ENGINEERING TRIPOS PART IIA

Saturday 10 May 2003 9 to 10.30

Module 3F4

DATA TRANSMISSION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain why inter-symbol interference (ISI) may occur in a baseband digital transmission system and what is the effect of ISI on system bit error rate (BER) performance.

Briefly explain why an equaliser is often introduced ahead of the data slicer and discuss the relative merits of *zero-forcing* and *minimum mean squared error* (MMSE) approaches for equaliser design. [30%]

(b) The 3-tap finite impulse response (FIR) filter of Fig. 1 is to be used to equalise a binary uni-polar transmission system, having the unit pulse response shown in Fig. 2, where T is the bit period and the sampling time is such that it maximises the value of the pulse response in each bit period. Assuming the zero-forcing approach, calculate the filter coefficients, $b_0 \ldots b_2$, such that the equalised output in response to a single received pulse is ideally a single unit-valued sample.

Determine the BERs with and without equalisation if the channel noise is white and Gaussian with a standard deviation of 0.1 V and a mean of zero. [40%]

(c) Calculate the BER if instead a decision-feedback equaliser is used in the above system. State any assumptions made. [30%]

Note the Gaussian error integral function:

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

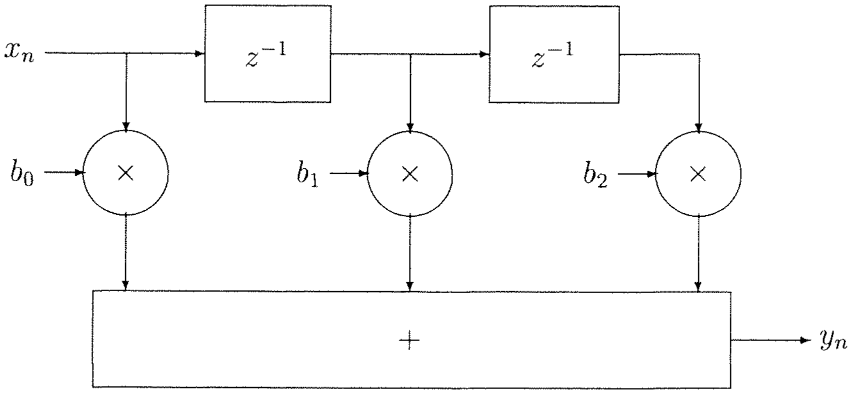
x_n
z^{-1}
z^{-1}
b_0
b_1
b_2
×
×
×
+
y_n

Fig. 1

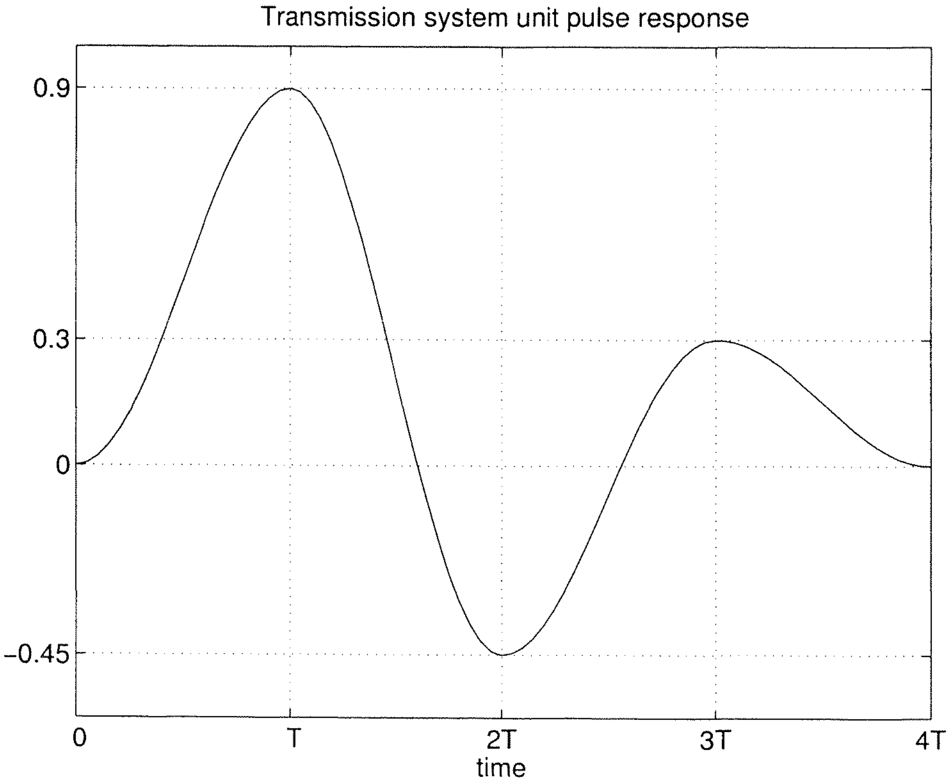
Transmission system unit pulse response
0.9
0.3
0
-0.45
0
T
2T
3T
4T
time

Fig. 2

2 (a) A pulse amplitude modulated (PAM) communication system transmits data in the form of a weighted impulse train

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \, \delta(t - nT_s)$$

where a_n are the line-coded symbols and T_s is the symbol period.

(i) Show that the power spectral density (PSD) of the transmitted symbols is

$$S_x(\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} R(m) \; \exp(jm\omega T_s)$$

where $R(m) = E[a_n \, a_{n+m}]$. [30%]

(ii) A bipolar line coding scheme, with nominal signal levels of -1 V, 0 V and 1 V, is used to generate the transmitted symbols. Unfortunately a fault in the line coding circuit results in a d.c. shift of $+0.2$ V in the actual transmitted voltage levels. Determine an expression for the PSD of the transmitted signal, assuming that the input data bits are random with equal probability of ones and zeros. [30%]

(b) Prior to transmission, the line-coded signal is passed through a filter with the following impulse response

$$h(t) = \begin{cases} \cos(\pi t/T_s) & \text{for } -T_s/2 < t < T_s/2 \\ 0 & \text{elsewhere.} \end{cases}$$

Determine the PSD of the transmitted signal and sketch the result.

What advantages does this pulse shape offer compared with a rectangular pulse shape of equal duration? [40%]

3 (a) Define the basic criterion for choosing the optimum demodulated bit for the k^{th} bit period in a demodulator for a binary phase-shift keyed (BPSK) signal in noise; you should assume that the bit period is T_b, the received signal phasor waveform is $r(t)$, and the signal phasors corresponding to the two possible transmitted bit states are known to be $\pm g(t - kT_b)e^{j\phi_0}$.

Hence show how this criterion may be simplified to result in a matched correlation demodulator. [30%]

(b) The voltage signal-to-noise ratio at the threshold detector of such a demodulator can be shown to be given by

$$\frac{v_s}{\sigma} = \sqrt{\frac{2E_b}{N_0}}$$

Explain the meaning of the symbols in this equation. Calculate the bit error rate at the detector output if the bit rate is 56 kbit s^{-1} and, at the demodulator input, the signal phasor amplitude is 2 V and the noise power spectral density is 4×10^{-5} V^2 Hz^{-1}. [25%]

(c) Estimate the approximate bandwidth of this BPSK modulated signal, between -3 dB points of the spectrum. [20%]

(d) If it is desired to transmit this data through a radio channel of bandwidth 15 kHz, suggest an appropriate alternative modulation method, giving reasons and key parameters. Indicate qualitatively what effect this design change would have on the bit error rate of the system. [25%]

Note the Gaussian error integral function:

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

(TURN OVER

4 (a) Show how the modulation components of an amplitude and phase modulated signal of the form

$$s(t) = a(t)\cos(\omega_C t + \phi(t))$$

can be represented by a phasor waveform $p(t)$. What are the advantages of this form of representation for analysis of digital modulation schemes?

Write down an expression for the phasor waveform of a quadrature phase-shift keyed (QPSK) signal and sketch the modulation phasors on a 2-dimensional argand diagram. [30%]

(b) A Digital Audio Broadcast (DAB) signal employs QPSK modulation on a coded orthogonal frequency division multiplexed (COFDM) set of carriers. Determine the maximum bit rate that would be available to the user for a system with the following parameters:

Channel bandwidth: 1.6 MHz
Carrier frequency spacing: 800 Hz
Symbol rate on each carrier: 600 symbol s^{-1}
Error-correction code rate: 1:2

Suggest reasons why QPSK modulation is a good choice for COFDM systems designed for reception of good quality digital audio in vehicles. [45%]

(c) Practical transmission channels exhibit multipath effects. Giving reasons, estimate the maximum path delay variation that could be tolerated by the above system before the bit error performance against noise starts to become degraded. [25%]

END OF PAPER

Module 3F4, May 2003

DATA TRANSMISSION

Answers

1 (b) $b_0 = 1.1111, \;\; b_1 = 0.5556, \;\; b_2 = -0.0926$

Without equalisation, BER $= 0.2267$; with equalisation, BER $= 0.00109$

(c) DFE BER $= 3.4 \,.\, 10^{-6}$

2 (a) (ii) $$S_x(\omega) = \frac{0.04 \,.\, 2\pi}{T_s^2} \sum_{m=-\infty}^{\infty} \delta\left(\omega - m\frac{2\pi}{T_s}\right) + \frac{1}{T_s} \sin^2\left(\frac{\omega T_s}{2}\right)$$

(b) $$S_y(\omega) = S_x(\omega) \left| \frac{T_s}{2} \left[\operatorname{sinc}\left(\frac{\omega T_s - \pi}{2}\right) + \operatorname{sinc}\left(\frac{\omega T_s + \pi}{2}\right) \right] \right|^2$$

3 (a) Simplified result (matched correlation demodulator):

Detect polarity of $y(k) = \displaystyle\int_{kT_b}^{(k+1)T_b} Re[r(t)\; g(t - kT_b)\; e^{-j\phi_0}]\; dt$

(b) BER $= 0.0295$

(c) Bandwidth $\simeq$ 50 kHz

(d) 16-QAM

4 (a) QPSK phasor waveform for k^{th} symbol period:

$$p_k(t) = [b_{2k} + j\; b_{2k+1}]\; g(t - kT_s)\; e^{j\phi_0} \qquad \text{where } b_i = \pm 1$$

(b) User bit rate $= 1.2$ Mb s^{-1}

(c) Max path delay variation $\simeq 0.42$ ms