ENGINEERING TRIPOS PART IIA

Friday 9 May 2003 2.30 to 4.00

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A beam has a cross-section relative to the t, s axes as shown in Fig. 1. The lower edge is defined by the line $t = 10s^2$. The centroid G is located at s = 0.075 m, t = 0.24 m. The cross-sectional area A = 0.0533 m^2 and the second moments of area about the s- and t-axes are $I_{ss} = 0.00365$ m^4 and $I_{tt} = 0.000427$ m^4 respectively.

(a) Evaluate the product second moment of area I_{xy}. [20%]

(b) By plotting a Mohr's circle of second moment of area (or otherwise) find the principal axes of the section and the corresponding principal second moments of area. Show these on a sketch of the section. [30%]

(c) A 3 m long beam of this cross-section is mounted as a cantilever. It is made from a material with Young's Modulus E, and is loaded by a force P acting along the positive y direction through G at its tip. Calculate the magnitude and direction of the tip deflection and illustrate the deflection with a sketch. [50%]

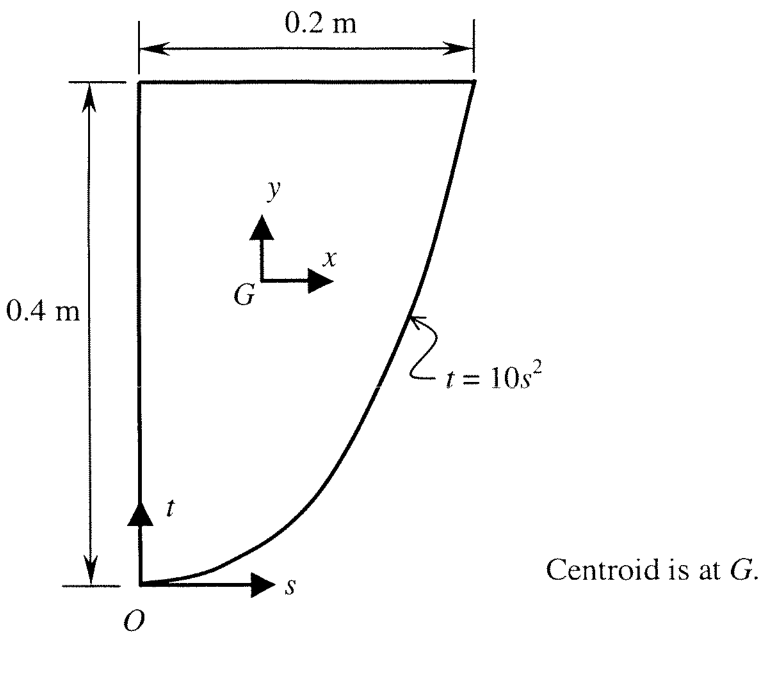

Fig. 1

2 A beam of length $3L$ is simply-supported at its two ends. The beam is continuous over an intermediate support which creates two spans, of length L and $2L$. It is loaded by a distributed load, of intensity w per unit length, which extends from the middle of one span to the middle of the other. The beam has a uniform flexural stiffness EI and is unstressed before the application of the load. In answer to the questions that follow, set up the appropriate equations but do not attempt to solve the resulting sets of simultaneous equations.

(a) Use Macaulay's method to produce a set of simultaneous equations that can be used to determine the support reactions. Indicate clearly which are the unknown variables in your equations. [50%]

(b) How would these equations be modified if the central support deflected by a fixed amount δ? [25%]

(c) How would your equations be altered if you wished to determine the influence line for bending moment at the centre of the shorter span. [25%]

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3 (a) Explain briefly why general elastic stability problems always lead to eigenvalue problems, and explain the physical significance of the eigenvalues and eigenvectors. [20%]

(b) Fig. 2 shows a strut made up of three rigid weightless bars each of length a. It is loaded axially by a force P. All joints are pinned. At internal joint B, a rotational spring connects the two bars on either side of the joint, and applies a couple of magnitude $G\theta_B$ between the two bars AB and BC, θ_B being the angle between the two bars. At the internal joint C a linear spring of stiffness k connects the pin to a rigid foundation. Both the linear spring and the rotational spring are unstressed when the vertical displacements w_B and w_c are zero. Vertical loads Q_B and Q_C are applied at joints B and C as shown. All deformations are confined to the plane of the paper.

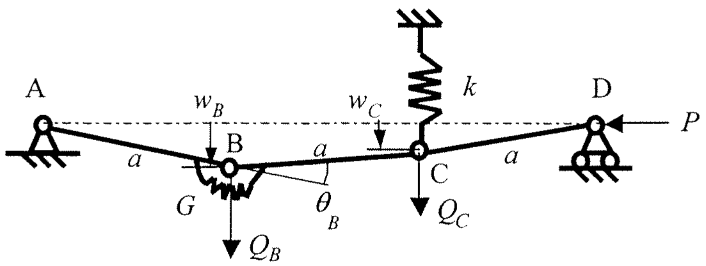

Fig. 2

i) For a general (but small) displacement of this structure, show that the total potential energy Π may be written as the sum of a linear plus a quadratic part, viz:

$$\Pi(w) = -w^T Q + \frac{1}{2} w^T K_{TOT} w$$

where $Q = [Q_B, Q_C]^T$, $w = [w_B, w_C]^T$ and K_{TOT} is a 2×2 matrix. Express K_{TOT} in terms of P, a, k and G. [40%]

(cont.

ii) If $G = \frac{1}{3}ka^2$ then $K_{TOT} = \begin{pmatrix} c & d \\ d & c \end{pmatrix}$, where $c = (4ka - 6P)/3a$ and $d = (-2ka + 3P)/3a$. Determine, as a function of k and a, the critical load P_{cr} at which buckling will first occur and the eigenvectors at that load. Explain briefly what buckling mode shape you would expect to see when $Q_B = Q_C = 0$, giving reasons. [40%]

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4 (a) Consider a simply-supported beam of length L with minor-axis flexural rigidity EI_y and torsional rigidity GJ. Let both ends be restrained against rotation about the longitudinal axis and let the flanges be unrestrained against warping.

The critical value M_{cr} of equal and opposite major-axis end-couples that will cause lateral-torsional buckling is then given by

$$M_{cr} = \frac{\pi}{L}\sqrt{GJEI_y}\left(1+\frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right)^{1/2}$$

where Γ is the warping constant and may be taken as equal to $I_y D^2/4$ for a Universal Beam, where D is the distance between flanges.

(i) Show that, for short spans, the critical value M_{cr} corresponds to that which causes Euler buckling of the compression flange about its own major axis. [30%]

(ii) Determine the value of M_{cr} in kNm for a $457 \times 152 \times 74$ kg/m steel Universal Beam of length 8 m, simply supported as described above. [30%]

(iii) Briefly explain what is meant by warping. The above equation for M_{cr} includes the effect of warping by means of the second term in the brackets. Since this term is positive, explain why including warping (which is a freedom to deform) appears to lead to a *higher* predicted critical moment than when warping is ignored. [20%]

(b) The "Column Paradox" arose when experiments on the buckling of columns made of *inelastic* materials showed better agreement with the Tangent Modulus Formula (based on assumptions of *elastic* material behaviour) than with the Double Modulus Formula (based on assumptions of *inelastic* material behaviour). Briefly explain the essential insight shown by Shanley which resolved this paradox. [20%]

END OF PAPER

3D4 2003. Answers:

1. (a) 0.0001067 m^4 , (b) 0.000602 m^4 , 0.000102 m^4 ,
   (c) $24.1 \times 10^3 P/E$ at 40.2° to vertical.
3. (c) $P_{cr} = 2ka/3$
4. (a) (ii) 156 kNm