ENGINEERING TRIPOS PART IIA

Thursday 24 April 2003 9 – 12

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) A square flat plate of area A_c is held stationary and normal to a uniform, steady, flow of air at a velocity of U_c. The flow is incompressible. Explain why the drag coefficient should be of order unity. [10%]

(b) The plate in (a) moves at a velocity U_c in a stationary fluid and has a drag coefficient of C_D. Show that the work done by the plate per unit time on the fluid is

$$\tfrac{1}{2}\rho U_c^3 C_D A_c.$$

[10%]

(c) The flat plate in (b) is now replaced by a typical passenger car that is found to have a drag coefficient of 0.3. This is substantially less than that of a flat-plate normal to the flow as considered in (a) and (b). Briefly explain why. [10%]

(d) Cars travelling at a velocity of U_c along a one-way section of road enter a long tunnel of length L, cross-sectional area, A_T and perimeter P_T. The cars each have a drag coefficient, C_D, and cross-sectional area A_c. The spacing between the cars, S_c, is large enough that their wakes do not interfere (S_c is from the front of one car to the front of the next). The cars set up a uniform flow in the tunnel. The tunnel walls have a skin-friction coefficient, C_f, that is independent of the Reynolds number of the flow in the tunnel. Develop an expression for the velocity of the flow, U_T through the tunnel. You may ignore end effects since the tunnel is very long. [40%]

(e) Why might you expect the skin-friction coefficient to be independent of the Reynolds number? [10%]

(f) Develop an expression for the temperature rise between the inlet and outlet of the tunnel, ignoring the heat added by the cars' engines. Assume that the tunnel walls are adiabatic. [10%]

(g) If the overall efficiency of the cars' engines is η_c, how does this change the expression for the temperature rise between the inlet and outlet of the tunnel? [10%]

2 A thin aerofoil at an angle of attack may be modelled as a flat plate of length equal to the chord with a vortex located at the *half-chord* point as shown in Fig. 1. The Kutta condition states that the flow should leave the trailing edge smoothly, i.e. there can be no flow "around" the trailing edge. In this model, the trailing edge is not a stagnation point.

(a) By considering the velocity components normal to the plate, at the trailing edge, find an expression for the lift, L, of the aerofoil as a function of the angle of attack α. [30%]

(b) This model of an aerofoil is now to be used to examine the lift of a biplane. Consider two such aerofoils located one above the other with a distance h between them as shown in Fig. 2. To simplify the algebra you may assume that α is very small when calculating distances and angles of vectors. Apply the same condition as in (a) at *one* of the trailing edges and hence calculate the lift of one of the pair (the assumptions lead to both aerofoils having the same lift). [50%]

(c) Compare the combined lift of the pair for finite h to the combined lift as $h \to \infty$ and comment. [20%]

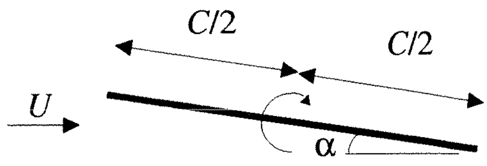

Fig.1

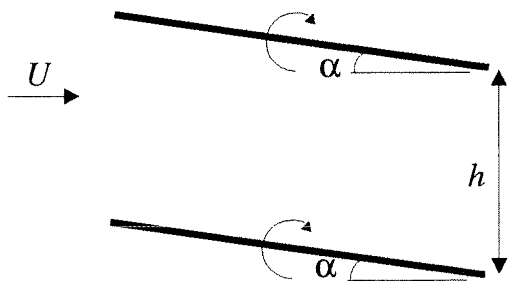

Fig. 2

3 A line vortex is located in the inviscid flow over a 90^o external corner (the corner is shown on the right-hand-side of figure 3). The complex potential and conformal transformation are used to find the velocity of this vortex by following the steps given below.

(a) Write down the complex potential for a vortex of strength Γ at a position $z = x + iy = 0 + ia$ relative to an infinite, plane wall located at $y = 0$ in a uniform flow of velocity U from left to right as shown in Fig. 3 (on the left-hand-side). [20%]

(b) The transformation $z = \zeta^{2/3}$ (where $\zeta = \xi + i\eta$) maps the points on this wall in the z-plane to points on a 90^o corner in the ζ-plane, as shown in Fig. 3. What point in the z-plane is mapped onto the corner point? What is the position of the vortex shown in Fig. 3. when mapped to the ζ-plane? (HINT: It may be useful to write the complex numbers in their polar form) [10%]

(c) Using the chain-rule, find the value of the circulation of the vortex that ensures that the velocity on the corner point is not infinite. [40%]

(d) Using the result from (c), derive an expression for the velocity of the vortex in the transformed (ζ) plane. [30%]

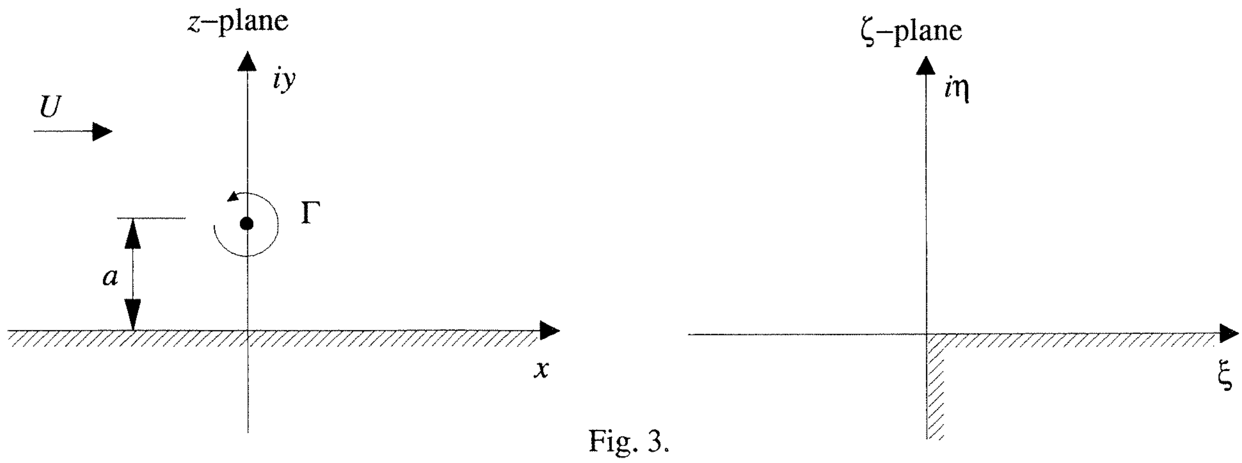

Fig. 3.

4 A wind meter is installed next to the runway of a small airport. There is concern about how a landing aircraft might affect the readings. You may assume the meter is located at ground level, exactly at the side of the runway, which is twice as wide as the aircraft wing-span. Assume that the flow is inviscid.

(a) Assuming that the aircraft of semi-span, s, approaches along the centreline of the runway and that the maximum influence occurs when the wing-tip is closest to the sensor as shown in Fig. 4, express the induced velocity as a function of aircraft height above the ground and the strength of the bound vortex. [50%]

(b) Determine the height of the aircraft where maximum influence occurs. [25%]

(c) Comment on the assumption that the maximum influence occurs when the wing-tip is closest to the sensor. [25%]

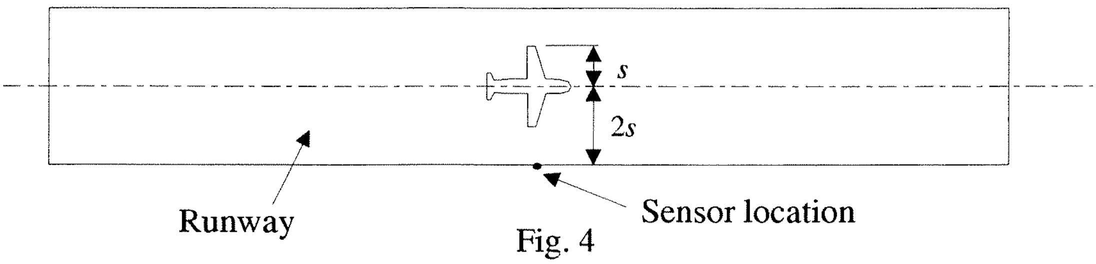

Fig. 4

5 Consider the case where the spanwise circulation distribution for a wing of semi-span, s is parabolic and given by

$$\Gamma(z) = \Gamma_o \left(1 - \frac{z^2}{s^2}\right),$$

where z is the distance along the semi-span and $z = s$ is the wing-tip.

(a) According to lifting-line theory, how is the strength of an incremental line vortex filament in the vortex sheet behind the wing related to the distribution of circulation along the wing? [20%]

(b) If the total lift generated by the wing with the parabolic distribution is to be equal to the total lift generated by a wing with an elliptical circulation distribution, what is the relationship between the Γ_o values of both distributions? [25%]

(c) How does the downwash velocity at the plane of symmetry compare for the two configurations? You may ignore the presence of the fuselage. [30%]

(d) Explain briefly how the planform shape of an untwisted wing with a parabolic wing loading would differ qualitatively from that of an untwisted wing with elliptic load. How would the angles of attack compare? [25%]

6 Measurements are made of a laminar boundary layer developing in a zero pressure-gradient and the velocity profile is found to be well described by a function of the form

$$u(x, y) = A \sin(By)$$

where y is the distance normal to the wall ($y = 0$ is on the wall) and $u(x, y)$ is the velocity component parallel to the wall in the x-direction. A and B are parameters to be determined.

(a) If δ is the thickness of the boundary layer and U_e is the velocity at the edge of the layer (i.e. $U_e = u(x, \delta)$), determine the values of the parameters A and B that give a profile that is consistent with appropriate boundary conditions. State clearly the boundary conditions used. Write this profile in non-dimensional form. [20%]

(b) Using the velocity profile from (a), derive an expression for;

(i) the displacement thickness, δ^*,

(ii) the momentum thickness, θ, and

(iii) the wall shear-stress, τ_w,

in terms of U_e, δ, the kinematic viscosity, ν, and the fluid density ρ. [30%]

(c) Write down the von Karman momentum integral equation for a zero pressure-gradient boundary layer and explain *briefly* what it expresses. [20%]

(d) Using the results in (b) find an expression for the variation of the boundary layer thickness, δ and the local skin-friction coefficient C_f in terms of the Reynolds number based on the distance along the plate, $R_x = U_e x/\nu$. [30%]

7 Liquid metal flows in a 2D channel with a transverse magnetic field as shown in Fig. 6. The Lorentz force is given by

$$\mathbf{f} = -\sigma B^2 \mathbf{U}.$$

The pressure-gradient in the z-direction is zero, the flow is steady and the liquid metal is a Newtonian fluid. The flow is fully-developed so that the velocity profile is the same at any x-position.

(a) Write down the x-component of the momentum equation. [10%]

(b) Using scaling analysis find the distance, L, from the wall where the viscous force is as important as the Lorentz force. [20%]

(c) Solve the equation from (a) analytically for the velocity profile, ensuring that the boundary conditions are satisfied. Explain how the scaling length, L, from part (b) enters the solution and find an expression for the pressure-gradient in the channel.

HINT: consider solutions of the form $\cosh(Az) + C$ where A and C are constants to be determined. [50%]

(d) Show that if $h = 4L$ (with L from part (b)) then the actual distance *from the wall* where the Lorentz force is equal to the viscous force is *approximately* $0.75L$. [20%]

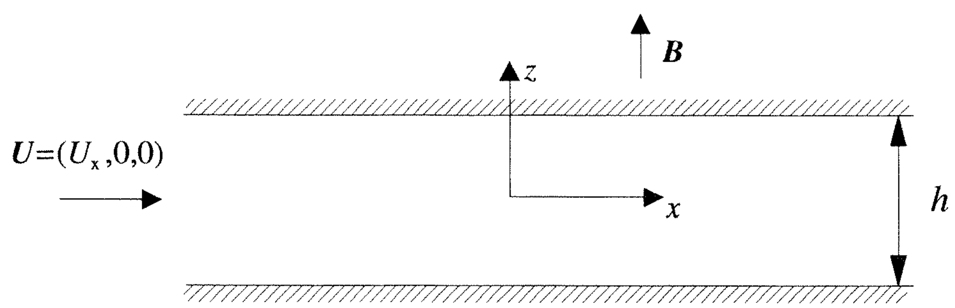

Fig. 6

8 A semi-circular Antarctic hut in a cross-wind, as shown in Fig. 7, is to be modelled by a cylinder of radius a in a uniform cross-flow of speed U. You may assume that the flow is inviscid.

(a) Write down the stream-function for a cylinder in a cross-flow of speed U using polar co-ordinates. [10%]

(b) An anemometer is installed at a distance a upstream of the hut on a tower of height a above the ground. If the anemometer measures only the horizontal component of the wind, what is the error in this measurement when compared with the actual windspeed far upstream. [30%]

(c) A vent is installed in the top of the hut, at the highest point, such that the pressure everywhere inside the hut is equal to the external static pressure at the position of the vent. The static pressure far upstream is p_a. Flow through the vent may be neglected.

Derive an expression for the vertical force per unit length acting on the hut. Where would you place the vent so that the pressure inside the hut is the same as the static pressure far upstream, p_a? How would this affect the vertical force on the hut? [50%]

(d) How would you expect viscosity to change the vertical force on the hut? [10%]

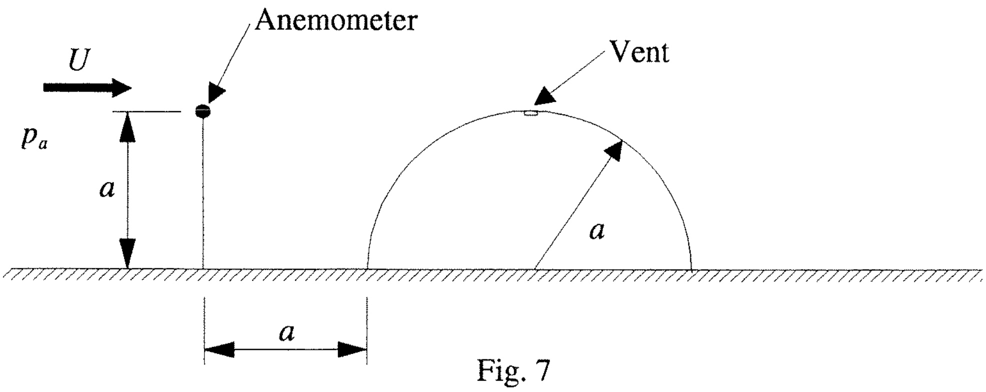

Fig. 7

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3A1 2003 Answers to exam questions

1. (a) see crib

   (b) see crib

   (c) see crib

   (d) $U_T/U_C=1/(1+C_fP_TS_C/C_DA_C)^{1/2}$

   (e) See crib

   (f) $\Delta T=(1/2C_DA_CL(U_C-U_T)^2U_C)/(A_TC_PU_TS_C)$

   (g) $\Delta T= \Delta T(\eta_c=1)*(2-\eta_c)$

2. (a) $L=\rho U^2\pi c\sin\alpha$

   (b) $L=2\rho U^2\pi c\sin\alpha(1+4(\tilde{h}/c)^2)/(4+8(\tilde{h}/c)^2)$

   (c) see crib

3. (a) $F(z)=Uz+(i\Gamma/2\pi)(\ln(z+ia)-\ln(z-ia))$

   (b) $\zeta=a^{3/2}\ e^{i3\pi/4}$ (and origin maps to corner)

   (c) $\Gamma=-\pi aU$

   (d) $u-iv=1/2Ua^{-1/2}e^{-i\pi/4}$

4. (a) $V_p4s^2\Gamma/\pi)h/(9s^4+10s^2h^2+h^4)$

   (b) h=0.86s

   (c) see crib

5. (a) $d\Gamma=(d\Gamma/dz)dz$

   (b) $\Gamma_{op}=(3\pi/8)\Gamma_{oe}$

   (c) $w_p/w_e=3/2$

   (d) see crib

6. (a) $u/U_e=\sin(\pi\eta/2)$

(b) (i) $\delta^*=(1-2/\pi)\delta$

(ii) $\tau=\delta(2/\pi-1/2)$

(iii) $T_w=\pi\rho\nu U_e/\delta$

(c) see crib

(d) $\delta = x\,(2\pi/(4/\pi-1))^{1/2}R_x^{-1/2}$

$C_f'=((4-\pi)/2)^{1/2}\,R_x^{-1/2}$

7. (a) see crib

(b) $L=(\rho\nu/\sigma B^2)^{1/2}$

(c) $U=\cosh(z/L)-\cosh(h/2L)$

(d) see crib

8. (a) $\Psi= U\sin\theta(r-a^2/r)$

(b) 12% error

(c) $F_y=-(4/3)\rho a U^2$ (down-force)

(d) see crib