ENGINEERING TRIPOS PART IIA

Friday 25 April 2003 9 to 12

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) For adiabatic flow of a perfect gas along a constant-area pipe with friction (i.e. a Fanno process) the change of impulse function is given by

$$d(\frac{F}{A}) = p(M^2 - 1)\frac{dV}{V}$$

where F/A is the impulse function per unit area, p the static pressure, A the cross-sectional area, M the Mach number and V the velocity. Sketch the range of possible duct flows on a T-s diagram, labelling the Mach number at critical points. State with reasons whether the velocity V, static pressure p and total pressure p_0 increase or decrease in the direction of the flow. State clearly any differences between the behaviour in subsonic and supersonic flow. [30%]

(b) A compressed air tank is located inside a nuclear power station. Air flows out of the tank down a pipe of diameter 0.5 m. The pipe is lagged and the flow can be assumed to be adiabatic. Due to safety restrictions only two points on the length of the pipe can be accessed to make measurements. At each location the total and static pressure and total temperature are measured as shown in the Table below. Due to corrosion in the tube the friction factor in the pipe is unknown.

Distance from pipe entry	Total pressure	Static pressure	Total temperature
150 m	120 kPa	96 kPa	300 K
160 m	102 kPa	68 kPa	300 K

Determine:

(i) The mass flow at either one of the two measurement locations. [20%]

(ii) The friction factor of the pipe, making use of the measurements at both locations. [20%]

(iii) The Mach number at the entrance of the pipe and the total pressure in the tank. [30%]

It can be assumed that the total pressure just inside the pipe is equal to the total pressure in the tank and that the friction factor of the pipe is constant along its length. The specific heat capacity at constant pressure c_p=1.005 kJkg^{-1}K^{-1}.

2 (a) By considering an infinitesimal sound wave in a frame of reference where the wave is stationary, or otherwise, show that the Riemann invariant across waves that are right-running relative to one-dimensional flow of a perfect gas is

$$V - \frac{2a}{\gamma - 1} = \text{constant}$$

where a is the speed of sound and V is the velocity of the gas. V is considered positive when directed to the right. [40%]

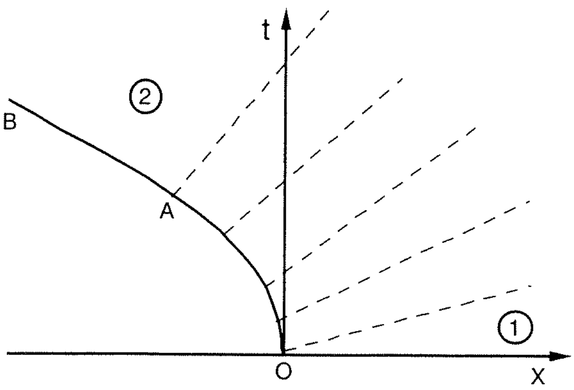

Fig. 1.

(b) Fig. 1 shows the position diagram for an unsteady flow in a cylinder of constant cross-sectional area. Initially a piston is located at x=0. To the right of the piston is stationary air with a pressure of 100 kPa. The piston accelerates to the left starting from zero velocity following the path OA and then continues to move at constant velocity along the path AB. The air to the right of the piston is expanded to state 2.

(i) The velocity of the piston along path AB is such that the density of the air in state 2 is half the density of the air in state 1. Determine the velocity of the piston along path AB in terms of the initial speed of sound a_1. Also determine the pressure at state 2. [30%]

(ii) If the piston is accelerated along path OA to the local speed of sound and then moves at constant velocity along the path AB, determine the pressure at state 2. [30%]

(TURN OVER

3 (a) Describe the various flow regimes which may be obtained in a converging-diverging nozzle as the pressure at the outlet is reduced relative to that at inlet. Include in your description sketches of the total pressure and static pressure distributions along the nozzle. The flow may be assumed to be adiabatic and inviscid. [30%]

(b) Air is discharged steadily through a convergent-divergent nozzle. The total pressure and total temperature at inlet to the nozzle are 180 kPa and 288 K and the total pressure and static pressure at the exit of the nozzle are 140 kPa and 125 kPa respectively. If the nozzle throat area is $0.01 \mathrm{m}^2$, determine:

(i) The mass flow rate and the Mach number and area at the exit of the nozzle. [30%]

(ii) The Mach number and area of the nozzle at the plane at which the shock is located. [20%]

(iii) The force exerted by the air on the diverging part of the nozzle. [20%]

4 Figure 2 shows a bend in a duct (exaggerated for clarity), designed to turn the upstream flow by 2° and to increase the cross-sectional area (perpendicular to the flow) by a factor 1.215. The segments AB, BC, CD, DE, PQ and QR are straight, DE and QR are parallel and the section BCD is designed to cancel any incident waves. The turning is isentropic and the flow downstream of the bend is uniform.

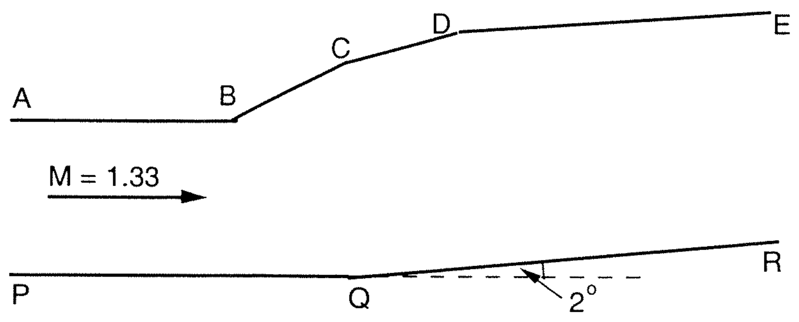

Fig. 2.

(a) Sketch the wave pattern in the duct. [20%]

(b) Find the downstream Mach Number and the angle between AB and BC. [30%]

(c) Use the method of characteristics, with a discretisation of 2°, to find the flow angle and Mach Number in each cell of your discretisation. [20%]

(d) Using a clearly labelled sketch, describe how you would estimate the position of Q. The angle relative to the upstream flow of all *relevant* lines on your sketch should be clearly labelled but *you need not* complete the calculation to find the coordinates of Q. You may assume that the angle of any characteristic is that appropriate to the flow immediately upstream of it. [20%]

(e) Describe what would happen at the corners Q, C and D if this design was analysed by the method of characteristics using a much smaller discretisation than 2°. [10%]

5 Figure 3 shows the wave and flow patterns immediately downstream of a two-dimensional supersonic nozzle for a range of back pressures p_b.

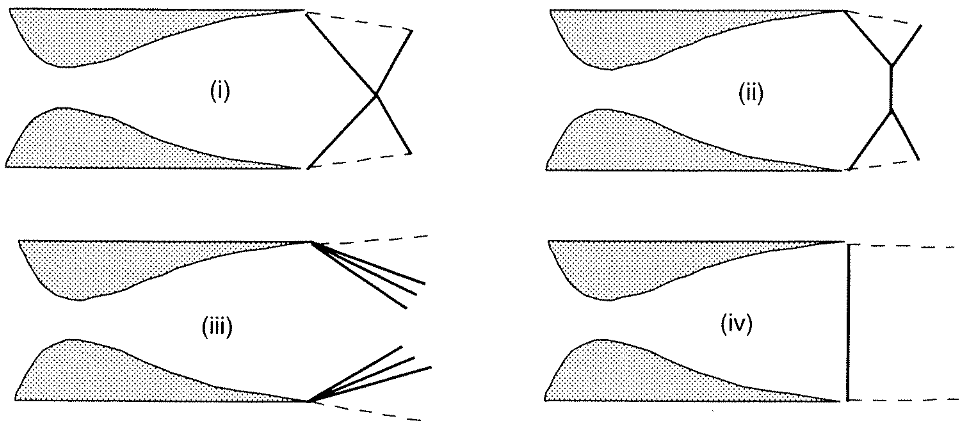

Fig. 3

(a) State the order in which these flow patterns appear as the back pressure p_b is reduced. [20%]

(b) For each case, sketch the flow pattern further downstream distinguishing between compression and expansion waves and showing the jet boundaries. [20%]

(c) Describe carefully the circumstances under which case (ii) appears. [20%]

(d) Show that, for a nozzle exit Mach Number of 2, case (ii) is possible for a range of pressure ratios given by $1.739 < \frac{P_0}{p_b} < 3.961$ where P_0 is the stagnation pressure in the nozzle. [40%]

6 (a) Describe a numerical method for calculating compressible potential flow in a duct. Your description should include the method of formulating the numerical scheme, the method of solving the resulting equations and the boundary conditions needed. [50%]

(b) Discuss the relative merits of using backswept and radial blades at the exit of the impeller of centrifugal compressors. [20%]

A centrifugal compressor working with air has impeller blades with radial exit angle. The impeller tip diameter is 0.155 m, the rotational speed is 48000 rpm and the slip factor is estimated to be 0.85. Calculate the work input per kg of air. If the stagnation temperature of the air at inlet is 288 K and the total-to-total efficiency of the whole compressor is 0.8 calculate the stagnation pressure ratio of the compressor. [30%]

7 An incompressible viscous fluid is confined between two parallel plates. The length of the plates is very large relative to the gap between them so that they can be considered as being of infinite extent. The lower plate moves in a direction parallel to its surface with a velocity which varies in an arbitrary manner with time.

(a) Show that the of velocity of the fluid between the plates satisfies the following equation of motion

$$\frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial y^2}$$

where V is the fluid velocity, ν is the kinematic viscosity of the fluid and y is the direction perpendicular to the surface of the plates. [15%]

(b) A finite difference solution to this equation is to be obtained by dividing the gap between the plates into a finite number of equally spaced intervals Δy, so that $y_i = i\Delta y$, and taking time steps of length Δt, so that $t^n = n\Delta t$. Show that the following finite difference approximation to the equation is first-order accurate in time and second - order accurate in space. [25%]

$$V_i^{n+1} = V_i^n + \frac{\nu \Delta t}{(\Delta y)^2}(V_{i+1}^n + V_{i-1}^n - 2V_i^n).$$

(c) By considering the change in amplitude of a small perturbation given by $V_i^n = (-1)^i \varepsilon$ over one time step, obtain an expression for the maximum stable time step of the finite difference method. [20%]

(d) In order to improve the accuracy in time a different approximation given by

$$V_i^{n+1} = V_i^n + \left[1.5\left(\frac{\partial V}{\partial t}\right)^n - 0.5\left(\frac{\partial V}{\partial t}\right)^{n-1}\right]\Delta t$$

is to be used, where the $\frac{\partial V}{\partial t}$ terms at time level n are calculated from the equation in part (a) and those at time level $n\text{-}1$ are stored from the previous time step. Show that this approximation is second order accurate in time. [15%]

(e) By considering the growth of a small perturbation which at time step n is given by

$$V_i^n = (-1)^i(-1)^n \varepsilon$$

find an expression for the maximum stable time step of the modified scheme. [25%]

(TURN OVER

8 A stationary gas turbine has a single-stage axial turbine and a multi-stage axial compressor. The air enters the compressor with an *absolute* stagnation pressure of 1 bar and an *absolute* stagnation temperature of 300 K. The combustion products, which may be assumed to behave as an ideal gas with the properties of air, enter the turbine with a stagnation temperature of 1350 K. The turbine has a total-total isentropic efficiency of 89%.

(a) The flow leaves the turbine stator blades with a velocity of 500 ms^{-1} inclined at 70° to the axial direction. The mean blade speed of the turbine rotor is 400 ms^{-1} , the axial velocity is constant across the stage and the absolute flow angle entering the stator and leaving the rotor is purely axial.

(i) Sketch the velocity triangles, determine the flow angle at the rotor inlet in the relative frame and find the turbine work output per kg of flow. [20%]

(ii) Determine the degree of reaction for this turbine stage. [20%]

(iii) What is the stagnation pressure ratio across the turbine ? [20%]

(b) The rotor blades in all the compressor stages have the same angles, but their height is reduced through the compressor to maintain the same axial velocity from inlet to outlet. The *absolute* flow into each rotor row is at an angle of +30° to the axial direction. The mean blade speed for each rotor is 280 ms^{-1}, the flow coefficient V_x/U is 0.45 and the *absolute* swirl velocity downstream of each rotor is 185 ms^{-1}.

(i) How many compressor stages can be driven by the turbine of part (a) ? [15%]

(ii) Calculate the static pressure and *relative* stagnation pressure at inlet to the first compressor rotor blade. If the stagnation pressure loss coefficient Y_p of the compressor blades is 0.04, what is the change in *relative* stagnation pressure across the first rotor blade ? [25%]

END OF PAPER

Paper 3A3 2003. Answers.

Q1 b) i $m = 44.9$ kg/sec.
b) ii $C_f = 0.0065$
b) iii $M_1 = 0.25$, $P_{o1} = 235$ kPa.

Q2 b) i $V/a_1 = -0.647$
b) ii $P = 24.7$ kPa

Q3) b) i $M_e = 0.41$, $m = 4.29$ kg/s, $A_e = 0.02\ m^2$.
b) ii $M = 1.88$, $A = 0.0153\ m^2$.
b) iii $F = 807$ N acting upstream.

Q4) b) $M = 1.535$, $\theta = 5.96^o$.

Q6) b) $m = 128.9$ kJ/kg. $P_{o2}/P_{o1} = 2.91$.

Q7) c) $\nu\ \Delta t/\Delta y^2 < 0.5$.
e) $\nu\ \Delta t/\Delta y^2 < 0.25$.

Q8) a) i 22.2^o, 187.9 kJ/kg .
a) ii 41.25%
a) iii $P_{o4}/P_{o3} = 0.55$
b) i 6 stages.
b) ii $P_1 = 0.882$ bar, $P_{o1rel} = 1.235$ bar, $\Delta P_{orel} = 0.014$ bar .