ENGINEERING TRIPOS PART IIA

Saturday 26 April 2003 9 – 12

Module 3A5

ENERGY AND POWER GENERATION

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Prove the Maxwell relation for a pure substance

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

By expressing changes in specific enthalpy dh in terms of changes in temperature dT and pressure dp, and using $Tds = dh - vdp$, show that

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p$$

Hence show that

$$\left(\frac{\partial T}{\partial p}\right)_h = -\frac{1}{c_p}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]$$

where c_p is the isobaric specific heat capacity. [40%]

(b) A *non-ideal* gas obeys the equation of state

$$p(v-b) = RT$$

where R and b are both constant, taking the values 0.45 kJ kg^{-1} K^{-1} and 0.04 m^3 kg^{-1} respectively. c_p is also constant and takes the value 1.75 kJ kg^{-1} K^{-1}. If the gas undergoes a steady-flow throttling process from a pressure of 10 bar and a temperature of 600 K to a pressure of 2 bar, calculate its final temperature. [20%]

(c) (i) Derive an expression for the specific entropy of the gas of part (b). [20%]

(ii) Calculate the change in the steady-flow specific exergy e during the throttling process. What would be the change in e of a *perfect* gas with the same value of R undergoing a throttling process from the same initial state to the same final pressure? Assume that the environment is at temperature $T_0 = 300$ K. [20%]

2 (a) A semi-perfect gas (*i.e.*, an ideal gas with an isobaric specific heat capacity c_p which varies only with temperature) undergoes a compression process 1→2. If η_c is the constant *polytropic efficiency* and R is the specific gas constant, show that

$$R \ln\left(\frac{p_2}{p_1}\right) = \eta_c \int_{T_1}^{T_2} \frac{c_p(T)}{T}\, dT$$

Write down the equivalent equation for an expansion process 3→4 with polytropic efficiency η_t. What is the main reason why polytropic, rather than isentropic, efficiencies are preferred for industrial use? [30%]

(b) Air enters the compressor of a gas turbine at 1 bar and 298.15 K and is compressed with a polytropic efficiency of 0.90 to 30 bar. The air behaves as a semi-perfect gas with $R = 0.287$ kJ kg^{-1} K^{-1} and,

$$c_p(T) = c_{p0}\left(\frac{T}{T_0}\right)^n$$

where $c_{p0} = 1.010$ kJ kg^{-1} K^{-1}, $T_0 = 298.15$ K and $n = 0.12$. Calculate the compressor exit temperature and the work of compression per kg of air entering. [35%]

(c) The compressed air now enters the combustor. Fuel with a lower calorific value (at 298.15 K) of 45 MJ kg^{-1} is supplied at 298.15 K. There is a 5% loss of total pressure in the combustor and the outlet temperature is 1500 K. The polytropic efficiency of the turbine expansion is 0.85 and turbine blade cooling may be neglected. The turbine exhaust pressure is 1 bar. The products of combustion also behave as a semi-perfect gas but with $R = 0.3$ kJ kg^{-1} K^{-1} and c_p varying according to the above equation with $c_{p0} = 1.05$ kJ kg^{-1} K^{-1}, $T_0 = 298.15$ K and $n = 0.13$.

Calculate the air/fuel ratio and the overall efficiency of the gas turbine. [35%]

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3 For this question, use only the data given below. Do not use the steam tables.

DATA:

At p = 40 bar, T = 450 °C;	$h = 3331.0$ kJ kg^{-1}	$s = 6.939$ kJ kg^{-1} K^{-1}
Saturated liquid at 30 °C:	$h_f = 125.7$ kJ kg^{-1}	$s_f = 0.437$ kJ kg^{-1} K^{-1}
Saturated vapour at 30 °C:	$h_g = 2556.4$ kJ kg^{-1}	$s_g = 8.455$ kJ kg^{-1} K^{-1}
Isobaric specific heat capacity of saturated liquid at 30 °C:		$c_{pf} = 4.18$ kJ kg^{-1} K^{-1}

(a) Figure 1 shows the T-s diagram of a steam cycle. Steam enters the turbine at pressure $p_3 = 40$ bar and temperature $T_3 = 450$ °C, and the steam temperature in the condenser is $T_4 = T_1 = 30$ °C. The cycle efficiency is $\eta_{st} = 0.33$. Calculate the dryness fraction x_{4s} corresponding to an isentropic expansion in the turbine. Neglecting the feed pump work, calculate also the actual specific enthalpy at turbine exit h_4 and the turbine isentropic efficiency η_t. [30%]

(b) The rate of change of saturated vapour pressure p_{sat} with temperature T is given by the Clausius-Clapeyron equation

$$\frac{dp_{sat}}{dT} = \frac{h_{fg}}{v_{fg}\,T} = \frac{(h_g - h_f)}{(v_g - v_f)T}$$

Consider a small change in temperature δT_4 of the steam in the condenser giving rise to changes in heat input δq and work output δw_t (both per unit mass of steam circulating). Assuming that the turbine isentropic efficiency η_t remains constant and neglecting v_f in comparison to v_g, show that

$$\delta q = -c_{pf}\delta T_4 \qquad \text{and} \qquad \delta w_t = -\eta_t\, x_{4s}\, h_{fg}\frac{\delta T_4}{T_4}$$

where h_{fg} and c_{pf} are evaluated at the condenser temperature T_4. [30%]

(c) The overall efficiency η_{cc} of a CCGT plant is given by the equation

$$\eta_{cc} = \eta_{gt} + \eta_b \eta_{st}(1-\eta_{gt})$$

(Cont.

State the meaning of the symbol η_b. If the steam cycle of part (a) forms the bottoming cycle of a CCGT with $\eta_{gt} = 0.35$ and $\eta_b = 0.80$, calculate η_{cc}. [10%]

(d) If the temperature of the steam in the condenser increases by 10 °C (due to an increase in cooling water temperature), use the results of parts (a), (b) and (c) to estimate the changes in η_{st}, η_b and η_{cc}. [30%]

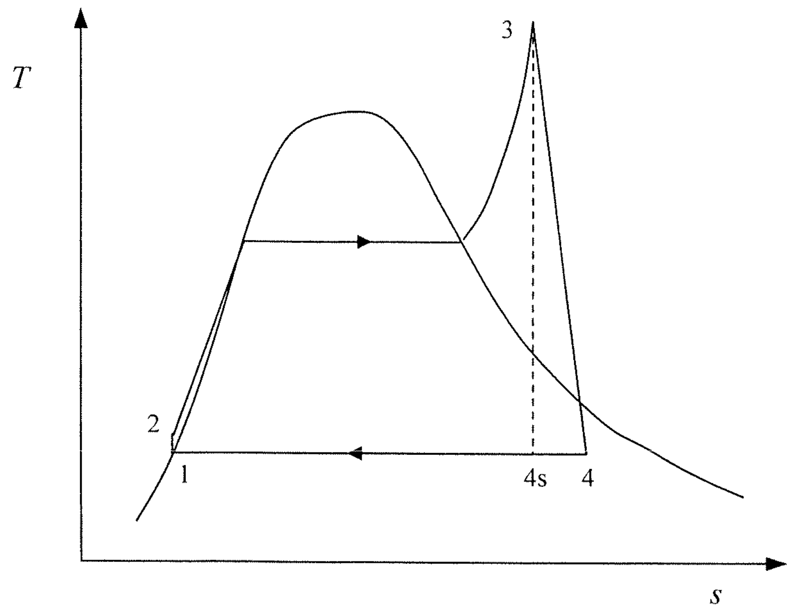

Fig. 1

4 (a) Explain what is meant by the terms *junction loss* and *spectrum loss* in the context of photovoltaic cell operation. What other factors limit photovoltaic cell efficiency? [30%]

The current density J_L passing through a resistive load attached to a photovoltaic cell is given by

$$J_L = J_S - J_0\left[\exp\left(\frac{eV_L}{kT}\right) - 1\right]$$

where J_S is the short-circuit current density, J_0 is the reverse-saturation current density (equal to 10^{-8} A m^{-2}), e is the electronic charge (1.6×10^{-19} C), k is Boltzmann's constant (1.381×10^{-23} J K^{-1}), T is the operating temperature and V_L is the voltage across the load.

(b) In the early morning the short-circuit current density J_S is 200 A m^{-2} and the operating temperature is 10 °C. Show that the power output from the photovoltaic cell is maximised when V_L is 0.504 V, and hence find, per unit area of the cell, the load resistance and this maximum power output. [40%]

(c) At midday, when the sunlight is at its strongest, the short-circuit current density J_S is 400 A m^{-2} and the operating temperature is 30 °C. Find the corresponding optimal operating conditions (load voltage, resistance and power per unit area) for the photovoltaic cell. Comment on the practical implications of these results. [30%]

5 (a) What is meant by the terms *laminar burning velocity*, *flammability limits* and *quenching distance* for a fuel-air mixture? What is the physical and/or chemical origin of the flammability limits? [30%]

(b) A premixed flame with laminar burning velocity S_L is stabilised on the rim of a vertical circular tube. Write down an expression for the angle $\theta(r)$ that the flame makes with the axis of the tube, if the flow velocity inside the tube obeys a parabolic velocity profile given by

$$V(r) = V_0(1 - r^2 / R^2)$$

where $V(r)$ is the velocity at a radial distance r, R is the radius of the tube and V_0 the centreline velocity. Sketch the flame shape. Assume that $V_0 > S_L$ and ignore the region close to the tube wall where $S_L > V(r)$. [20%]

(c) The products of rich methane-air combustion at an equivalence ratio of 2.0 are in chemical equilibrium at 10 bar and 1600 K. Calculate their volumetric composition, assuming that the only gases present are CO_2, CO, H_2O, H_2 and N_2. [50%]

6 In a premixed-combustion gas turbine, the hot air from the compressor is admitted into an adiabatic straight circular duct into which the fuel is to be mixed. The fuel is liquid $C_{10}H_{22}$ with density $\rho_{fu} = 800$ kg m^{-3} and is injected parallel to the air uniformly across the duct in the form of droplets of initial diameter $d_{in} = 40\times10^{-6}$ m and at a total mass flow rate of 0.02 kg s^{-1}. The air is at a constant temperature of 1000 K and a pressure of 10 bar, and it has a mass flow rate of 0.5 kg s^{-1}. The duct diameter is $D = 0.05$ m. As the droplets flow with the air, they evaporate and finally a flammable mixture is formed at the exit of the duct.

(a) Calculate the equivalence ratio of the mixture after complete evaporation, assuming no chemical reactions occur in the duct. [30%]

(b) It can be shown that the mass flux of fuel at the surface of an evaporating droplet of diameter d is given by

$$\dot{m}'' = \frac{\rho_{fu}\beta}{4d}$$

with the evaporation coefficient β taken as constant and equal to 10^{-6} m^2 s^{-1} for the present conditions. Derive an expression for the droplet diameter as a function of time and hence calculate the necessary length of duct to achieve complete evaporation. [40%]

(c) During tests, it is found that autoignition occurs half-way along the duct, which is a dangerous situation. In order to solve the problem, a designer proposes to lower the air temperature to 950 K. The reaction rate constant has an activation temperature of 15000 K. If all the other parameters stay the same, *estimate* the autoignition length with the suggested air temperature. Suggest additional measures to avoid autoignition. [30%]

7 Exhaust gas is recirculated to the intake manifold of a spark-ignition engine that follows an ideal Otto cycle. The residual gas fraction *without* exhaust gas recirculation (EGR) is 10% by mass (Case A). After introducing EGR, the total burnt gas fraction in the charge is 30% by mass (Case B). Assume that the total mass admitted into the cylinder is unchanged and that operation remains stoichiometric. The initial temperature is 300 K and the compression ratio is 9. Assume that the mass of the fuel in the charge, m_f, is much smaller than both that of air, m_a, and that of the residual gas.

(a) Show that the temperature *rise* after combustion can be approximated by

$$\frac{m_f}{m_a}\frac{Q(1-x_r)}{c_v}$$

where Q is the lower calorific value of the fuel, x_r the burnt gas fraction in the charge and c_v the specific heat capacity of the mixture, taken as constant and equal to that of air. [30%]

(b) The temperature rise due to combustion in Case A is 2100 K. Estimate the maximum burned gas temperature for Case B (*i.e.*, after EGR is introduced). [20%]

(c) Determine the percentage change in gross indicated work per cycle between Cases B and A. [20%]

(d) Determine the percentage change in indicated fuel conversion efficiency between Cases B and A. [20%]

(e) Suggest a method by which the indicated work of the cycle in Case B might be restored to that of Case A, while keeping EGR induction. [10%]

8 An inter-cooled turbocharged compression-ignition engine operates with a compression ratio of 14 and a displacement volume of 0.5×10^{-3} m^3 per cylinder. The inlet manifold is at 325 K and 2.0 bar, while the exhaust is at 700 K and 1.8 bar. The fuel is a light diesel with lower calorific value of 43 MJ kg^{-1} and the fuel-air ratio is 0.03 by mass. The cycle can be represented by an ideal limited-pressure cycle where 80% of the fuel energy is released during the constant volume part of the heat addition. The ambient temperature and pressure are 285 K and 1 bar respectively. The compressor efficiency is 80% and the mechanical efficiency of the turbocharger 100%. You may take c_v = 0.8 kJ kg^{-1} K^{-1} and γ = 1.4.

(a) Draw a p–V diagram for the turbocharged cycle, indicating the relevant parameters. [20%]

(b) Calculate the pressures, temperatures and volumes for each point in the power section of the cycle. [30%]

(c) Calculate the gross and net indicated mean effective pressure for the ideal cycle. [25%]

(d) Determine the necessary turbine efficiency. [25%]

END OF PAPER

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ANSWERS

1. (b) 618.3 K. (c) i. $s = c_p \ln T - R \ln P + \text{constant}$; ii. –233.1 kJ/kg, -217.3 kJ/kg (perfect gas)

2. (b) 818.7 K, 564.6 kJ/kg. (c) 49.62, 42.4%.

3. (a) x_{4s}= 0.8112, h_4=2096.8 kJ/kg, η_t=85.7% . (c) 52.2% . (d) -1.31, -1.04, -0.9 percentage points.

4. (b) 96.2 W/m^2 , 2.64x10^{-3} Ω m^2 . (c) 0.557 V, 212.8 W/m^2 , 1.46x10^{-3} Ω m^2

5. (c) Volume fractions of CO_2, CO, H_2O, H_2, N_2: 2.77%, 12.03%, 12.03%, 17.57%, 55.68%

6. (a) 0.6 . (b) 0.12 m

7. (b) 2356 K . (c) 22.2 % reduction over Case A. (d) Unchanged at 58.5%.

8. (b) 2 bar, 0.538x10^{-3} m^3, 325 K; 80.46 bar, 0.0385x10^{-3} m^3, 934 K; 188.3 bar, 0.0385x10^{-3} m^3, 2186 K; 188.3 bar, 0.0424x10^{-3} m^3, 2410 K; 5.37 bar, 0.538x10^{-3} m^3, 871.7 K. (c) Gross IMEP: 16.85 bar; Net IMEP: 16.65 bar. (d) 70%.