ENGINEERING TRIPOS PART IIA

Tuesday 6 May 2003 9 to 10.30

Module 3C5

DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 An assembly comprises a square flat plate ABCD of mass m and side $2a$ with two thin uniform bars each of mass $m/2$ and length $2a$ attached at corners B and D as shown in Fig. 1.

(a) For the plate ABCD alone, find the principal moments of inertia using x, y, and z axes as shown. In what sense can the plate be considered to be a circular disc? [25%]

(b) Show that the inertia matrix at the centre G of the assembly using x, y, and z axes as shown is

$$\frac{ma^2}{3}\begin{pmatrix} 5 & ? & 3\sqrt{2} \\ ? & 11 & ? \\ ? & ? & 8 \end{pmatrix}$$

and find the values marked "?" in the matrix. [40%]

(c) Find the principal moments of inertia of the assembly at G and show that the plate diagonal AC is one of the principal axes. [30%]

(d) In what sense is the assembly equivalent to a cylinder? [5%]

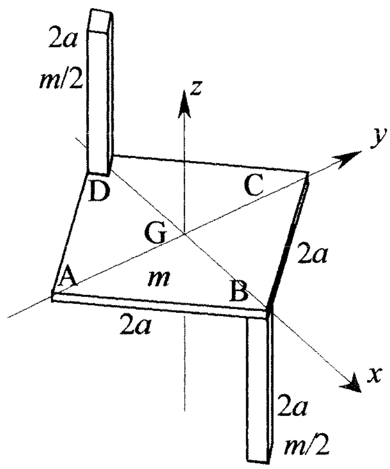


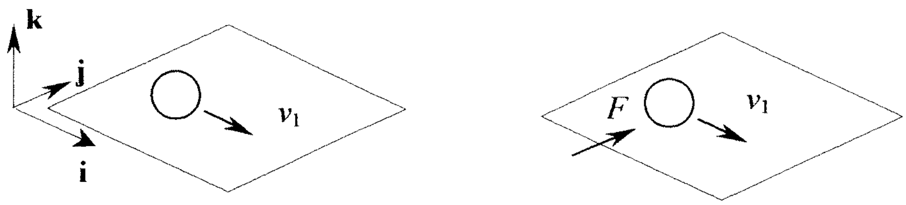
Fig. 1

2 A uniform solid rigid sphere of mass m and radius a rolls without slip on a flat horizontal table as shown in Fig. 2(a). Its initial velocity is $v_1\mathbf{i}$ as shown in the figure.

(a) Show that the ball rolls with constant velocity $v_1\mathbf{i}$ in a straight line and that any amount of spin ω_3 about the vertical axis $\mathbf{k}$ does not affect the motion. [20%]

(b) A constant sideways force $F\mathbf{j}$ now acts to deviate the ball from its straight-line motion as shown in Fig. 2(b). Find the acceleration of the ball and demonstrate that the ball follows a parabolic path. [60%]

(c) Does spin ω_3 about the vertical axis $\mathbf{k}$ affect the parabolic motion in (b)? [20%]



(a) (b)

Fig. 2

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3 Cuspoidal nutation of a gyroscopic pendulum is to be studied using an AAC rotor of mass m spinning at angular speed ω about an axis BD within a light framework as shown in Fig. 3. At B there is a ball joint which is fixed in space. The centre of the rotor is at G and the distance BG is a. Motion of the rotor is described by Euler's angles θ and ϕ. The angular velocity ω of the rotor is "fast" and constant with respect to the framework. The framework is released from rest with its axis horizontal ($\theta = \pi/2$).

(a) Sketch the form of the variation of θ with time:

(i) assuming no energy loss; [15%]

(ii) allowing for some energy loss due to friction. [15%]

(b) Use conservation of energy and conservation of moment of momentum about the vertical axis to estimate the maximum value of θ reached during the cuspoidal motion. [40%]

(c) Find the steady state value of θ reached after oscillations have died away. [30%]

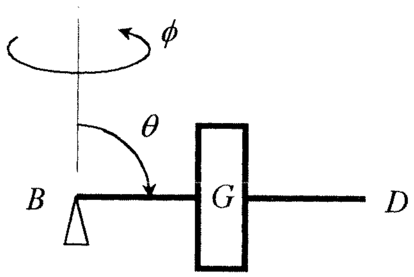

Fig. 3

4 A crane mounted on an offshore supply vessel has a light cable of unstretched length L and stiffness k to which a mass M is attached, as shown in Fig. 4. The angle of the cable to the vertical is θ and the stretch of the cable is $x(t)$, so that the instantaneous length is $L+x(t)$. Due to the motion of the supply vessel, the top of the cable has a prescribed (known) motion $y(t)$.

(a) Derive expressions for the kinetic and potential energies of the system. Do not assume that θ is small. [10%]

(b) By using Lagrange's equation, derive the equations of motion for x and θ. [45%]

(c) By considering the accelerations of the mass, and employing D'Alembert's principle, show that the equations of motion are statements of force equilibrium along the cable and moment equilibrium about the top point. [25%]

(d) Derive an expression for the cable motion $\theta(t)$ when the top motion has the form $y(t) = A\sin\omega t$, under the condition that the cable is very stiff, so that $x = 0$. You may assume that θ is small in deriving this expression, but comment on the validity of your result. [20%]

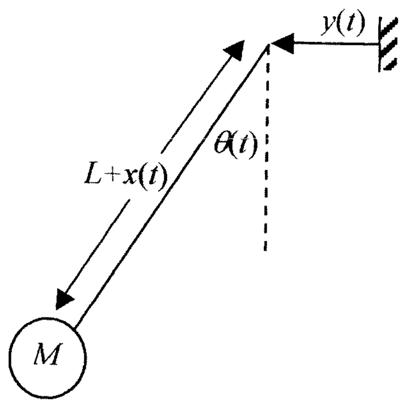

Fig. 4

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5 (a) A linear system with N degrees of freedom undergoes small amplitude vibrations. Write down expressions for the mass and stiffness matrices of the system in terms of the kinetic and potential energies. [5%]

(b) A three-degree-of-freedom pendulum system is shown in Fig. 5. Each of the three strings is of length L, each of the lumped masses is of mass M, and the lowest mass is subjected to a horizontal force F as shown in the Figure. Derive the equations of motion of the system, assuming from the outset that the angles θ_1, θ_2, and θ_3 are small. [45%]

(c) The top mass is now held in a fixed position so that $\theta_1 = 0$. Write down the equations of motion that govern the remaining degrees of freedom, and proceed to calculate the natural frequencies and mode shapes of the system. [50%]

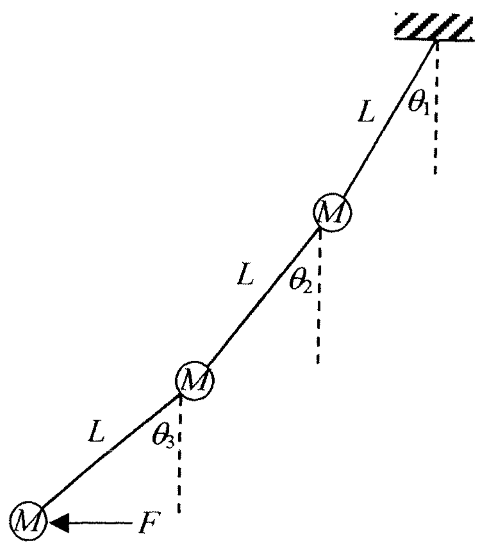

Fig. 5

END OF PAPER

ENGINEERING TRIPOS PART IIA 2003

MODULE 3C5: ANSWERS

1. (a) $ma^2/3,\ ma^2/3,\ 2ma^2/3$
   (b) Missing terms are zero, except the 3-1 entry is $3\sqrt{2}$.
   (c) $11ma^2/3,\ 11ma^2/3,\ 2ma^2/3$

2. (b) $\dot{v}_2 = Fa^2/A,\quad A = 7ma^2/5$
   (c) No

3. (b) $2mga(A+ma^2)/C^2\omega^2$
   (c) $mga(A+ma^2)/C^2\omega^2$

4. (a)
$$T = (m/2)\left\{\dot{x}^2 + \dot{y}^2 + (L+x)^2\dot{\theta}^2 + 2\dot{y}\dot{\theta}(L+x)\cos\theta + 2\dot{x}\dot{y}\sin\theta\right\}$$
$$V = -mg(L+x)\cos\theta + kx^2/2$$
   (b)
$$m\ddot{x} - m(L+x)\dot{\theta}^2 + m\ddot{y}\sin\theta - mg\cos\theta + kx = 0$$
$$m(L+x)^2\ddot{\theta} + 2m\dot{x}\dot{\theta}(L+x) + m\ddot{y}(L+x)\cos\theta + mg(L+x)\sin\theta = 0$$
   (d) $\theta = \alpha\cos(\omega_n t + \phi) + A\sin\omega t/[L(\omega_n^2 - \omega^2)]$

5. (b)
$$\begin{pmatrix}3&2&1\\2&2&1\\1&1&1\end{pmatrix}\begin{pmatrix}\ddot{\theta}_1\\\ddot{\theta}_2\\\ddot{\theta}_3\end{pmatrix} + (g/L)\begin{pmatrix}3&0&0\\0&2&0\\0&0&1\end{pmatrix}\begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix} = (F/mL)\begin{pmatrix}1\\1\\1\end{pmatrix}$$
   (c)
$$\omega_1^2 = (g/L)(2-\sqrt{2}),\quad \mathbf{u}_1 = \begin{pmatrix}1\\\sqrt{2}\end{pmatrix};\quad \omega_2^2 = (g/L)(2+\sqrt{2}),\quad \mathbf{u}_2 = \begin{pmatrix}1\\-\sqrt{2}\end{pmatrix}$$