ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3A1
Fluid Mechanics I

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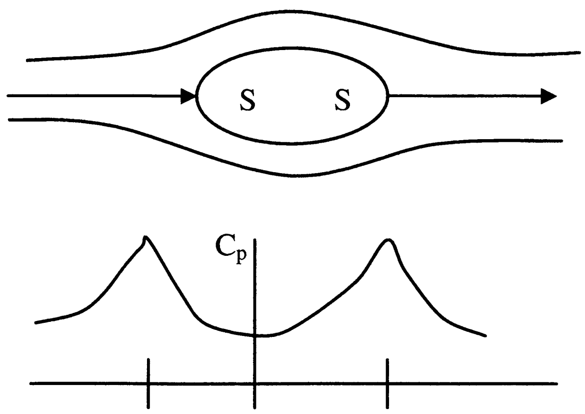
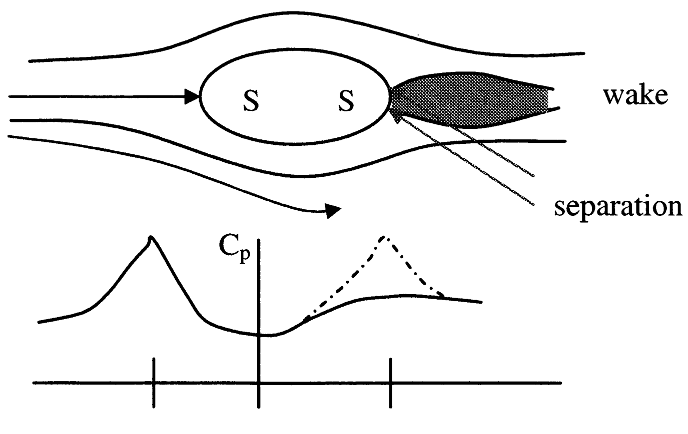
3A1: FLUID MECHANICS I EXAMINATION CRIBS

1. (a) (i) inviscid flow



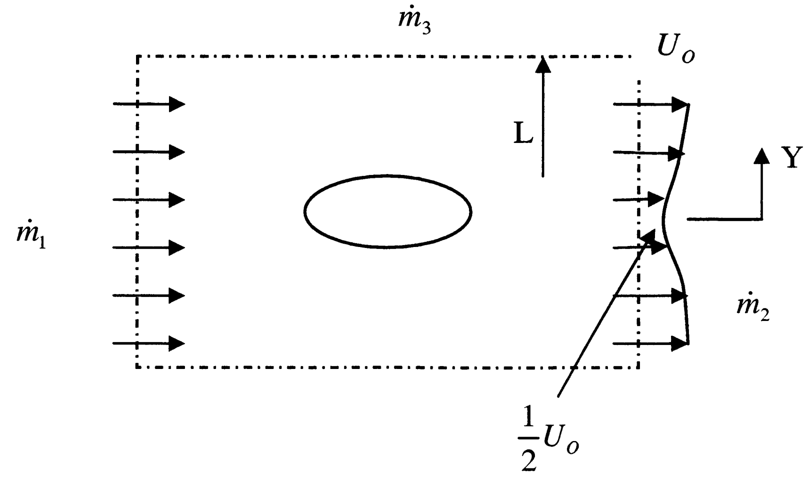
Symmetrical flow and pressure distribution

(ii) viscous flow



B'layer separation and wake; no rear stagnation point ⇒form drag

(b)



Mass: $\dot{m}_1 = 2\rho U_o L$

$$\dot{m}_2 = 2\int_o^L \rho u d\rho Y = 2\int \rho\left(\frac{u_o}{2} + \frac{\gamma}{L}\frac{u_o}{2}\right) dY = \frac{3}{2}\rho u_o L$$

$$\therefore \dot{m}_3 = \frac{1}{2}\rho u_o L$$

Momentum:

$$D = 2\int_o^L \rho u^2 dY\Big|_{IN} - 2\int_o^L \rho\left[\frac{U_o}{2}\left(1 + \frac{Y}{L}\right)\right]^2 dY - \dot{m}_3 U_o$$

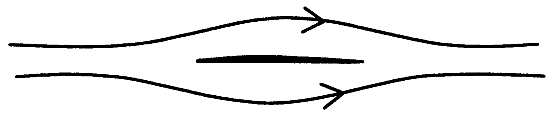
$$= 2\rho u_o^2 L - \frac{1}{2}\rho U_o^2 L - \rho\frac{U_o^2}{2}\left[Y + 2\frac{Y^2}{2L} + \frac{Y^3}{3L}\right]_o^L = \rho U_o^2 L\left(2 - \frac{1}{2} - \frac{7}{6}\right)$$

$$\therefore D = \frac{2}{3}\frac{\rho U_o^2}{2} L$$

$$\therefore D = 4.3 \quad KN/m \quad and \quad C_D = 2/3(L/D).$$

(c) two drag contributions: viscous drag due to skin friction over wetted surfaces
: form drag associated with lower pressure in separated wake region

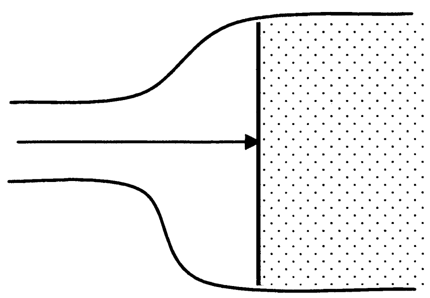
D<<L(~flat plate)



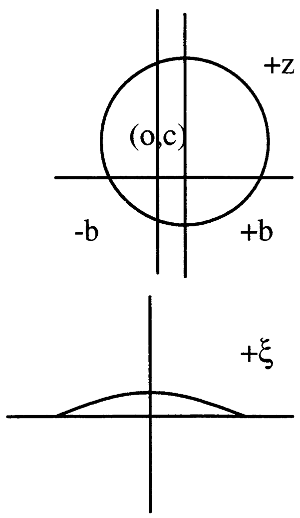
Mostly friction drag
C_D<<1

Mostly form drag from wake
C_D~O(1)

D>>L (eg bluff body)



2 (a)



Substitute z = ±b into the transformation:

$$z = +b \rightarrow \xi = b + b^2/b = 2b$$

$$z = -b \rightarrow \xi = -b + b^2/-b = -2b$$

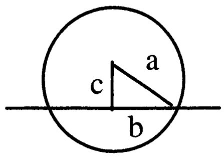
$\therefore$ chord = 4b

(b) $$F_c(z) = Ue^{-i\alpha}(z - ic) + \frac{Ua^2}{z - ic}e^{i\alpha} - \frac{i\Gamma}{2\pi}\log(z - ic)$$

TE velocity in the ξ-plane is $\left.\frac{dF_c}{d\xi}\right|_{z=b} = \frac{dF_c}{dz}\frac{dz}{d\xi} = \frac{dF_c/dz}{d\xi/dz}$

$$\frac{d\xi}{dz} = 1 - \frac{b^2}{z^2} = 0 \text{ @ } z = b \quad \therefore \quad \left.\frac{dF_c}{d\xi}\right|_{z=b} = 0 \quad also \quad (Kutta..)$$

$$\frac{dFc}{dt} = Ue^{-i\alpha} - \frac{Ua^2 e^{i\alpha}}{(z-ic)^2} \bullet \frac{i\Gamma}{2\pi} - \frac{1}{z-ic} = 0 \text{ @ } z = b$$

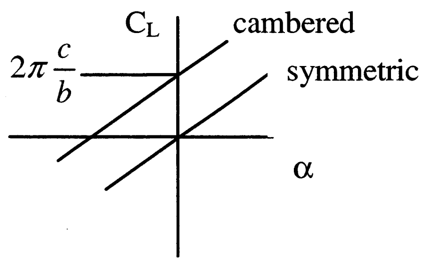


$a^2 = b^2 + c^2$ ----▶ $\frac{i\Gamma}{2\pi}\frac{1}{b-ic} = Ue^{-i\alpha} - \frac{Ua^2 e^{i\alpha}}{(b-ic)^2}$

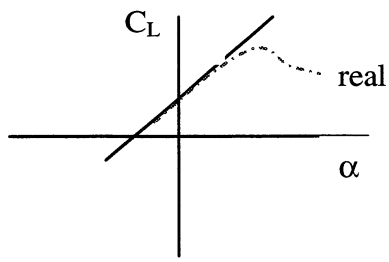
$$\therefore \Gamma = 4\pi U(b\sin\alpha + c\cos\alpha)$$

(c) $$C_c = \frac{\rho u \Gamma}{\frac{1}{2}\rho u^2 (4b)} = 2\pi\left(\sin\alpha + \frac{c}{b}\cos\alpha\right)$$

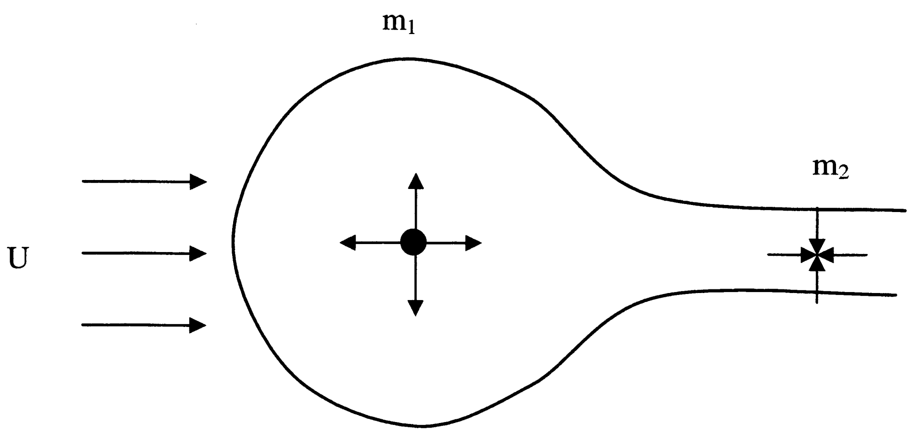
(d) (i) the airfoil is cambered so has lift even a zero angle of attack

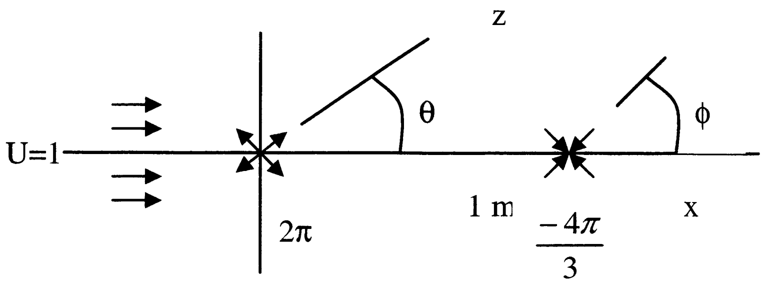


(ii) on a real airfoil, b'layer growth, especially on the suction side, would lead to b'layer blockage progressively reducing C_L and then, eventually, stalled flow.



3.





(a) $F(z)=Uz+\frac{m_1}{2\pi}\log(z)-\frac{m_2}{2\pi}\log(z-a)=z+\log(z)-\frac{2}{3}\log(z-1)$

(b) $\frac{dF}{dz}=1+\frac{1}{z}-\frac{2/3}{z-1}=0$ @ stagnation point

$\therefore z(z-1)+z-1-\frac{2}{3}z=0$

$\therefore z^2-\frac{2}{3}z-1=0$

$\therefore z=0.333\pm1.05\Rightarrow$ upstream stagnation @ (-0.72,0)

(c) need the streamline that passes through stagnation point:

$F(z)=\phi+i\psi,\quad F(z)=U(x+iY)+\log(re^{i\theta})-\frac{2}{3}\log(re^{i\phi})$ (r=1, $r=\sqrt{2}$)

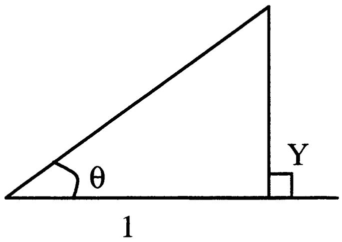
$\therefore \psi=UY+\theta-\frac{2}{3}\phi$

on the stagnation point $\left.\begin{matrix}Y=0\\ \theta=\pi\\ \phi=\pi\end{matrix}\right\}\therefore \psi^*=\frac{\pi}{3}$

hence body surface $\psi = \frac{\pi}{3} = UY + \theta - \frac{2}{3}\phi \qquad (U = 1)$

∴ d downstream θ, ϕ → 0 hence Y → π/3 ⇒ thickness →2π/3

(d) @ x = 1



$\phi = \frac{\pi}{2}$

Body:

$$\frac{\pi}{3} = UY + \theta - \frac{2}{3}\phi$$

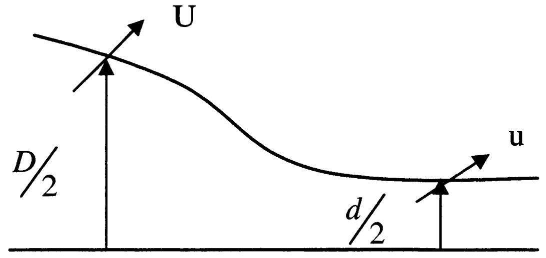
$$= UY + \tan^{-1} Y - \frac{\pi}{3}$$

hence by substitution the half-thickness @ X = 1 is ~ 1.21 m.

4. a) $\frac{D\Gamma}{Dt} = 0$; the circulation around a closed loop in an **in**viscid fluid is constant as the loop moves with the fluid (Kelvin).

b) Suppose at some time there was a region of rotational fluid, ie a region where vorticity $|\overline{w}| \neq 0$. A loop around this region would have non-zero circulation $\left(\Gamma = \oint \overline{u} \bullet d\overline{\ell} = \oint \overline{w} \bullet dS\right)$. Kelvin's heaven says that this loop must always have had this circulation. However, if the original state of the fluid was irrotational then $\overline{w}$ and hence Γ was zero. Hence, in the absence of viscosity, an irrotational flow must remain irrotational.

(c)



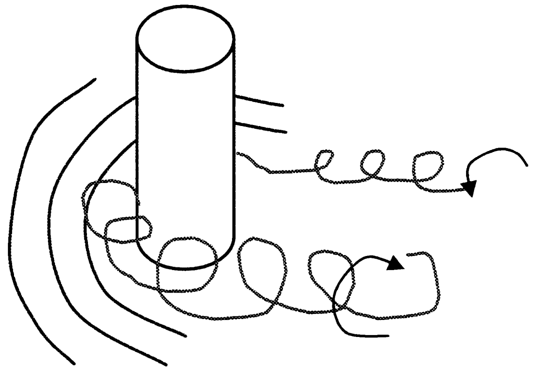
circulation,

$\Gamma = \oint \bar{u} \bullet d\bar{\ell} = \pi DU =$ constant

(outside b'layer). (Faster swirl in smaller diameter pipe.)

$$\therefore u = UD/d$$

d)



Vortex lines upstream are perpendicular to the flow (these are in the b'layer). As they approach the pole these lines "bunch up" and form a vortex tube. This tube is stretched around the front (upstream) side of the pole increasing the vorticity and, hence, the swirling velocity. This leads to intense velocities which scour away the snow. Downstream the vortices form long trailing airs that are not stretched much and are less intense.

5. (a) laminar boundary layer

$$\delta \sim (\nu t)^{\frac{1}{2}}, \quad t = x/U, \quad \delta \sim \left(\nu x/U\right)^{\frac{1}{2}} \quad \frac{\delta}{x} \sim \left(\frac{Ux}{\nu}\right)^{-\frac{1}{2}} = \mathrm{Re}_x^{-\frac{1}{2}} \Bigg\| \delta \sim x^{\frac{1}{2}}$$

Turbulent boundary layer – for more rapid growth; turbulent exchange of mass in contrast to molecular effect δ grows nearly linear with x | $\delta \sim x^{0.8}$ actually.

(b) velocity profile

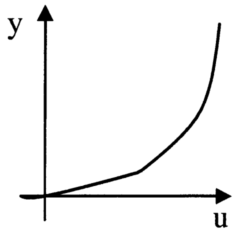
Laminar – smooth – nearly parabolic/cubic

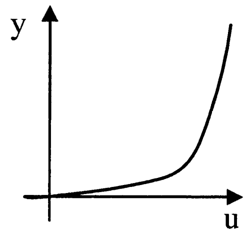
Turbulent – nearly uniform over much of the profile due to intense mixing; logarithmic region near surface; laminar sublayer very close to wall. Often approximated by $u \sim y^{1/7}$.

(c) velocity gradient near wall

following b) →small gradient for laminar ~ parabolic

→ large gradient for turbulent due to near uniform and laminar sublayer





d) momentum deficit will be proportional to δ; surface stress proportional to derivative of momentum deficit ∴ stress ~ rate of growth of δ.
Stress – small for laminar
Stress – large for turbulent

e) <u>laminar</u> $q \sim k\dfrac{\Delta T}{\delta_T}; h \sim \dfrac{k}{(\alpha t)^{\frac{1}{2}}} \sim \dfrac{k}{\alpha\left(\dfrac{x}{U}\right)^{\frac{1}{2}}}$ $\quad h \sim \dfrac{kU^{\frac{1}{2}}}{\alpha x^{\frac{1}{2}}}$

<u>turbulent</u> much larger heat transfer coefficient due to large gradient close to wall (due to more uniform profile elsewhere).

f) separation is a result of the adverse pressure gradient (or the external flow decelerating) and these effects, being transferred down through the boundary layer directly by the pressure. This tendency to separate is mitigated by the shear stresses in the fluid dragging forward the slow moving fluid near the wall.

The internal shear stress is much larger in the turbulent flow due to mass exchanges than in the turbulent flow due to molecular effect, (viscosity).

6. (a) $\delta \sim (\upsilon t)^{\frac{1}{2}} \qquad t \sim x/U_\infty$

$$\delta \sim \left(\upsilon x/U_\infty\right)^{\frac{1}{2}} \qquad \frac{\delta}{x} \sim \mathrm{Re}_x^{-\frac{1}{2}}$$

$$\delta\big|_{x=L} \sim \left(\upsilon L/U_\infty\right)$$

(b) $\delta_T \sim (\alpha t)^{\frac{1}{2}} \qquad t \sim x/U_\infty$

$$\delta_T \sim \left(\alpha\, x/U_\infty\right)^{\frac{1}{2}} \qquad \upsilon = \alpha$$

$$\delta_T \sim \left(\upsilon L/U_\infty\right)^{\frac{1}{2}}$$

(c) $\upsilon = \alpha$

$$\tau_o \sim \mu\, U_\infty/\delta$$

$$q \sim k\frac{\Delta T}{\delta_T}$$

$$h = q/\Delta T \sim k/\delta_T$$

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho U_\infty^2} \sim 2\frac{\mu U_\infty}{\delta \rho U_\infty^2} \qquad St = \frac{h}{\rho c_p U_\infty} \sim \frac{k}{\delta_T} \bullet \frac{1}{\rho c_p U_\infty}$$

$$C_f \sim 2\, {}^{\upsilon}\!/\!_{\delta U_\infty} \qquad St \sim \frac{\alpha}{\delta_T U_\infty}$$

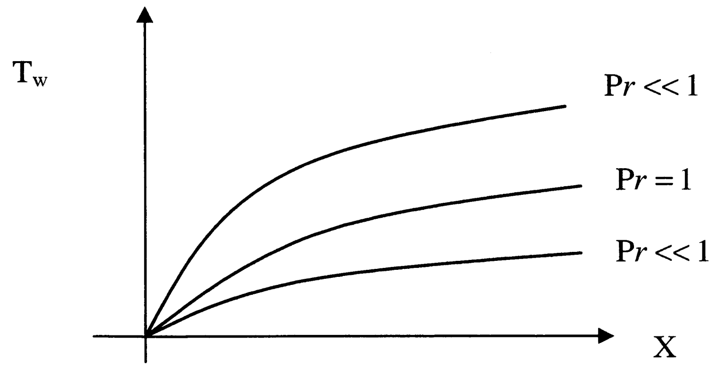
$$\upsilon = \alpha \qquad \delta = \delta_T$$

The ~ in both arguments are equivalent and, therefore, $St=C_f/2$

(d) $$q \sim k\frac{\Delta T}{\delta_T}$$

$$\Delta T \sim \frac{q\delta_T}{k} \sim \frac{q\left(\alpha\, {}^{x}\!/\!_{U_\infty}\right)^{\frac{1}{2}}}{k} = \frac{q}{\rho c_p}\left(\frac{x}{\alpha U_\infty}\right)^{\frac{1}{2}}$$

$$T_{wall} - T_{ambient} \sim \frac{q}{\rho c_p}\left(\frac{x}{\alpha U_\infty}\right)^{\frac{1}{2}}$$



(e)

$$\nu/\alpha << 1; \quad \delta << \delta_T$$

$$\delta_T \sim \left(\alpha \frac{x}{U_\infty}\right)^{\frac{1}{2}}$$

analysis follows d)

$$T_{wall} - T_{ambient} = \frac{q}{\rho c_p}\left(\frac{x}{\alpha U_\infty}\right)^{\frac{1}{2}}$$

(f) $\nu/\alpha >> 1; \quad \delta >> \delta_T$

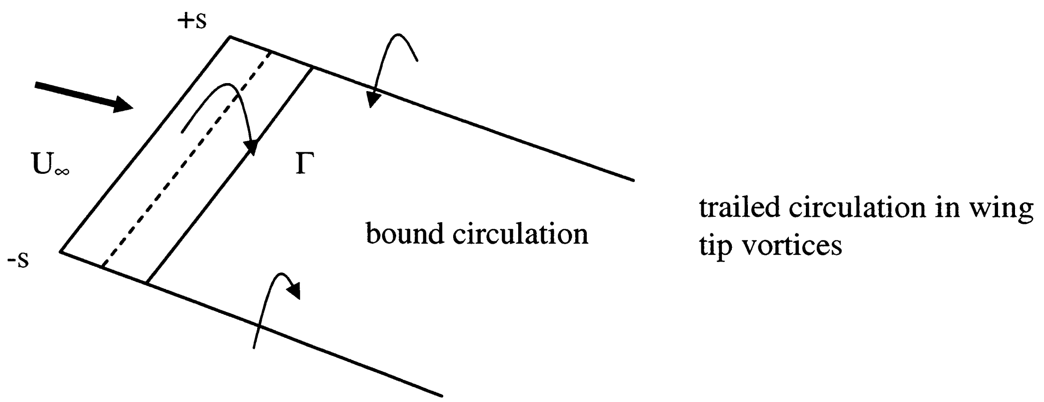
Two effects

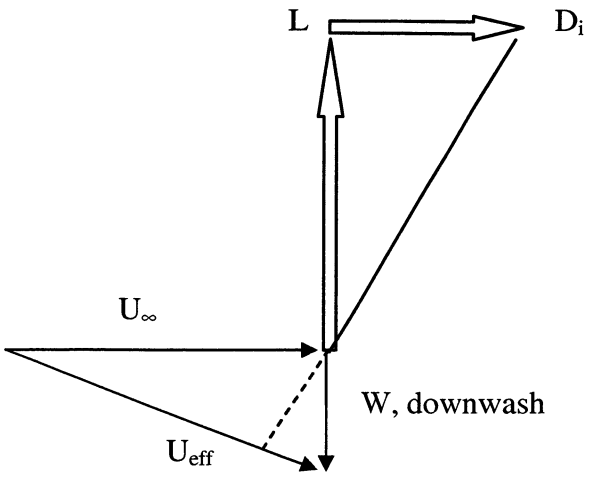
a) Thermal boundary layer will be much smaller and following previous arguments $T_{wall} - T_{ambient}$ whill be larger.

b) However the velocity of the edge of the thermal boundary layer will not be U_∞ but will be smaller (by $\alpha^{\frac{1}{2}}$ if the velocity profile was linear.

Thus $T_{wall} - T_{ambient}$ will be increased.

7. (i)

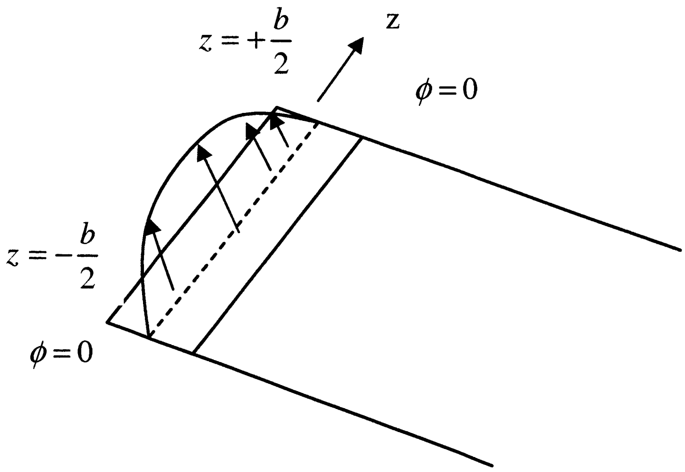




- The trailed vortex structure induces a downwash, w, along the wing which rotates the effective flow direction.
- Lift generated by circulation is perpendicular to the flow velocity vector – hence the downwash generates a component of lift in the direction of the undisturbed free-stream – this is the so-called **induced drag**.
- Physically, the trailed tip vortices represent secondary kinetic energy which dissipates into entropy at rate ~ $D_i U_\infty$
- $\left.\begin{matrix} w \sim \Gamma \quad L \sim u\Gamma \therefore D_i \sim w\Gamma = \Gamma^2 \\ w \sim \dfrac{1}{s} \qquad\qquad D_i \sim \dfrac{1}{AR} \end{matrix}\right\} D \sim L^2/AR$

  so for fixed chord aspect ratio = s/c

(ii) for general distribution $\Gamma(\phi) = 2bU_\infty \sum_{i=1}^{n} A_i \sin i\phi$ with $z = -\frac{b}{2}\cos\phi$ $\left.\begin{matrix} z = -\frac{b}{2} \to +\frac{b}{2} \\ \phi = 0 \to \pi \end{matrix}\right\}$



induced drag, $D_i = \rho \int_{-b/2}^{+b/2} \Gamma(z_o) w(z_o) dz_o$

$$w(z) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma}{dz} \bullet \frac{dz}{z - z_o}$$

8. (i) - an airfoil lifts because it has associated circulation, Γ

- for an initially undisturbed flow Kelvin's theorem states $D\Gamma/Dt=0$, ie the circulation wrto. a given loop does not change

- hence the lift on an airfoil is balanced by the shed circulation of a "starting vortex" this vortex is made possible by intense viscous action at the airfoil TE

Hence: $$D_i = \rho \int_o^{\pi} \underbrace{\left(2bU_\infty \sum_i A_i \sin i\phi \right)}_{\Gamma} \underbrace{\left(U_\infty \sum_i iA_i \frac{\sin i\phi}{\sin\phi} \right)}_{w} \underbrace{\frac{b}{2}\sin\phi d\phi}_{dz}$$

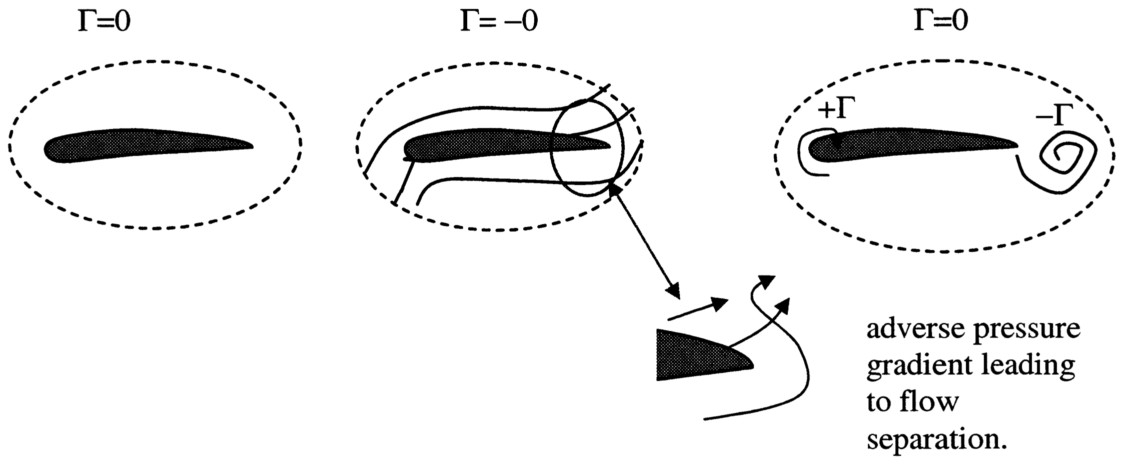
$$= \rho b^2 U_\infty^2 \int_o^{\pi} \underbrace{\sum_i A_i \sin i\phi \sum_j jA_j \sin i\phi A\phi}_{\substack{=\pi/2 \text{ if } i=j \\ =0 \text{ if } i\neq j}}$$

$$\therefore D_i = \frac{\pi}{2}\rho b^2 U_\infty^2 \sum_{i=1}^{n} iA_i^2$$

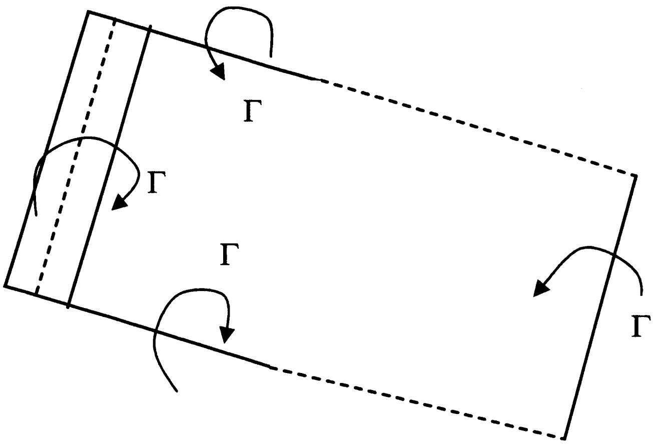
$$= \underbrace{\frac{\pi}{2}\rho b^2 A_1^2 U_\infty^2}_{L} \left[1 + \underbrace{\sum_{i=2}^{n} i\frac{A_i^2}{A_1^2}}_{+ve} \right]$$

Hence D_i minimum if $A_i = 0$ for i = 2→n

Hence minimum drag for elliptic loading.

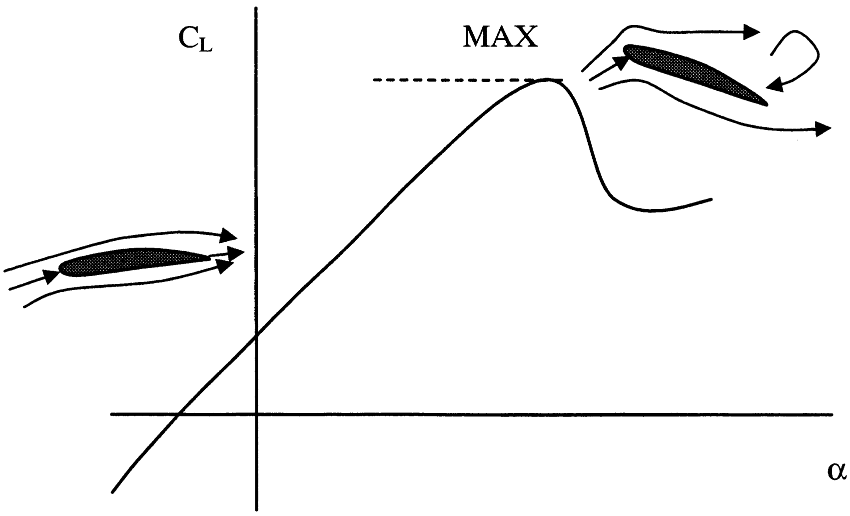


- a finite wing has a horseshoe-type vortex structure which causes induced drag via an induced downwash on the wing ⇒ for the designer this forces compromises over the wing aspect ratio.

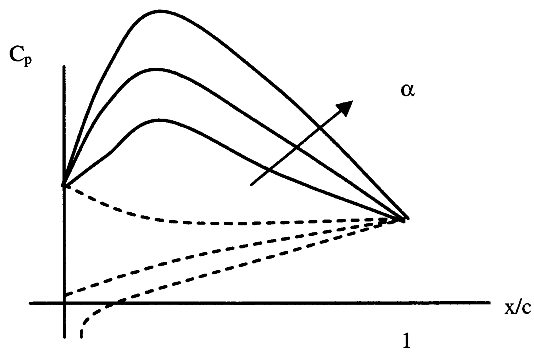


(ii) - stall occurs at large angles of attack – just after maximum lift ⇒ clearly a key design concern as maximum lift is like to be at take-off!!!

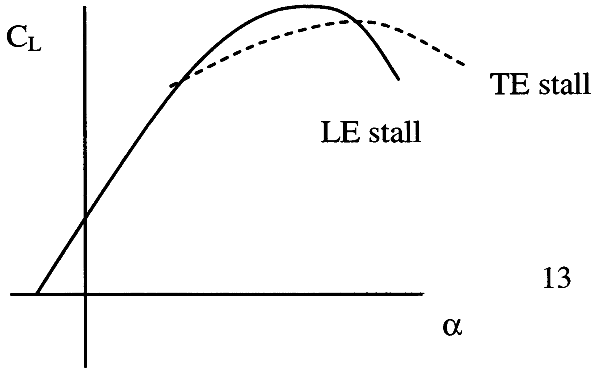
- it is a much better design if stall occurs gradually (with some "warning") rather than in an abrupt manner.



- Key design issue is diffusion over the aft of the airfoil suction surface ⇒ eventually boundary layer can withstand no further diffusion.
- Key design compromise is lift (ie difference between suction and pressure side pressures) and drag (associated with diffusion levels – and risk of stall-along airfoil

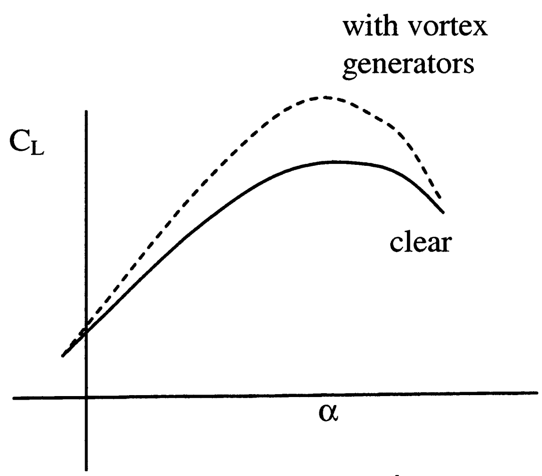


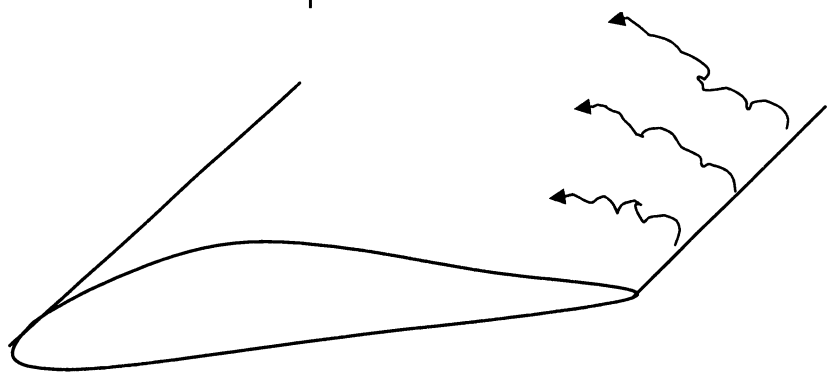
- Additional complications arise from the state of the boundary layer: a laminar layer separates more readily but many then undergo transition to a turbulent boundary which reattaches (result = laminar bubble) or which simply stalls; a turbulent layer can withstand adverse pressure gradients better but produces more drag.



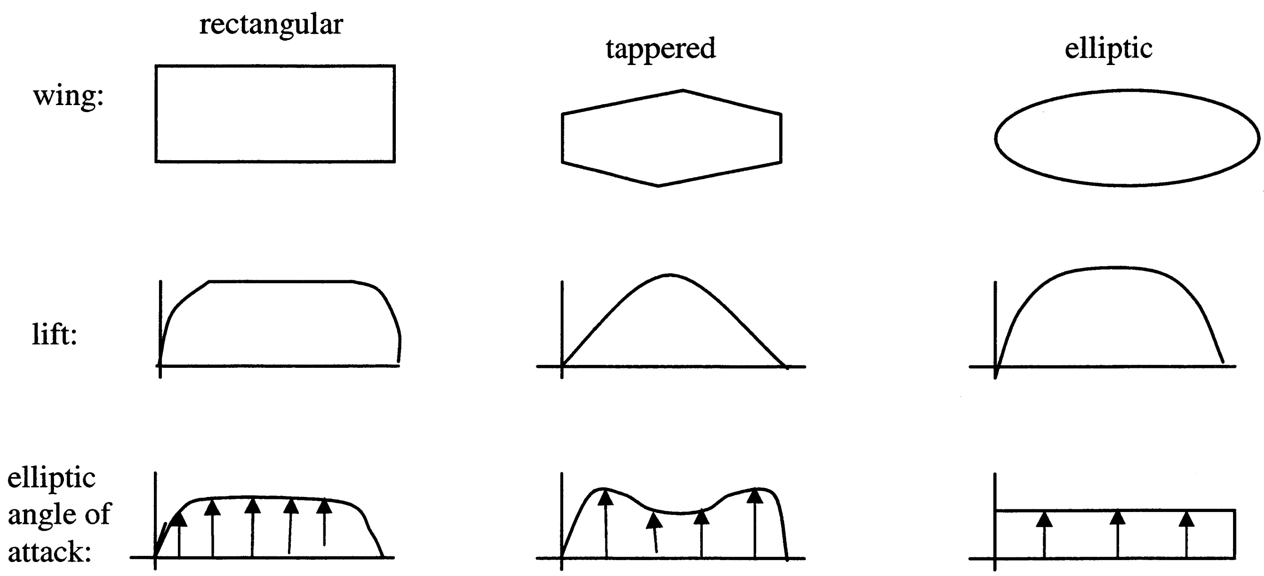
- Geometry influences Cp's and hence stall: a thick air foil will tend to stall from the TE (milder, more progressive, better "flying" qualities) whereas a thin airfoil will tend to stall from the LE (sudden, unpleasant)

- Suction side vortex generators can delay stall by re-energising the boundary layer



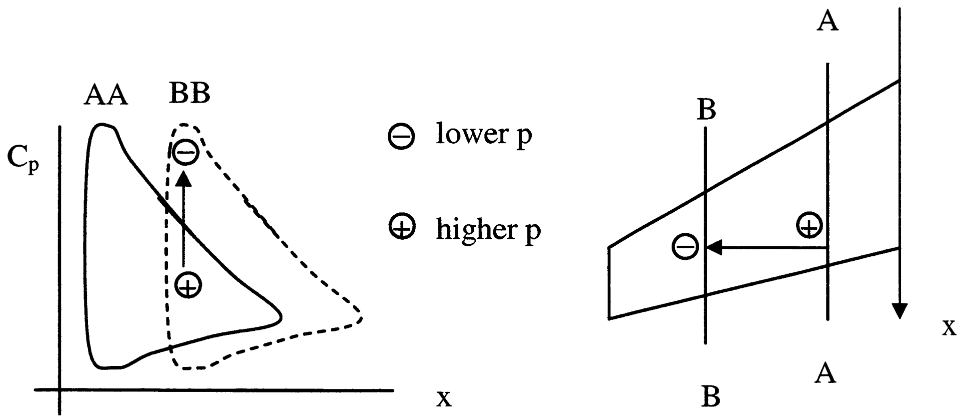


(iii) - the spanwise loading distribution on a finite wing influences the stalling behaviour:



Stall occurs first in centre	Stall occurs first near tip	Stall occurs simultaneously
⇒ good as low roll moment	⇒ bad & v. dangerous	⇒ V. bad!

- tip stall is very dangerous – in practice one tip would stall first and create a very large rolling movement ⇒ most stalls take place near the ground (at max. lift) and so this could be disastrous. In practice the designer goes to some trouble to make sure stall takes place more progressively and without the risk of large rolling movements.
- Wing sweep is introduced mainly to control drag onset at transonic speeds
- When a wing is swept a spanwise pressure variation is introduced



- as a result of these spanwise pressure gradients boundary layer flow near the wing surface is diverted towards the tips
– these secondary flows benefit the stalling behaviour near the wing root but thickness the boundary layers towards the tip increasing the danger of tip stall
- boundary layer fences can be installed to prevent this – but with a drag penalty (the A310 was the first large civil transport with a "clean" wing).