ENGINEERING TRIPOS PART IIA

Thursday 8 May 2003 9.00 to 10.30

Module 3C7

MECHANICS OF SOLIDS

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) What is the fundamental assumption underlying continuum mechanics? Using an example, discuss the importance of length scale in calculation of stresses in real structural elements. [20%]

(b) List the three basic principles that are needed to solve any problem in solid mechanics. [20%]

(c) The water dam shown in Fig. 1 is subjected to hydrostatic pressure on its left surface and uniform pressure p (per unit area) on its top; its right surface is traction free. The density of water is ρ, and the gravitational acceleration is g.

(i) Write down the boundary conditions on the top and two lateral surfaces of the dam in terms of $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$. [30%]

(ii) The following stress field has been proposed for the dam

$$\sigma_{xx} = C_1 x^2 (H - y), \quad \sigma_{yy} = C_2 (H - y)^3, \quad \sigma_{xy} = C_3 x (H - y)^2$$

where C_1, C_2 and C_3 are constants having units of Nm^{-5}. For what ratios C_2 / C_1 and C_3 / C_1 does this stress field represent a state of equilibrium? [20%]

(iii) Can this stress field satisfy the boundary conditions in (i)? [10%]

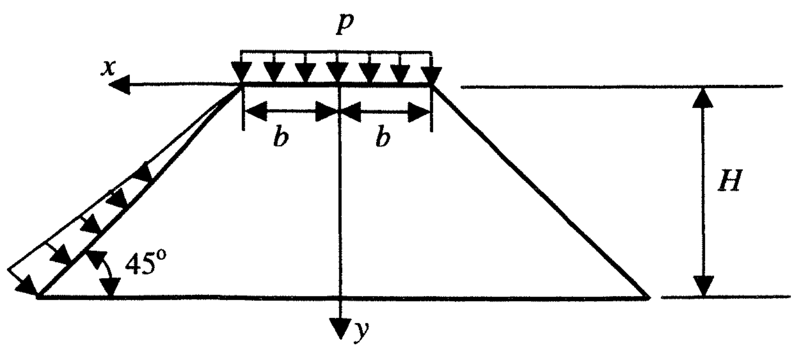

Fig. 1

2 A compound cylindrical disk consists of two concentric disks which nest inside each other. Before assembly, the outer radius b of the inner disk (with inner radius a) is made larger by an amount δ than the inner radius of the outer disk (with outer radius c). After assembly, a contact pressure p_c is developed between the two cylindrical disks. The two disks are made of the same material, with Young's modulus E and Poisson's ratio ν, which yields according to the Tresca criterion. Plane stress is assumed for the compound disk.

(a) Find the circumferential strain in the compound disk assuming that both disks remain elastic. [25%]

(b) With help from the above solution, show that

$$\frac{p_c}{E} = \frac{\delta}{b}\left(\frac{b^2/a^2+1}{b^2/a^2-1} + \frac{c^2/b^2+1}{c^2/b^2-1}\right)^{-1}$$ [35%]

(c) An internal pressure p is now applied to the compound disk.

(i) Determine the complete stress field in the compound disk. [25%]

(ii) Find the conditions under which the two disks yield simultaneously due to the contact pressure p_c alone (ie. with $p = 0$). [15%]

(TURN OVER

3 (a) Show that

$$\phi = r\left[A\cos\theta + B\sin\theta + C\theta\cos\theta + D\theta\sin\theta\right]$$

is a valid Airy stress function. [20%]

(b) The stress function in (a) has been proposed to determine the elastic field in an elastic tapered cantilever beam with semi-angle α , loaded at one end as shown in Fig. 2. The beam has unit thickness into the plane of the paper.

(i) Write down the stress boundary conditions to be satisfied on the tapered faces AB and AC. [10%]

(ii) Show that stresses derived from $\phi(r,\theta)$ satisfy the above boundary conditions. [20%]

(c) For an end force P as shown in Fig. 2 determine the stresses σ_{rr} in the cantilever as a function of r and θ. [30%]

(d) Discuss under what conditions the stress σ_{rr} determined in (c) is a good approximation to the stress field in the beam. [20%]

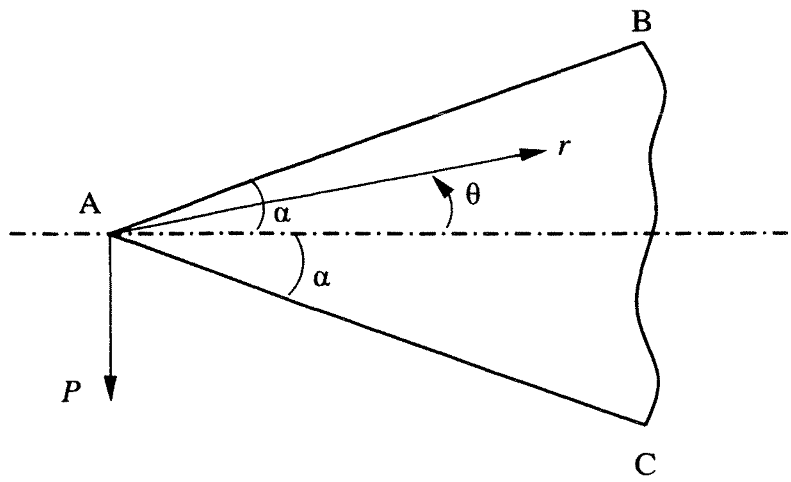

Fig. 2

4 (a) State briefly the upper and lower bound theorems of plasticity and discuss their applicability. [30%]

(b) Figure 3 shows an apparatus for the plane strain "back extrusion" of a slab in which the thickness is reduced from $6h$ to $2h$ by a pair of rigid $45°$ dies. The material being extruded has a yield strength k in pure shear, and friction is negligible.

(i) Using the upper bound method, with the lines of tangential velocity discontinuity shown as dotted, determine the total force F per unit width required to perform the extrusion. Give your answer in terms of the variable x shown in Fig. 3. [40%]

(ii) Using the estimate in (i) determine the minimum force required to perform the extrusion. [10%]

(iii) Determine the force F in terms of x if friction forces (friction factor f) acted on the sloping faces of the die. [20%]

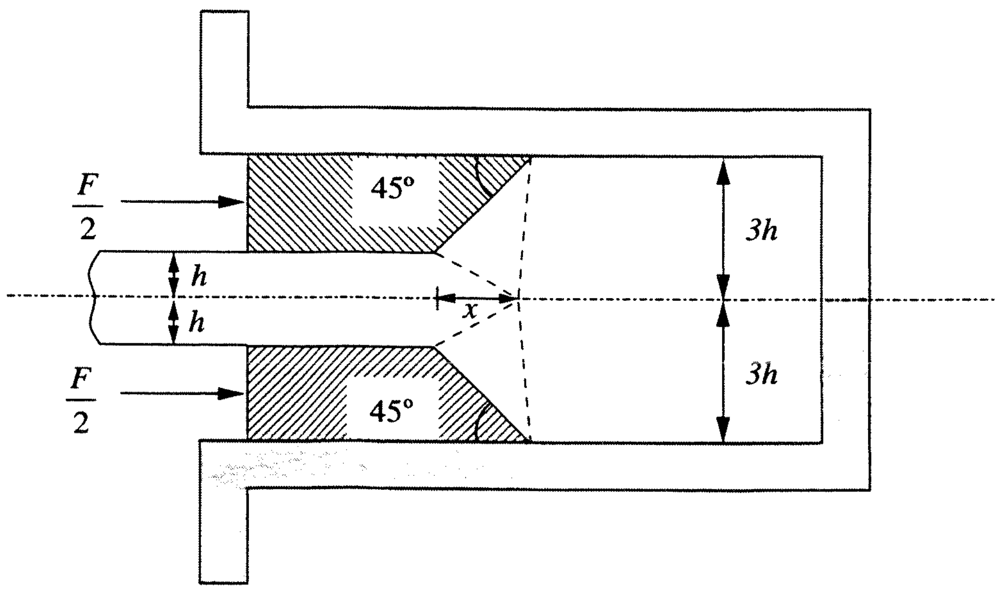

Fig. 3

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Answers for 3C7

1. (c) (ii) $\dfrac{C_2}{C_1} = \dfrac{1}{3}$ and $C_3 = C_1$

   (c) (iii) not possible

2. (a) inner cylinder $-\dfrac{p_c b^2}{E(b^2 - a^2)}\left(1 + \dfrac{a^2}{r^2} - \nu + \nu\dfrac{a^2}{r^2}\right)$

   outer cylinder $\dfrac{p_c b^2}{E(c^2 - b^2)}\left(1 + \dfrac{c^2}{r^2} - \nu + \nu\dfrac{c^2}{r^2}\right)$

   (c) (ii) $\dfrac{a}{b} = \dfrac{b}{c}$

3. (c) $\sigma_{rr} = -\dfrac{2P\sin\theta}{r(\sin 2\alpha - 2\alpha)}$

4. (b) (i) $\dfrac{F}{2k} = \dfrac{4(x^2 - hx + 4h^2)}{h + x}$

   (b) (ii) $F_{\min} \approx 15.2kh$