ENGINEERING TRIPOS PART IIB

Module 3C6 Examination, 2004

**Answers**

1. See crib

2. (a) $w = 0 \quad and \quad \dfrac{\partial^2 w}{\partial x^2} = 0 \quad$ at $x = 0$ and $x = L$;

Modes: $u = \sin\left(\dfrac{n\pi x}{L}\right); \quad \omega_n = \left(\dfrac{n\pi}{L}\right)^4 \dfrac{Eh^2}{12\rho}$

(b), (c) See crib

(d) With this approximation, a tapered beam has same natural frequencies as a uniform beam of thickness $h_0 + \varepsilon L/2$.

3. (a) $[M] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \quad [K] = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad with \quad k = P/L$

(b) $\omega_1^2 = \left(2-\sqrt{2}\right)\dfrac{k}{m}; \quad \omega_2^2 = 2\dfrac{k}{m}; \quad \omega_1^2 = \left(2+\sqrt{2}\right)\dfrac{k}{m}$

(c) 4.7% decrease in lowest natural frequency

4. (a) $[M] = m\begin{bmatrix} 2 & -L/2 & L/2 \\ -L/2 & L^2/3 & 0 \\ L/2 & 0 & L^2/3 \end{bmatrix} \quad [K] = k\begin{bmatrix} 3 & -L & L \\ -L & L^2 & 0 \\ L & 0 & L^2 \end{bmatrix}$

(b) $\omega_1^2 = \left(3-\sqrt{3}\right)\dfrac{k}{m}; \quad \omega_2^2 = 3\dfrac{k}{m}; \quad \omega_1^2 = \left(3+\sqrt{3}\right)\dfrac{k}{m}$

(c), (d) See crib