ENGINEERING TRIPOS PART IIA

Tuesday 29 April 2003 2.30 to 4.00

Paper 3C3

MACHINE DESIGN – TRIBOLOGY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Explain briefly the need for a positive entraining velocity and a convergent geometry for the generation of load support in hydrodynamic bearings. [20%]

(b) A square, tilting pad bearing of length L has a film thickness at entry of h_1 and at exit h_0. The bearing is supplied with a lubricant of viscosity η and the lower plane surface moves with speed U as illustrated in Fig. 1. The pivot is located a distance χL from the entry of the bearing.

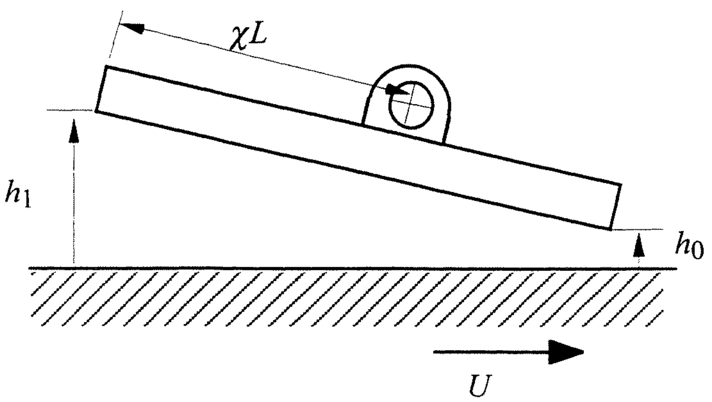

Fig. 1

Calculations give the following data: H is the ratio of inlet film thickness to outlet film thickness, $\bar{p}$ is the mean pressure on the bearing, f is the friction force per unit area and Q the volume flow of lubricant under the pad.

χ	0.55	0.60	0.65	0.70	0.75	0.80
H	1.47	2.26	3.77	6.21	10.8	17.4
$W^* = \bar{p}h_0^2 / \eta UL$	.0585	.0703	.0585	.0400	.0236	.0105
$F^* = fh_0 / \eta U$	.870	.700	.565	.460	.380	.300
$Q^* = Q / Uh_0 L$	.650	.917	1.37	2.14	3.38	5.35

What are the values of χ, W^*, F^* and Q^* for $H = 1$? [15%]

(c) The mean pressure on the bearing shown in Fig. 1 is to be fixed.

(i) Where should the pivot be placed to maximise the minimum film thickness? Show that in such a design

$$\mu \approx 2.64\sqrt{\frac{\eta U}{\bar{p}L}}$$

where μ is the overall coefficient of friction. [25%]

(ii) If it can be assumed that all the heat generated is carried away by the oil passing under the pad, how does the outlet oil temperature vary with the position of the pivot? Estimate the factor by which the temperature increase may be reduced by changing χ from 0.55 to 0.65. [25%]

(iii) Does bearing with $\chi = 0.5$ represent a realistic design? How could it be made so? [15%]

2 Figure 2 shows a journal bearing; C is the centre of the shaft and O the centre of the bearing. The lubricant film thickness h can be taken as $h = c(1+\varepsilon\cos\theta)$, where c is the radial clearance, ε is the eccentricity ratio and the angle θ is measured from the line joining O and C. The rotating shaft carries a steady radial load.

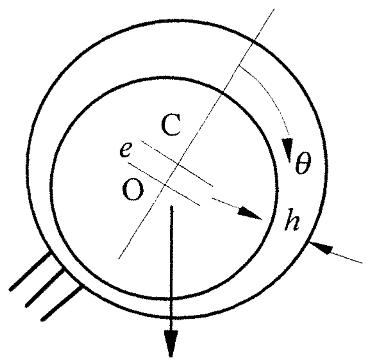

Fig. 2

(a) Explain briefly, when modelling the performance of such a device, what is meant by the 'full' and 'half' Sommerfeld boundary conditions, illustrating with a sketch the form of the variation of hydrodynamic pressure with angle θ for each case. [20%]

(b) An axially long 360° journal bearing of radius R runs at rotational speed ω and is lubricated by a film of air, of viscosity η, drawn in from the surrounding environment. Explain why, in analysing this situation the full Sommerfeld solution conditions are the more relevant. [20%]

(c) If the Sommerfeld number S is defined as $\left(\eta\omega/\bar{p}\right)\left(R/c\right)^2$ where $\bar{p}$ is the mean nominal pressure on the bearing, then the relation between S and ε is

$$\frac{1}{S} = \frac{6\pi\varepsilon}{\left(1-\varepsilon^2\right)^{1/2}\left(2+\varepsilon^2\right)} .$$

If the bearing runs with an eccentricity ratio approaching unity, obtain a relation between the Sommerfeld number, the minimum film thickness $h_{\min}$ and the clearance c. [40%]

(d) Explain why for this type of bearing the direction of the line OC is perpendicular to the direction of the load vector. [20%]

Version 4

3 (a) Explain briefly the idealisations that are implicit in the Hertzian analysis of contacts. [25%]

(b) A ball is rotating in contact with a plane surface under a contact force P. Show that the motion is resisted by a torque of magnitude $3\pi\mu Pa/16$ where μ is the coefficient of friction between the ball and the plane and a is the radius of the contact spot. Hertzian conditions can be assumed.

Note that $\int_0^{\pi/2} \sin^2\theta\cos^2\theta \, d\theta = \frac{\pi}{16}$. [25%]

(c) A test rig, shown schematically in Fig. 3, consists of two flat plates, A and B, which are separated by three steel balls, each of radius r. The balls are located symmetrically by a cage C which is at rest. Plates A and B each rotate about the central axis with angular velocities of magnitude Ω but of opposite senses. In operation a normal force P is applied to the plates which is sufficient to ensure that there is no skidding at any of the points of contact.

(i) Show that there must be spin at a minimum of three of the points of contact between the balls and the plates. [15%]

(ii) Plate A is aluminium and plate B is steel. If the coefficient of friction of steel on steel is 0.2 and of aluminium on steel 0.18 at which of the contacts is slip most likely? [20%]

(iii) If the total normal load is 5 N and the balls are of radius 5 mm, estimate the torque required to maintain plates A and B at their steady rotational speeds. [15%]

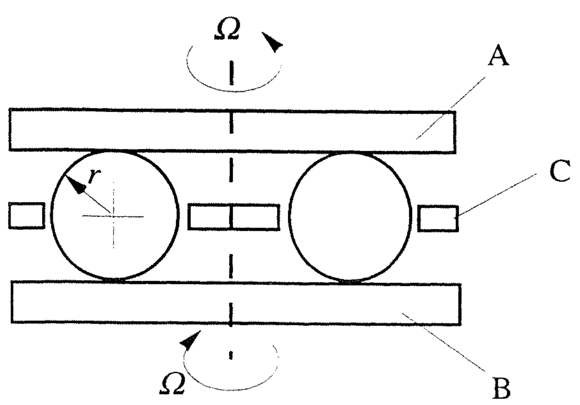

Fig. 3

4 (a) Two spur gears with module m, pressure angle ϕ and tooth numbers N_1 and N_2 mesh together. Derive the following expression for the maximum Hertz stress p_0 for single tooth contact at the pitch point due to a line load P' acting normally to the tooth faces:

$$p_0 = \sqrt{\frac{P'E^*}{\pi} \frac{2(N_1 + N_2)}{mN_1 N_2 \sin\phi}} \ .$$

Make use of the data sheet formulae for Hertzian contacts as necessary. E^* has its usual meaning. [20%]

(b) Explain why the bending stress σ_b at the root of a spur gear of module m due to a line load P_T' acting tangentially to the pitch circle can be expressed, using the AGMA geometry factor J, in the form:

$$\sigma_b = \frac{P_T'}{Jm} \ .$$

Explain, with reference to the relevant figure on the data sheet, how J depends on the assumed tooth load distribution. [20%]

(c) Suggest a suitable spur gear design for the power train of a ride-on garden mower, taking into account the following design requirements:

(i) low cost;
(ii) 15 teeth on the pinion;
(iii) 3:1 speed ratio;
(iv) compact design, minimising the wheel diameter;
(v) power of 20 kW at a pinion speed of 1200 r.p.m.

State clearly any design assumptions that you have made and make use of the data sheet where necessary. Use the result of part (a) to calculate the maximum Hertzian pressure where appropriate. [50%]

(d) How might the gear design for part (c) be modified in practice for a better performance, higher-cost gear? [10%]

END OF PAPER

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ANSWERS

1 (b) $\chi = 0.5$, $W^* = 0$, $Q^* = 0.5$, $F^* = 1$

(c) (i) $\chi \approx 0.6$, (ii) temperature increase reduced to about 31%.

2 $h_{\min} \approx 2\pi^2 S^2 c$

3 (c) Spin most likely at steel/steel contacts where potential friction torque is lower than at larger Al/steel contacts.

Required torque 2.23×10^{-5} Nm

4 (c) Appropriate standard module 4.5 mm; reasonable design might have face width of 67 mm through hardened gears, pinion with 15 teeth wheel with 45.

(d) Possibilities include – helical teeth, increase tooth numbers, better material etc.