ENGINEERING TRIPOS PART IIA

Saturday 10 May 2003 2.30 to 4.00

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) In the game of craps, the player rolls a pair of dice and sums the numbers showing. If the total is 7 or 11 on the first roll the player wins, whereas if the total is 2, 3 or 12 the player loses. Any other number is called the 'point'. In such a case the player must roll the dice again. If he rolls the same 'point' number, he wins. If he throws a 7, he loses. Any other number requires another roll. The process continues until either a 7 or the 'point' is thrown. Describe a Markov chain representing the game of craps process. In particular define the states and the events, draw the transition network and calculate the transition matrix. [40%]

(b) Consider the following transition matrix (%):

From / To	A	B	C	D	E
A	20	80	0	0	0
B	0	90	10	0	0
C	30	0	20	10	40
D	0	0	10	90	0
E	0	25	0	0	75

(i) Draw the transition network and determine the classes. Does the Markov chain have absorbing classes? [20%]

(ii) If the system is initially in state B, calculate the transient probability vector of the states after two, and after three transitions. [20%]

(iii) Are the sufficient conditions for convergence of the transient probabilities to steady-state probabilities satisfied? Explain. If so, calculate the limiting distribution. [20%]

2 (a) Explain the meaning of Kendall's notation (U/V/s/k/W) for the conventional description of a queuing system. What are the implicit assumptions when Kendall's notation is used? [15%]

(b) Define the utilization factor and the term 'steady state' of a queuing system. Does every queuing system eventually reach a steady state? Explain. [15%]

(c) Define the lack of memory property of a random variable. What is the reason for the prevalence of the exponential distribution in queuing models? [10%]

(d) Consider an M/M/1 queue. Derive formulas for the expected number of customers (L) in the system, the expected queue length (L_q), the average waiting time (W) for a customer in the system, and the average waiting time (W_q) for a customer in the queue. [40%]

(e) Define a Poisson arrival process and state (but do not derive) formulas for the mean and variance of the number of customers arriving in the time interval [0,t]. [20%]

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3 Consider the linear regression framework and do the following tasks:

(a) Derive the equations for determining the least squares estimates of the intercept and slope of the linear model $y = a + bx + \varepsilon$. [20%]

(b) Explain why the least squares estimates of the slope and intercept of the line are random variables. [20%]

(c) Derive formulas for the expected value and variance of the estimate of the slope of $y = a + bx + \varepsilon$. [40%]

(d) Calculate the least squares line $y = a + bx + \varepsilon$ for the following data: [20%]

x	y
39	144
47	220
45	138
47	145
65	162
46	142
67	170
42	124
67	158
56	154
64	162
56	150
59	140
34	110
42	128

4 A queuing system with 3 servers is observed for a long period of time and data are collected on the proportion of time the system is in each of the states. Assume an infinite calling population. Capacity is limited, so whenever there is an arrival when 6 customers are present in the system (three customers being served and three waiting in queue), the arriving customer balks and goes elsewhere for service. Each state, denoted by n ($0 \leq n \leq 6$), represents the number of customers present in the system. In the following table estimates of steady state probabilities are given.

State, n	Probability, π_n
0	0.068
1	0.170
2	0.212
3	0.177
4	0.147
5	0.123
6	0.103

(a) What is the probability that all servers are idle? [10%]

(b) What is the probability that a customer will not have to wait? [15%]

(c) What is the probability that a customer will have to wait in the queue? [15%]

(d) What is the probability that an arriving customer will be lost? [15%]

(e) What is the expected number of customers in the queue? [15%]

(f) What is the expected number of customers in service? [15%]

(g) What is the utilization of the servers? [15%]

END OF PAPER

3E3 Modelling Risk
Exam Paper 2003

Short Crib

1(a)

From / To	Start	Win	Lose	P4	P5	P6	P8	P9	P10
Start	0	0.222	0.111	0.083	0.111	0.139	0.139	0.111	0.083
Win	0	1	0	0	0	0	0	0	0
Lose	0	0	1	0	0	0	0	0	0
P4	0	0.083	0.167	0.75	0	0	0	0	0
P5	0	0.111	0.167	0	0.722	0	0	0	0
P6	0	0.139	0.167	0	0	0.694	0	0	0
P8	0	0.139	0.167	0	0	0	0.694	0	0
P9	0	0.111	0.167	0	0	0	0	0.722	0
P10	0	0.083	0.167	0	0	0	0	0	0.75

1(b)(i)
N/A

1(b) (ii)
[0.039, 0.763, 0.102, 0.020, 0.074].

1(b) (iii)
[0.034, 0.638, 0.091, 0.091, 0.145]

3(d)
$a = 95.612$
$b = 1.047$

4(a) 0.068

4(b) 0.450

4(c) 0.447

4(d) 0.103

4(e) 0.702

4(f) 2.244

4(g) 74.8%