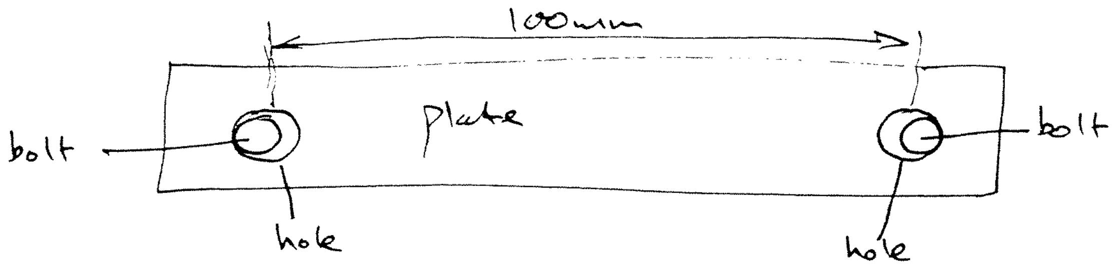
1. (a) 'Safety factor' ratio of material strength / working stress

'Safety margin' difference between material strength / working stress. [10%]

For the plate and u-bolt:



If bolt centre-centre distance exceeds nominal distance (100mm), then 'failure' occurs if the bolt cannot fit in the holes.

$\therefore$ working stress $\equiv$ nominal distance
material strength $\equiv$ maximum (or minimum) distance. [10%]

(b) 'Safety factor' $= \dfrac{108}{107} = \underline{1.01}$ nominal case

this is one of two cases, ie. bolt wider than plate holes.

$= \dfrac{108 - 0.6}{107 + 1.1} = \underline{0.99}$ worst case [20%]

Note answers can vary depending on the nominal distance chosen, ie centre-centre pitch or 'half'-pitch.

1\. cont. (c)(i) Percentage comes from safety margin. Note calculating margin resolves shortcoming of calculating safety-factor (variation due to choice of nominal distance)

margin, m = hole max distance − bolt max distance

$$= (100 + 8) - (100 + 7)$$

need to include effect of variation

$$\mu \quad \sigma$$

$$= (100, 0.5) + (8, 0.1) - (100, 1) - (7, 0.1)$$

$$= (1, 1.13)$$

Converting to a normalised normal function:

$$z = \frac{m - \mu_m}{\sigma_m} = -0.887$$

$$p(m < 0) = 0.187$$

[20%]

Since bolt distance is as likely to be too small:

probability of no fit $= 2 \times 0.187 = \underline{37.5\%}$

(ii) If bolt can bend;

$$z = \frac{m - \mu_m}{\sigma_m} = \frac{-1 - 1}{1.13} = 1.775$$

$$P(m < -1) = 0.038$$

∴ probability of no fit $= 2 \times 0.038 = \underline{7.6\%}$

[20%]

1. cont. (d) Need a case where $2 \times P(m < -1) = 0.1\%$

Hence $z \approx 3.283$

This leads to a hole size of <u>9.7mm</u> [20%]

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2 a) 1 Establish a starting point (given by user, or e.g. shotgun search)
2 Choose a search (descent) direction ($\mathbf{s}_k$)
3 Decide how far to travel along it (i.e. calculate α_K, or perform a line search)
4 Stop if convergence test is met (e.g. Grad ≈ 0) or return to 2

$\mathbf{x}_k$ and $\mathbf{x}_{k+1}$ are the successive points reached at stage 4 above; $\mathbf{s}_k$ is the new search direction, and α_k is the 'distance' to be travelled along it.

All gradient descent methods (e.g. Conjugate, Steepest Descent) will always converge if the function is *unimodal*, provided the analytical derivatives (Grad and Hessian) are available. If there are any constraints, then the feasible region must also be *robust*, and *convex*.

N.B. If the Grad has to be calculated by numerical differentiation, then gradient methods effectively become Direct Search methods. As such they can fail, even on a strongly or linearly unimodal function, if the local 'valley' leading to the global minimum is too narrow to be detected by the numerical differentiation sampling. [30%]

b) Since both methods start along the line of steepest descent, the first iteration is the same for each:

$$\nabla = [3x_1^2, 4x_2, 10x_3]^T \text{ so } \mathbf{s}_0 = -\nabla_0 = [-3, -4, -10]^T$$

$$H = \begin{bmatrix} 6x_1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{ so } H_0 = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\alpha_0 = \frac{125}{1118} = .1118$$

$$\mathbf{x}_1 = [.6646, .5528, -.1181]^T$$

$$\nabla_1 = [1.325, 2.211, -1.181]^T$$

So for Steepest Descent, $\mathbf{s}_1 = [-1.325, -2.211, 1.181]^T$

and for Conjugate Gradient, $\mathbf{s}_1 = [-1.325, -2.211, 1.181]^T + \dfrac{8.039}{125} \times [-3, -4, -10]^T$
$= [-1.518, -2.468, 0.538]^T$

(borrowing the calculation for β_0 from the Fletcher Reeves method, to save time). [40%]

c) In the Steepest Descent, $\mathbf{s}_k$ and $\mathbf{s}_{k+1}$ are both directions of steepest descent, so $\mathbf{s}_{k+1}$ should be perpendicular to $\mathbf{s}_k$; whereas in the Conjugate Gradient method $\mathbf{s}_{k+1}$ is intended to be *conjugate* to $\mathbf{s}_k \ldots \mathbf{s}_0$, when the function is quadratic.

Thus, in the Steepest Descent we would expect $\mathbf{s}_k \cdot \mathbf{s}_{k+1} = 0$, and in the Conjugate Gradient method we would expect $\mathbf{s}_k^T \mathbf{H} \mathbf{s}_{k+1} = 0$. Using the results from Part (b):

For Steepest Descent, $[-3, -4, -10] \cdot [-1.325, -2.211, 1.181] = 1.009$ (meaning that $\mathbf{s}_k$ and $\mathbf{s}_{k+1}$ make an angle of 89.8° with each other)

For Conjugate Gradient, $H_1 = \begin{bmatrix} 3.988 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

4C4

$$\text{So } \mathbf{s}_k^T \mathbf{H} \mathbf{s}_{k+1} = [-3, -4, -10] \begin{bmatrix} 3.988 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} -1.518 \\ -2.468 \\ 0.538 \end{bmatrix} = 3.849$$

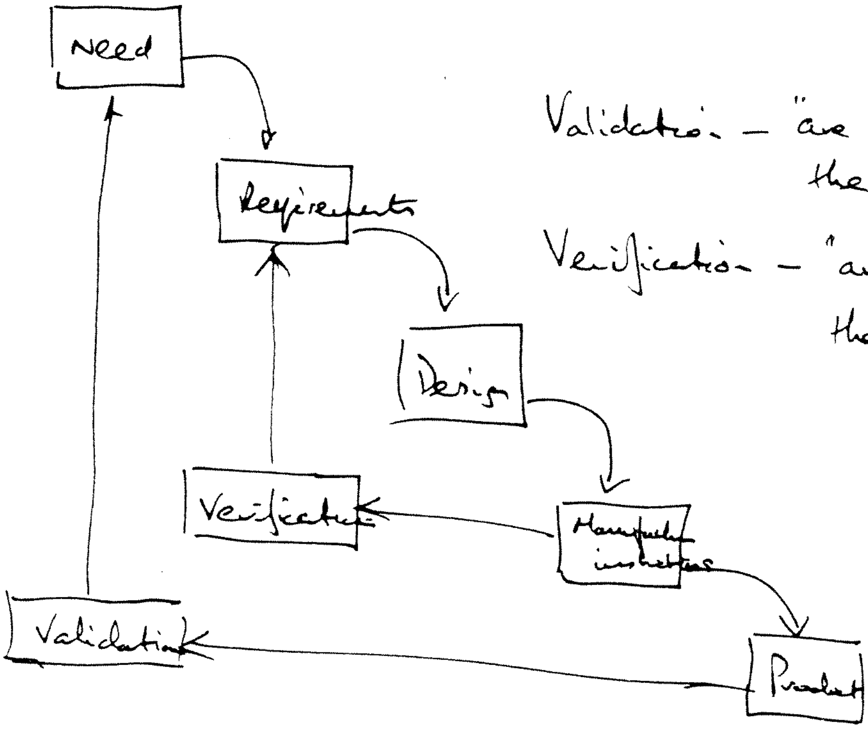
These are not exactly zero partly because of rounding errors, but mainly because the function is not quadratic – which means that the calculation of α_0 does not minimise the function along $\mathbf{s}_0$; and the calculation of β_0 does not produce an exactly conjugate direction, since **H** is not constant.

The Steepest Descent method will in general never find a search direction which passes through the Global Minimum, even if the function is quadratic; whereas the Conjugate Gradient method should do so after a number of moves equal to the number of dimensions in the search space. The Conjugate Gradient method should thus converge more quickly - but not after just 3 moves in this case, because the function is not quadratic. (Actually, a Univariant search would out-perform both of the above methods in this case, because each term in the function involves only one variable.)

[30%]

4C4

3 (a)

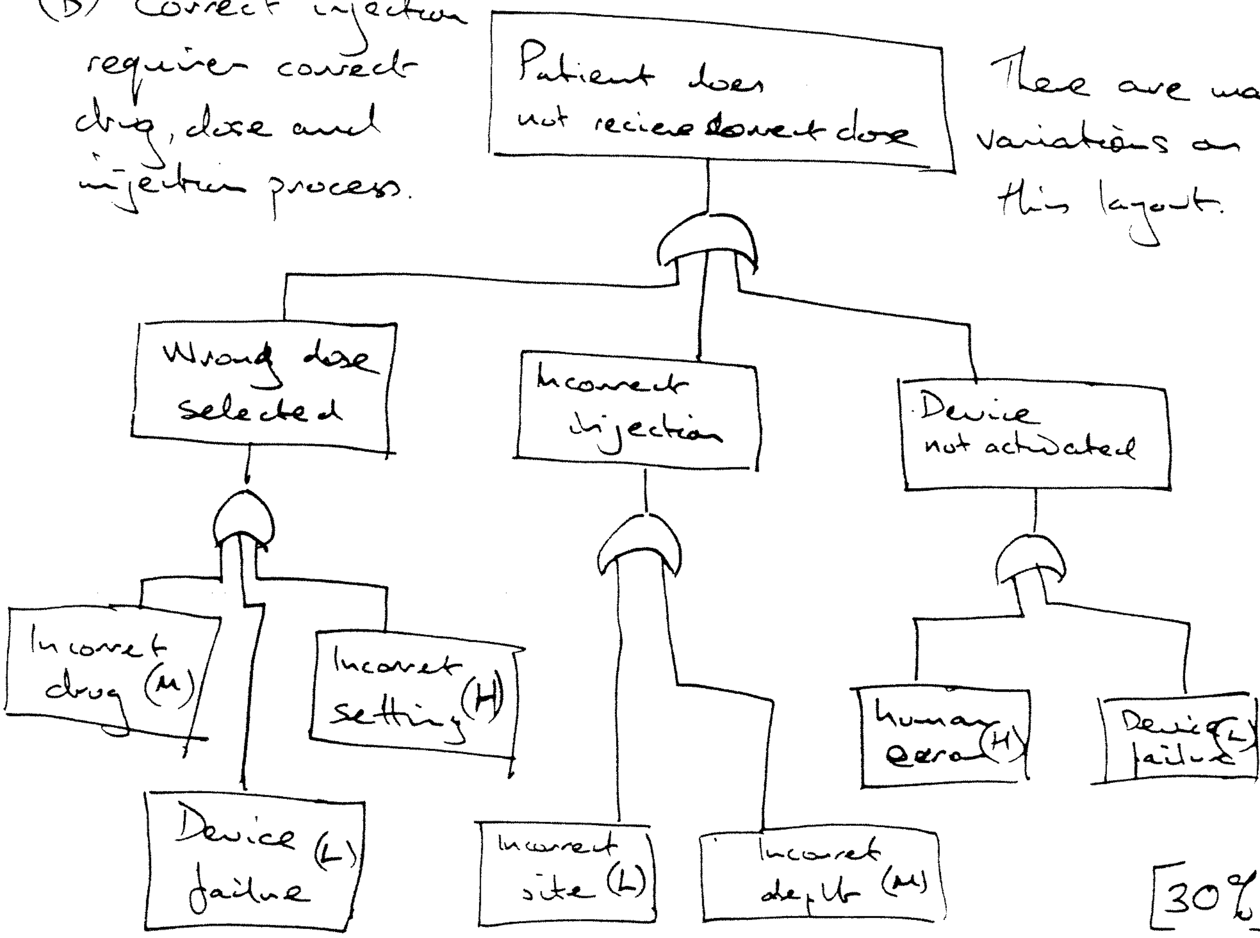


Validation – "are we doing the right thing?"

Verification – "are we doing the thing right?"

[10%]

(b) Correct injection requires correct drug, dose and injection process.



There are many variations on this layout.

[30%]

4C4

3 (c) High risk steps generally involve human intervention. Possible changes:

- only correct drug to fit injector, or drug pre-installed in a one-shot device.
- drug pre-dosed, or device dose can only be set by doctor/pharmacist.
- needle must be selected to ensure correct depth of injection. Automatic injection would be possible; ie needle is automatically extended (and retracted?)
- Activation must be simple - one-handed operation.

[40%]

A number of issues here relate to whether or not the device is to be disposable.

The sequence represents one of many real devices!

(d) Verification: does the device inject the correct dose on demand. Use check-weighing to test device performance - measuring ejected volume of drug.

Validation: does the device relieve symptom. Test with patient trials.

[20%]

4 (a) Solution-neutral problem statement:

"transport person up the hill".

It is important not to use words (at this stage) that imply the use of a particular solution. Avoid 'drag', 'lift', 'rope' etc.

[10%]

(b) Typical requirements might include:

Portability: possible for two people to lift.

Size: compact for storage.

Safety: must ensure safe use.

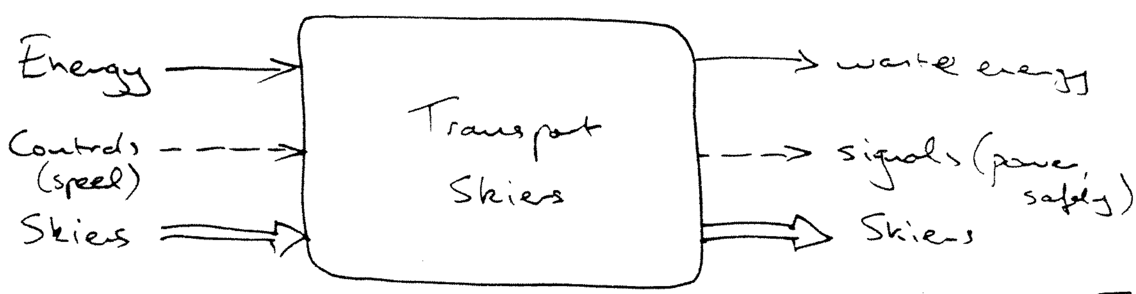
Portable power source

Easy to set up.

There are many other possible requirements, quantify where possible. A set of 10 sensible requirements would earn the marks.

[20%]

(c) A number of possibilities here:

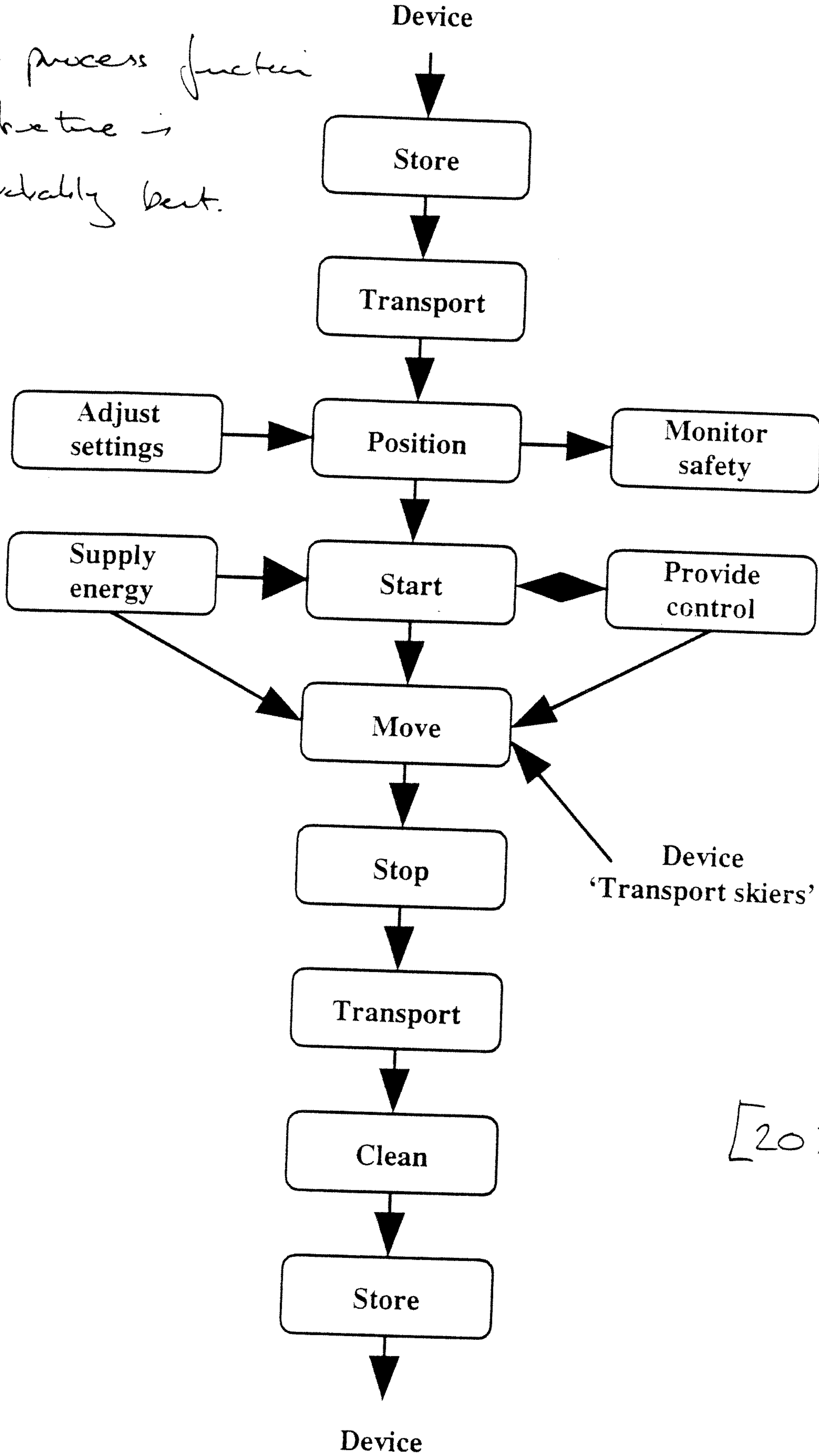


[10%]

4C4

4(c) cont.

A process function structure is probably best.



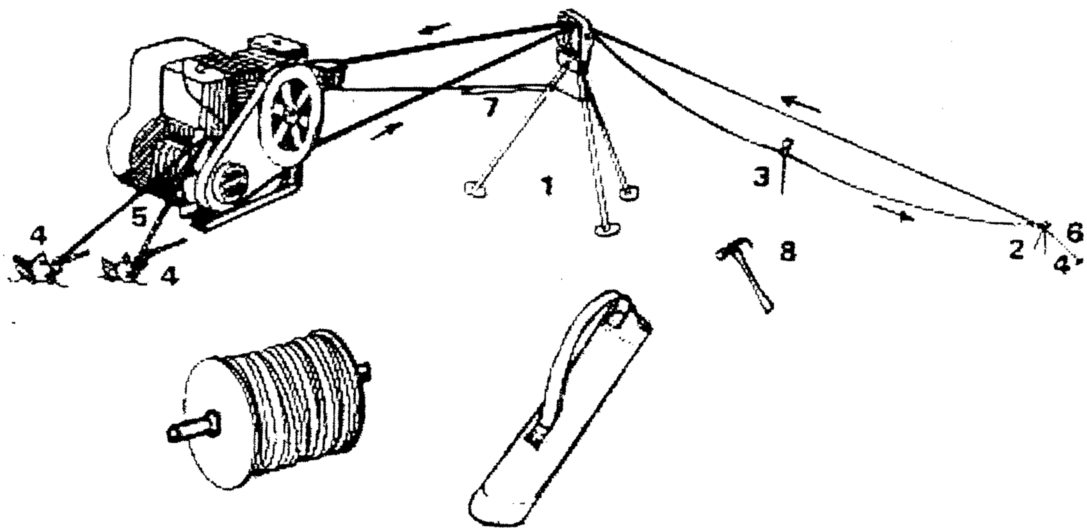
[20%]

4C4

4(d) One example below:

Britonlift Portable Tow

The Britonlift is an ultra-light portable ski tow which opens up new horizons in downhill skiing. Its unique design is so simple anyone can use it. It makes you independent and able to ski anywhere you like. You can enjoy hours more fun on the snow.

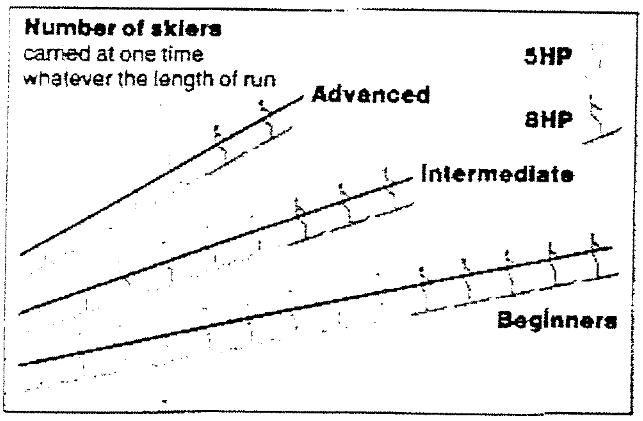


BritonLift portable complete with:

1. One top pulley tripod
2. One bottom pulley and bipod assembly
3. One guide pulley
4. Three stakes
5. One engine tie rope
6. One bottom pulley tie rope and pulling block/jamming cleat
7. One safety sensor rope
8. One purpose designed hammer with tool for removing stakes
9. Two Britonhooks

[40%]

Length Variable (up to 440m)
Application Lightweight portable ski lift
Capacity / hr (max) See diagram below



Comprising an engine unit, rope reel and a bag of running gear - all of which can be easily carried by two people, even on skis - the system takes just 10 minutes to set up, with no requirement for technical knowledge or additional tools. The entire system weighs less than 45kg! The BritonLift portable has now been used for the aerial events at two Winter Olympics in Nagano and Tignes.