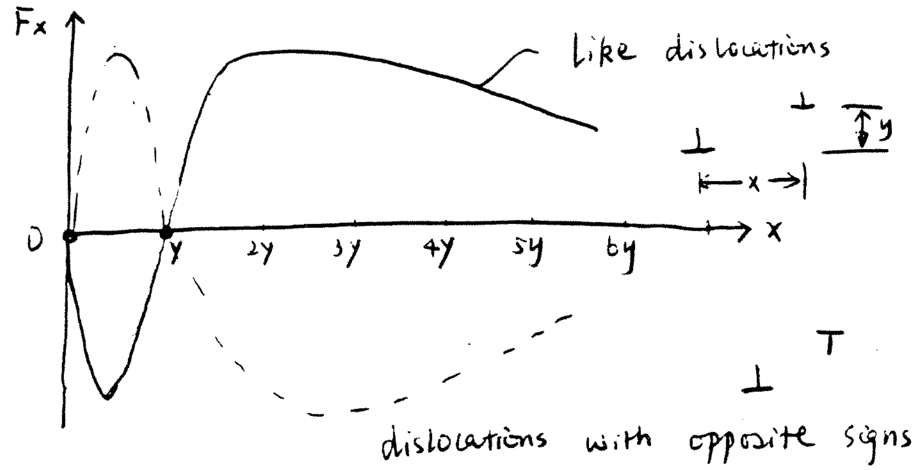
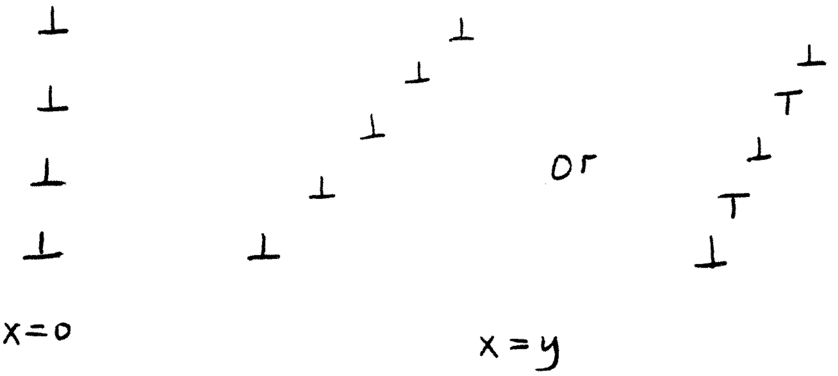
1. (a) For two screw dislocations, the interaction force is independent of the orientation angle θ, and inversely proportional to the distance r. For two edge dislocations, the interaction force is a function of both r and θ, and is also dependent upon the sign of the burgers vector.



From the above figure, it can be seen that there are two possible equilibrium configurations for edge dislocations:



(25%)

(b) $\sigma_{xx} = -\frac{Gb}{2\pi(1-\nu)r}\sin\theta(2+\cos2\theta)$, $\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)r}\sin\theta\cos2\theta$,

$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$, $\sigma_{xy} = \frac{Gb}{2\pi(1-\nu)r}\cos\theta\cos2\theta$.

$\sigma_{xz} = \sigma_{yz} = 0$

(20%)

1. (c) $\sigma_m = \dfrac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = -\dfrac{Gb(1+\nu)}{3\pi(1-\nu)r}\sin\theta$

$\Rightarrow \dfrac{d\sigma_m}{d\theta} = 0 \quad \Rightarrow (\sin\theta)' = \cos\theta = 0 \quad \Rightarrow \theta = \pm\dfrac{\pi}{2}$

At $\theta = -\dfrac{\pi}{2}$, $\sigma_m = \dfrac{Gb(1+\nu)}{3\pi(1-\nu)r}$ is the maximum

For $r = 10b$, $\sigma_m = \dfrac{1+\nu}{1-\nu}\dfrac{G}{30\pi} \approx 0.02G$

This is a large stress compared to σ_Y (30%)

(d) For a mixed dislocation whose Burger's vector makes an angle θ with the dislocation line, its energy is equal to that associated with a screw dislocation having a Burger's vector $b\cos\theta$ plus that due to an edge dislocation having a Burger's vector $b\sin\theta$:

$$U = \frac{Gb^2}{4\pi}\left(\frac{\sin^2\theta}{1-\nu} + \cos^2\theta\right)$$

$\dfrac{dU}{d\theta} = 0 \Rightarrow \quad \sin 2\theta - (1-\nu)\sin 2\theta = 0 \quad \Rightarrow \theta = \dfrac{\pi}{4}$ or $-\dfrac{\pi}{4}$

This result implies that a dislocation loop tends to be circular.

(25%)

2(a) Mean stress - needed to describe brittle failure of rocks, crazing & shear failure of polymers etc.

Mises equivalent stress - needed to describe yielding of metals

Max. shear ~~sh~~ stress - "

Max. principal strain - crazing of polymers

Max principal stress - crack propagation : cleavage failure.

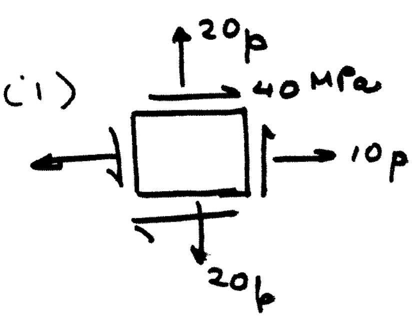
(25%)

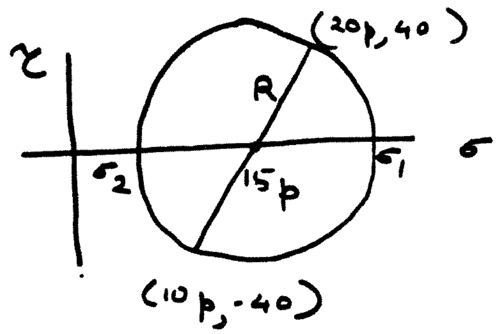
(b) $\sigma_h = \dfrac{pr}{t}$, $\sigma_\ell = \dfrac{pr}{2t}$, $\tau = \dfrac{T}{2A_e t}$

$r = 200\text{mm}$, $t = 10$ mm, $T = 100\text{kNm}$

$\sigma_h = 20p$, $\sigma_\ell = 10p$, $\tau = \dfrac{100\times10^3\times10^3}{2\pi(200)^2\times10} \approx 40\text{MPa}$

(i)





(30%)

$R = \sqrt{25p^2 + 40^2}$

$\sigma_1 = 15p + \sqrt{25p^2 + 40^2}$, $\sigma_2 = 15p - R$

Tresca criterion $\Rightarrow \sqrt{25p^2 + 40^2} = \dfrac{\sigma_f}{2} = 100 \Rightarrow p = 18.3\text{MPa}$

2(b)

(ii) Max principal stress criterion

$$\sigma_1 = \sigma_f$$

$$\Rightarrow 15p + \sqrt{25p^2 + 40^2} = 200$$

$$p^2 - 30p + 192 = 0$$

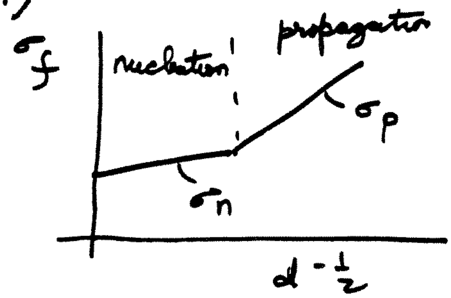
$$p = \frac{30 - \sqrt{30^2 - 4 \times 192}}{2}$$

$$p = 9.26\,MPa$$

$$\tan 2\theta = \frac{40}{5 \times 9.26} \Rightarrow \theta = 20.4^\circ$$

$\therefore$ crack propagation at 69.6° from longitudinal axis. (30%)

(iii)



In coarse grained mtls, nucleation controlled

In fine grained mtls, yielding occurs first & then failure after a certain amt. of plastic flow $\Rightarrow$ propagation controlled

(15%)

3 (a) G is the rate of change of potential energy with crack length

$$G = -\frac{\partial U}{\partial a}$$

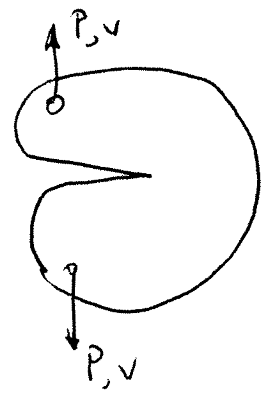
where U is the potential energy & a the crack length.

Fracture occurs when $G = G_c$, where G_c is material dependent.

K is a measure of the magnitude of the stress singularity in the vicinity of a sharp crack in a ~~ma~~ nominally elastic material. Fracture occurs when $K = K_c$, where K_c is a material specific value (20%)

(b) $$G = \frac{1}{2B} P^2 \frac{dc}{da}$$

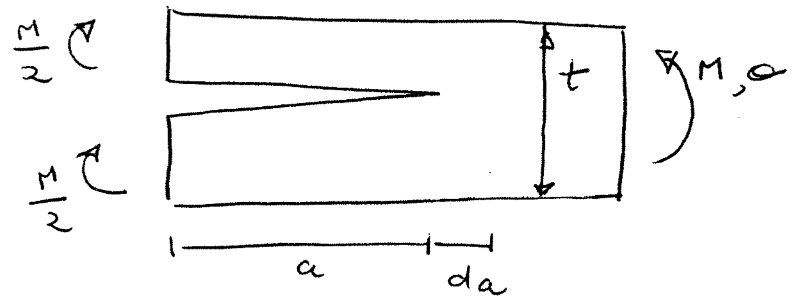
where c is the compliance of the specimen $c \equiv \frac{v}{P}$ & a the crack length.



Perform a compliance measurement as a function of crack length & extract G from above formula. (20%)

c (i)

Consider 1/2 specimen



$L >> a >> t$

Consider crack growth at constant θ

$$U = \frac{1}{2}\frac{M^2 L}{EI} \quad , \quad I = \frac{Bt^3}{12}$$

For crack growth da

$$du = \left[\frac{1}{2}\left(\frac{M}{2}\right)^2 \frac{12\times 8 \times 2}{EBt^3} - \frac{1}{2}\frac{M^2}{E}\frac{12}{Bt^3}\right] da$$

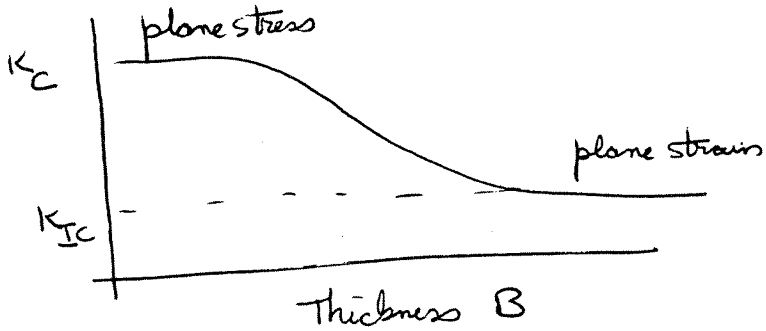
$$G = \frac{18M^2}{Et^3}$$ (40%)

(ii) $G_{IIc} = \frac{1}{2}\sigma_u \delta_u$ (20%)

$$M = \sqrt{\frac{\sigma_u \delta_u E t^3}{36}}$$

$$= \frac{1}{6}\sqrt{\sigma_u \delta_u E t^3}$$

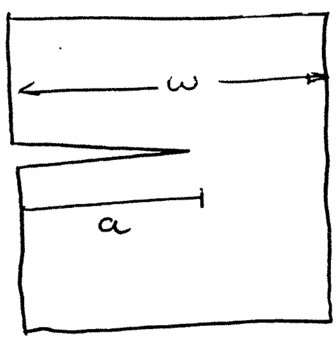
4(a)



Usually plane stress $K_C \approx 2 K_{IC}$ for ductile metals.

(20%)

(b)



Consider for example edge cracked specimen shown above.
The conditions for a valid K_{IC} test are.

$a, \; w - a > 2.5 \left(\frac{K_{IC}}{\sigma_Y}\right)^2$ → plastic zone smaller than a, w

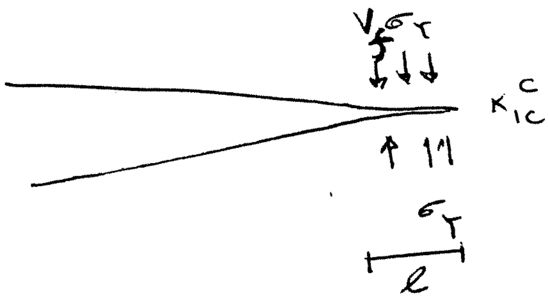
$B > 2.5 \left(\frac{K_{IC}}{\sigma_Y}\right)^2$, where B = thickness of specimen

→ so that plane strain K_{IC} is measured (see fig above)

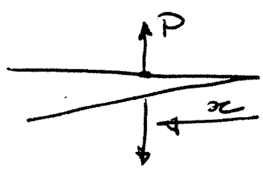
(20%)

(c) (i) Crack bridging occurs with stretching of the fibres in the wake of the crack. Load is transmitted across the crack faces by the fibres. The bridged zone increases in length with increasing crack size and this results in a increase in the remote ~~#~~ measured K_c or G_c. Therefore the reinforced ceramic exhibits R-curve effects. (20%)

(ii)



From data sheet



$$K = \sqrt{\frac{2}{\pi}} \frac{P}{\sqrt{x}}$$

$$K^{\infty} = K_{IC}^{c} + \sqrt{\frac{2}{\pi}} \int_0^{l} \frac{V_f \sigma_Y}{\sqrt{x}} \, dx$$

$$K^{\infty} = K_{IC}^{c} + \frac{2\sqrt{2} V_f \sigma_Y}{\sqrt{\pi}} \sqrt{l} .$$

(40%)