ENGINEERING TRIPOS PART IIA

Tuesday 6 May 2003 9.00 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Explain the physical significance of the wavefunction of a particle and how the basic postulates of quantum mechanics govern its form. [20%]

(b) Show that

$$\psi = A_0 \exp\left(\frac{-\alpha^2 x^2}{2}\right)$$

is a valid wavefunction for a particle of mass m in a one-dimensional simple harmonic oscillator potential well where the potential $V(x)$ varies only with position x as

$$V(x) = \frac{cx^2}{2}$$

and c and A_0 are constants and

$$\alpha^2 = \frac{(mc)^{1/2}}{\hbar}$$ [45%]

Hence, determine an expression for the total energy of the particle in this state. [10%]

(c) Sketch how the probability of finding the particle varies with position x. Given that this is the ground state for the particle in this system, explain how the quantum mechanical behaviour of this system differs from that of the classical simple harmonic oscillator. [25%]

2 (a) List the principle assumptions of the Free Electron Theory. [15%]

(b) Show that the average energy of an electron $\langle E \rangle$ in a metal at absolute zero is related to the Fermi energy E_F according to

$$\langle E \rangle = \frac{V}{N}\frac{(2m)^{3/2}}{5\pi^2\hbar^3}E_F^{5/2}$$

where $\frac{N}{V}$ is the number density of free electrons and m is the free electron mass. [25%]

(c) By calculating the number density of free electrons $\frac{N}{V}$, demonstrate that the expression above may be simplified to

$$\langle E \rangle = \frac{3E_F}{5}$$ [25%]

(d) Calculate $\langle E \rangle$ for sodium, which is in Group I of the Periodic Table and has an atomic number density of 2.54×10^{28} m^{-3}. [20%]

Calculate the wavelength of an electron with this energy $\langle E \rangle$. [15%]

Note: the free electron energy density of states for a material of volume V is given by

$$g(E)\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m)^{3/2}E^{1/2}\mathrm{d}E$$

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3 (a) Explain how the wavenumber, energy, group velocity, effective mass and charge of a hole in the valence band of a semiconductor can be related to the equivalent properties of an electron occupying the same state. [25%]

(b) A semiconductor has a band gap of 1.10 eV, an intrinsic carrier concentration of 10^{16} m^{-3} and an effective density of states in the valence band of 7×10^{24} m^{-3}.

(i) What density of acceptors are required to dope this semiconductor such that at room temperature (298 K) the Fermi energy lies 0.05 eV above the top of the valence band? State any assumptions made. [10%]

(ii) An equal density of dopants, but this time donors, is used to dope another sample of the same semiconductor. Calculate the position of the Fermi energy with respect to the bottom of the conduction band in this sample. [15%]

(c) A pn junction diode is fabricated using the p-type and n-type materials of part (b) above. Starting from the Poisson equation, show that the depletion width of the junction w is given by

$$w = 2\left(\frac{\varepsilon_0 \varepsilon_r V_0}{eN_A}\right)^{1/2}$$

where V_0 is the built in potential and N_A is the doping density of acceptors in the p-type material, which is equal to the doping density N_D in the n-type material. [40%]

Calculate the depletion width for this particular pn junction, where ε_r is 12. [10%]

4 (a) Sketch the distribution of minority carriers either side of the depletion width in a pn$^+$ junction for a positive bias applied to the p-side of the junction. [20%]

(b) Starting from the continuity equation, show that the excess hole concentration $p - p_{n0}$ on the n-side of a uniformly doped pn junction under a forward bias voltage V is given by

$$p - p_{n0} = p_{n0}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_h}\right)$$

where p_{n0} is the intrinsic hole concentration in the n-type semiconductor, L_h is the hole diffusion length and x is the distance from the edge of the depletion region. State all assumptions made. [45%]

(c) Hence, using the result from part (b), show that the current flow associated with this junction is given by

$$I = I_s\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

including an expression for the reverse saturation current I_S in terms of the electron and hole diffusion lengths, the electron and hole lifetimes, the minority carrier densities either side of the depletion width and the junction area. [35%]

5 (a) Distinguish between ohmic and rectifying contacts to n-type semiconductors with the aid of energy band diagrams. [30%]

(b) A Schottky Barrier Diode is made from n-type silicon and gold. The donor density in the silicon is $5 \times 10^{22}\ \mathrm{m}^{-3}$ and the effective density of states in the conduction band is $4 \times 10^{25}\ \mathrm{m}^{-3}$. The work function of gold is $4.7\ \mathrm{eV}$ and the electron affinity of silicon is $4.05\ \mathrm{eV}$.

Estimate:

(i) the barrier to electron flow from semiconductor to metal; [25%]

(ii) the barrier to electron flow from metal to semiconductor; [15%]

(iii) the reverse saturation current of the diode at room temperature if the diameter of the metal contact is $0.25\ \mathrm{mm}$. Assume that the electrons have an effective mass equal to the free electron mass. The Richardson-Dushman equation for thermionic emission may be assumed to be

$$J = 1\cdot 204 \times 10^{6}\ T^{2} \exp\left(\frac{-\phi}{kT}\right)\ \mathrm{A\ m^{-2}}$$ [30%]

END OF PAPER

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1 (b) $E = \frac{\hbar}{2}\left(\frac{c}{m}\right)^{1/2}$

2 (d) $\langle E \rangle = 1.89$ eV
$\lambda = 8.92 \times 10^{-10}$ m

3 (b) (i) $N_A = 1 \times 10^{24}\ \mathrm{m}^{-3}$
(ii) $E_F - E_C = -0.10$ eV
(c) $w = 50$ nm

5 (b) (i) $\mathrm{Barrier}_{\mathrm{Si}\to\mathrm{m}} = 0.48$ eV
(ii) $\mathrm{Barrier}_{\mathrm{m}\to\mathrm{Si}} = 0.65$ eV
(iii) $I = 53.7$ nA

AJF
May 2003