

ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3A5
Energy and Power Generation
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1.(a) From $Tds = dh - vdp$ we have :

$$\left(\frac{\partial h}{\partial p}\right)_T = v + T\left(\frac{\partial s}{\partial p}\right)_T$$

Hence, using the Maxwell relationship $(\partial s/\partial p)_T = -(\partial v/\partial T)_p$:

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p$$

From the definition of $g = (h - Ts)_p$:

$$\begin{aligned} \left(\frac{\partial g}{\partial p}\right)_T &= \frac{\partial}{\partial p} \left[\left(\frac{\partial h}{\partial T}\right)_p \right]_T = \frac{\partial}{\partial T} \left[\left(\frac{\partial h}{\partial p}\right)_T \right]_p \\ &= \frac{\partial}{\partial T} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p \right]_p \\ &= -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p \end{aligned}$$

(b) From the (p, v, T) equation of state :

$$v = \frac{RT}{p} - \frac{k}{T^3}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3k}{T^4}$$

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -\frac{12k}{T^5}$$

$$\text{Hence, } \left(\frac{\partial g}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p = \frac{12k}{T^4}$$

$$\therefore g_p = \frac{12kp}{T^4} + f_0(T)$$

As $p \rightarrow 0$, $g_p \rightarrow A + BT$, hence $f_0(T) = A + BT$

$$\therefore g_p = A + BT + \frac{12kp}{T^4}$$

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} - \frac{k}{T^3} - \frac{RT}{p} - \frac{3k}{T^3} = -\frac{4k}{T^3}$$

$$\therefore h = -\frac{4k}{T^3}p + f_1(T)$$

$$\text{Now, } C_p = \left(\frac{\partial h}{\partial T}\right)_p = \frac{12kp}{T^4} + \frac{df_1}{dT}$$

$$\therefore \frac{df_1}{dT} = A + BT \rightarrow f_1(T) = AT + \frac{BT^2}{2} + h_0 \quad (h_0 = \text{const.})$$

$$\therefore h = AT + \frac{BT^2}{2} - \frac{4k}{T^3}p + h_0$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p = -\frac{R}{p} - \frac{3k}{T^4}$$

$$\therefore s = -R \ln p - \frac{3k}{T^4}p + f_2(T)$$

$$\text{Now, } C_p = T\left(\frac{\partial s}{\partial T}\right)_p = \frac{12kp}{T^4} + T \frac{df_2}{dT}$$

$$\therefore T \frac{df_2}{dT} = A + BT$$

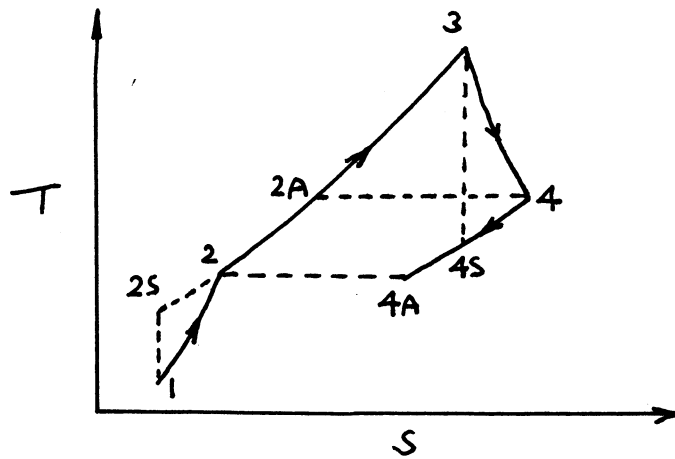
$$\frac{df_2}{dT} = \frac{A}{T} + B \rightarrow f_2(T) = A \ln T + BT + s_0 \quad (s_0 = \text{const.})$$

$$\therefore s = A \ln T + BT - R \ln p - \frac{3k}{T^4}p + s_0$$

Finally from $g = h - Ts$:

$$g = A(T - T \ln T) - \frac{BT^2}{2} + RT \ln p - \frac{kp}{T^3} + h_0 - Ts_0$$

2. (a)



$$\theta = T_3/T_1$$

$$\alpha = T_{2s}/T_1 = T_3/T_{4s}$$

$$\begin{aligned} \text{Net work output } W &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= c_p \eta_c (T_3 - T_{4s}) - c_p \frac{(T_{2s} - T_1)}{\eta_c} \\ &= c_p \eta_c T_3 \left(1 - \frac{1}{\alpha}\right) - \frac{c_p T_1}{\eta_c} (\alpha - 1) \end{aligned}$$

$$\therefore \frac{W}{c_p T_1} = \eta_c \theta \left(1 - \frac{1}{\alpha}\right) - \frac{(\alpha - 1)}{\eta_c}$$

$$\text{Heat input (with } \epsilon = 1) = c_p (T_3 - T_{2A}) = c_p (T_3 - T_4)$$

$$\therefore \frac{Q}{c_p T_1} = \eta_c \theta \left(1 - \frac{1}{\alpha}\right)$$

$$\frac{\partial}{\partial \alpha} \left(\frac{W}{c_p T_1} \right) = \frac{\eta_c \theta}{\alpha^2} - \frac{1}{\eta_c} \rightarrow \alpha = (\eta_c \eta_c \theta)^{1/2} \text{ at maximum.}$$

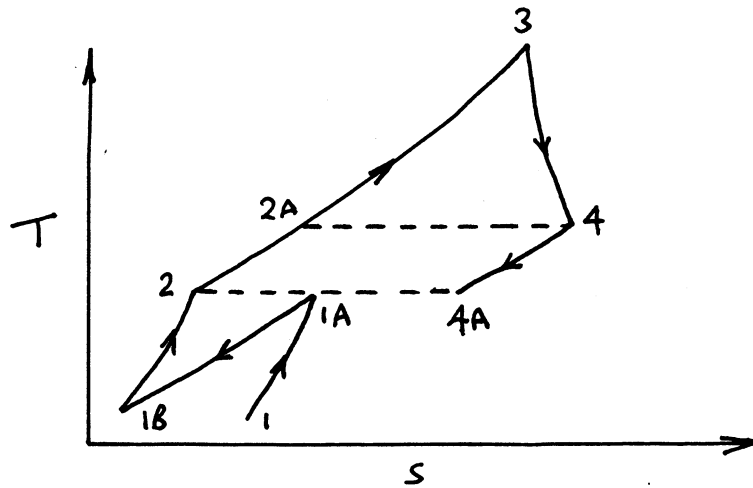
$$\text{Hence, for } \eta_c = \eta_e = 0.9 \text{ and } \theta = 5.5, \alpha = 2.111$$

$$\text{Compressor pressure ratio} = \alpha^{1/\gamma-1} = 13.66 \text{ (for } \gamma = 1.4).$$

$$\frac{W}{c_p T_1} = 0.9 \times 5.5 \left(1 - \frac{1}{2.111}\right) - \left(\frac{2.111 - 1.0}{0.9}\right) = 2.605 - 1.234 = 1.371$$

$$\frac{Q}{c_p T_1} = 0.9 \times 5.5 \left(1 - \frac{1}{2.111}\right) = 2.605 \rightarrow \eta_{\text{cycle}} = \frac{1.371}{2.605} = 52.6\%$$

(b)



$$\frac{W}{c_p T_1} = \eta_c \theta \left(1 - \frac{1}{x}\right) - \frac{(x_1 - 1)}{\eta_c} - \frac{(x_2 - 1)}{\eta_c}$$

For maximum specific work, $x_1 = x_2 = x^{1/2}$. Hence,

$$\frac{W}{c_p T_1} = \eta_c \theta \left(1 - \frac{1}{x}\right) - \frac{2(x^{1/2} - 1)}{\eta_c}$$

$$\frac{\partial}{\partial x} \left(\frac{W}{c_p T_1} \right) = \frac{\eta_c \theta}{x^2} - \frac{x^{-1/2}}{\eta_c} \rightarrow x = (\eta_c \eta_t \theta)^{2/3} \text{ at maximum}$$

For $\eta_c = \eta_t = 0.9$ and $\theta = 5.5$, $x = 2.707$

Overall pressure ratio = $x^{4/3} = 32.66$ (for $\delta = 1.4$).

$$\frac{W}{c_p T_1} = 0.9 \times 5.5 \left(1 - \frac{1}{2.707}\right) - \frac{2(2.707^{1/2} - 1)}{0.9} = 3.121 - 1.434 = 1.687$$

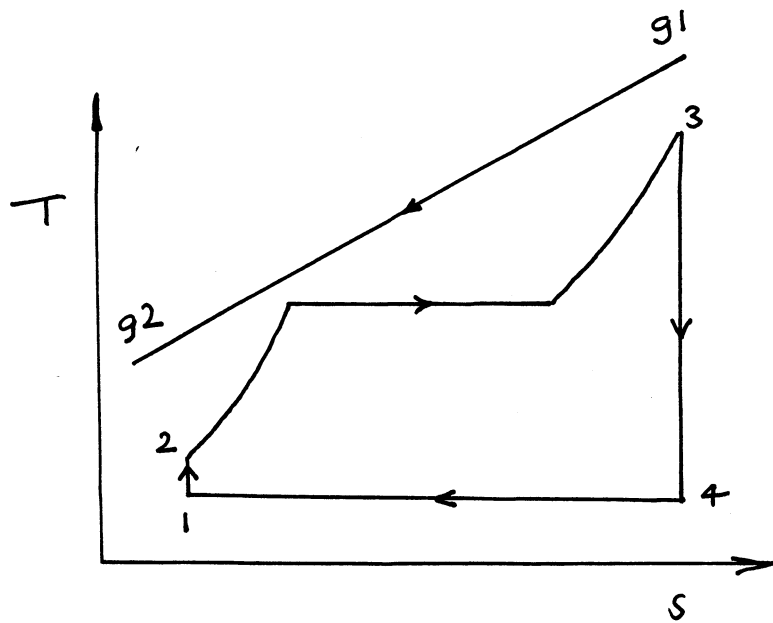
$$\frac{Q}{c_p T_1} = 0.9 \times 5.5 \left(1 - \frac{1}{2.707}\right) = 3.121 \rightarrow \eta_{\text{cycle}} = \frac{1.687}{3.121} = 54.1\%$$

(c) Intercooling recuperative cycles - turbine work and heat input stay about the same but the compressor work requirement is reduced. Hence there is an increase in both specific work output and cycle efficiency.

Intercooling non-recuperative cycles - turbine work stays the same, heat input increases and compressor work is reduced. Specific work output increases but cycle efficiency normally drops (it is like adding an extra cycle of lower pressure ratio to the main cycle).

In both cases, the turbine cooling flows are reduced because the compressor exit temperature is lower.

3.



- (a) Maximum work available from turbine exhaust is when the gas is cooled down to temperature T_0 .

$$W_{\max} = (h_{g1} - T_0 s_{g1}) - (h_{g0} - T_0 s_{g0})$$

No pressure drop on the gas side so:

$$W_{\max} = c_{pg}(T_{g1} - T_0) - c_{pg} T_0 \ln\left(\frac{T_{g1}}{T_0}\right)$$

$$= 1.2 \left[(823.15 - 298.15) - 298.15 \ln\left(\frac{823.15}{298.15}\right) \right]$$

$$= 266.66 \text{ kJ/kg exhaust gas.}$$

- (b) From tables (at 0.08 bar), $h_1 = 173.8 \text{ kJ/kg}$

$$\begin{aligned} \text{For feed pump, } (h_2 - h_1) &\simeq (p_2 - p_1) v_f = (60 - 0.08) \times 10^5 \times 0.001008 \\ &= 6039 \text{ J/kg} = 6.04 \text{ kJ/kg} \end{aligned}$$

$$\therefore h_2 = 173.8 + 6.04 = 179.84 \text{ kJ/kg}$$

From tables, $h_3 = 3423.1 \text{ kJ/kg}$. From SFEE:

$$\frac{m_s}{m_g} = \frac{c_{pg}(T_{g1} - T_{g2})}{(h_3 - h_2)} = \frac{1.2 \times (550 - 160)}{(3423.1 - 179.84)} = 0.1443$$

From tables, $S_3 = 6.883 \text{ kJ/kg K} = S_4$

From tables, $S_f = 0.592$, $S_g = 8.227 \text{ kJ/kg K}$ (at 0.08 bar)

$$\therefore x_4 = \frac{6.883 - 0.592}{8.227 - 0.592} = 0.8240$$

From tables, $h_f = 173.8$, $h_g = 2576.2 \text{ kJ/kg}$ (at 0.08 bar)

$$\therefore h_4 = (1 - 0.8240) \times 173.8 + 0.8240 \times 2576.2 = 2153.38 \text{ kJ/kg}$$

$$\therefore \text{Turbine Work, } (W)_{ST} = 0.1443(3423.1 - 2153.38) = 183.22 \text{ kJ/kg exhaust}$$

$$\text{Feed pump work, } (W)_{FP} = 0.1443 \times 6.04 = 0.87 \text{ kJ/kg exhaust.}$$

(c) HRSG lost work = $T_0(\Delta S)_{CR}$

$$\begin{aligned} \therefore (LW)_{HRSG} &= T_0 \left[C_{pg} \ln \left(\frac{T_{g2}}{T_{g1}} \right) + \frac{m_s}{m_g} (S_3 - S_2) \right] \\ &= 298.15 \left[1.2 \ln \left(\frac{433.15}{823.15} \right) + 0.1443(6.883 - 0.592) \right] \\ &= 40.94 \text{ kJ/kg exhaust} \end{aligned}$$

Condenser lost work = $Q \left(1 - \frac{T_0}{T_{cond}} \right)$ ($T_{cond} = 41.51^\circ\text{C}$)

$$\begin{aligned} \therefore (LW)_{COND} &= \frac{m_s}{m_g} (h_4 - h_1) \left(1 - \frac{T_0}{T_{cond}} \right) \\ &= 0.1443(2153.38 - 173.8) \left(1 - \frac{298.15}{314.66} \right) \\ &= 14.99 \text{ kJ/kg exhaust} \end{aligned}$$

Stack gas lost work = $(h_{g2} - T_0 S_{g2}) - (h_{g0} - T_0 S_{g0})$

$$\begin{aligned} (LW)_{STACK} &= C_{pg} \left[(T_{g2} - T_0) - T_0 \ln \left(\frac{T_{g2}}{T_0} \right) \right] \\ &= 1.2 \left[(433.15 - 298.15) - 298.15 \ln \left(\frac{433.15}{298.15} \right) \right] \\ &= 28.37 \text{ kJ/kg exhaust} \end{aligned}$$

(d) CHECK: $W_{max} = (W)_{ST} - (W)_{FP} + (LW)_{HRSG} + (LW)_{COND} + (LW)_{STACK}$

$$\begin{aligned} &= 183.22 - 0.87 + 40.94 + 14.99 + 28.37 \\ &= 266.65 \text{ kJ/kg exhaust} \quad \checkmark \end{aligned}$$

$$4 \text{ (a) Power } P = \int \Sigma_f V_c \phi_{ave} \\ = \int N_F \sigma_f V_c \phi_{ave}$$

where \int = energy per fission = $3.2 \times 10^{-11} \text{ J}$ (given)

ϕ_{ave} = average flux = $1.2 \times 10^{17} \text{ m}^{-2} \text{ s}^{-1}$ (given)

σ_f = microscopic fission cross-section

$$= 579 \text{ barns (given)} = 579 \times 10^{-28} \text{ m}^2$$

$N_F V_c$ = number of U-235 atoms in the core (i.e. in 100 tonnes of natural uranium)

Atomic mass of natural uranium =

$$0.99285 \times 238 + 0.00715 \times 235 = 238 \text{ kg/kmol}$$

$$\therefore N_F V_c = \frac{100 \times 10^3}{238} \times 6.022 \times 10^{26} \times 0.00715 \\ = 1.809 \times 10^{27}$$

$$\therefore P = 3.2 \times 10^{-11} \times 1.809 \times 10^{27} \times 579 \times 10^{-28} \times 1.2 \times 10^{17} \\ = \underline{\underline{402 \text{ MW}}}$$

Rate of consumption of U-235 through fission $R_f = N_F \sigma_f V_c \phi_{ave}$

$$\therefore R_f = 1.809 \times 10^{27} \times 579 \times 10^{-28} \times 1.2 \times 10^{17} \\ = 1.257 \times 10^{19} \text{ atoms s}^{-1} \\ = \frac{1.257 \times 10^{19}}{6.022 \times 10^{26}} \times 235 \times 3600 \times 24 = 0.424 \text{ kg/day}$$

But U-235 is also consumed through capture

$$R_c = N_F \sigma_c V_c \phi_{ave} = \frac{\sigma_c}{\sigma_f} R_f$$

$$\therefore R_c = \frac{101}{579} \times 0.424 = 0.074 \text{ kg/day}$$

$$\therefore R_{tot} = R_c + R_f = 0.074 + 0.424 = \underline{\underline{0.498 \text{ kg/day}}}$$

$$(b) \phi_{ave} = \frac{\int_{core} \phi \, dV}{\int_{core} dV}$$

For a spherical core $\int_{core} dV = \frac{4}{3} \pi R^3$ and $dV = 4\pi r^2 dr$

$$\begin{aligned}\therefore \phi_{ave} &= \frac{1}{\frac{4\pi R^3}{3}} \int_0^R \frac{\phi_{max} R_0}{\pi r} \sin\left(\frac{\pi r}{R_0}\right) 4\pi r^2 dr \\ &= \frac{3}{\pi R^3} \phi_{max} R_0 \int_0^R r \sin\left(\frac{\pi r}{R_0}\right) dr\end{aligned}$$

Integrating by parts

$$\begin{aligned}\therefore \phi_{ave} &= \frac{3}{\pi R^3} \phi_{max} R_0 \left[-\frac{rR_0}{\pi} \cos\left(\frac{\pi r}{R_0}\right) + \int \frac{R_0}{\pi} \cos\left(\frac{\pi r}{R_0}\right) dr \right]_0^R \\ &= \frac{3}{\pi R^3} \phi_{max} R_0 \left[-\frac{rR_0}{\pi} \cos\left(\frac{\pi r}{R_0}\right) + \frac{R_0^2}{\pi^2} \sin\left(\frac{\pi r}{R_0}\right) \right]_0^R \\ &= \frac{3}{\pi R^3} \phi_{max} R_0 \left[\frac{R_0^2}{\pi^2} \sin\left(\frac{\pi R}{R_0}\right) - \frac{RR_0}{\pi} \cos\left(\frac{\pi R}{R_0}\right) \right] \\ &= 3\phi_{max} \left[\frac{R_0^3}{\pi^3 R^3} \sin\left(\frac{\pi R}{R_0}\right) - \frac{R_0^2}{\pi^2 R^2} \cos\left(\frac{\pi R}{R_0}\right) \right]\end{aligned}$$

Writing $x = \frac{\pi R}{R_0}$

$$\therefore \phi_{ave} = 3\phi_{max} \left[\frac{\sin(x) - x \cos(x)}{x^3} \right]$$

(c) If $\frac{\phi_{max}}{\phi_{ave}} \leq 2.0$, then

$$\frac{3(\sin(x) - x \cos(x))}{x^3} \geq 0.5$$

\therefore At equality

$$\begin{aligned}x^3 &= 6(\sin(x) - x \cos(x)) \\ \text{or } x &= \sqrt[3]{6(\sin(x) - x \cos(x))}\end{aligned}$$

This can be used as the basis for an iterative solution

$$x_{n+1} = \sqrt[3]{6(\sin(x_n) - x_n \cos(x_n))}$$

Using the suggest first guess $x_1 = 0.8\pi = 2.5133$, successive iterands are

2.5133	2.5054	2.5017	2.4999	2.4990
2.4986	2.4984	2.4983	2.4983	

$$\text{Thus } \frac{R}{R_0} = \frac{X}{\pi} = \frac{2.4983}{\pi} = \underline{\underline{0.7952}}$$

R_0 can be made significantly greater than R by surrounding the core with a reflector, a material with a low neutron capture cross-section and a high scattering cross-section. This reflects back into the core a proportion of the neutrons leaking from the core, increasing the flux level within the core and thus, in effect, increasing R_0 .

5 (a) - Fuel NO : From 10 to 50% of N contained in oil (or coal) will react with hydrocarbon radicals to form HCN, which then gives NO. Not important for gasoline, could be for diesels.

- Prompt NO : Inside flame region, N_2 reacts with CH to give HCN, which then gives NO. Very quick reaction

- Thermal NO : $N_2 + O \rightarrow NO + N$ Zeldovich mechanism
 $O_2 + N \rightarrow NO + O$

This occurs at high temperatures only. To reduce NO, reduce T, $[O_2]$ or $[N_2]$, or time spent in high-T regions.

- Techniques used to reduce NO emission:

Gas turbines : Lean premixed combustion (lean \Rightarrow low T)
 Rich-Quench-Lean (minimize time in high-T region)

Steam injection (to lower T)

Spark ignition : Lean burn
 Exhaust gas recirculation (dilution reduces T)
 Catalyst (post treatment)

- If we overdo lowering T, we'll begin to have CO and unburnt hydrocarbon emission. In diesels, we will also have soot formed. We will also encounter flame stability problems (e.g. misfire), which increases smoke emissions a lot. Careful balance is necessary!

5 (b) At blow-off, $\frac{\tau_{chem}}{\tau_{res}} = C$

$$\tau_{chem} = \frac{\lambda}{P_0 C_p S_L^2} = \frac{\lambda_{ref} \left(\frac{T_0}{T_{ref}}\right)^{1/2}}{\left(\frac{P}{P_0}\right) C_p S_{Lref}^2 \left(\frac{T_0}{T_{ref}}\right)^2}$$

$$\tau_{res} = \frac{L}{U} = \frac{L}{\frac{\dot{m}}{\rho_0 A}} = \frac{L}{\dot{m}} \frac{P}{P_0} \cdot A$$

A = area of burner
P = constant pressure

$$\begin{aligned} \Rightarrow \frac{\tau_{chem}}{\tau_{res}} &= \frac{\lambda_{ref} / T_{ref}^{1/2}}{C_p S_{Lref}^2 / T_{ref}^4} \cdot \frac{T_0^{1/2}}{T_0^4} \cdot \frac{\dot{m}}{L} \cdot \frac{1}{A} \cdot \frac{1}{(P/P_0)^2} \\ &= C' \frac{\dot{m}}{L T_0^{3/2}} \end{aligned}$$

Hence, if blow-off occurs at conditions 1 & 2:

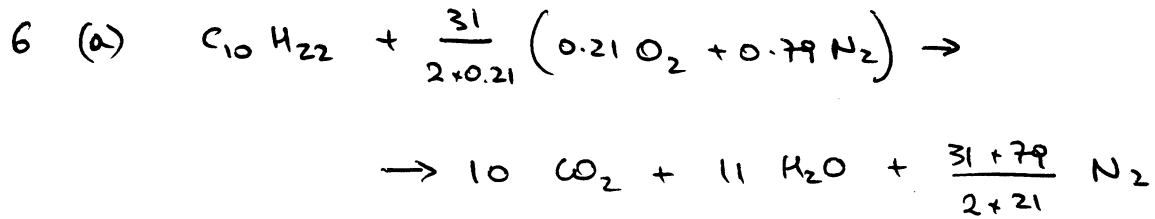
$$\frac{\dot{m}_1}{L_1 T_{01}^{3/2}} = \frac{\dot{m}_2}{L_2 T_{02}^{3/2}} \quad (\Rightarrow) \quad \frac{\dot{m}_2}{\dot{m}_1} = \frac{L_2}{L_1} \left(\frac{T_{02}}{T_{01}}\right)^{3/2}$$

$$\text{For } \frac{L_2}{L_1} = 2 \quad \& \quad \frac{T_{02}}{T_{01}} = 2, \quad \underline{\underline{\dot{m}_2 = 5.66 \dot{m}_1}}$$

So % increase is 466%.

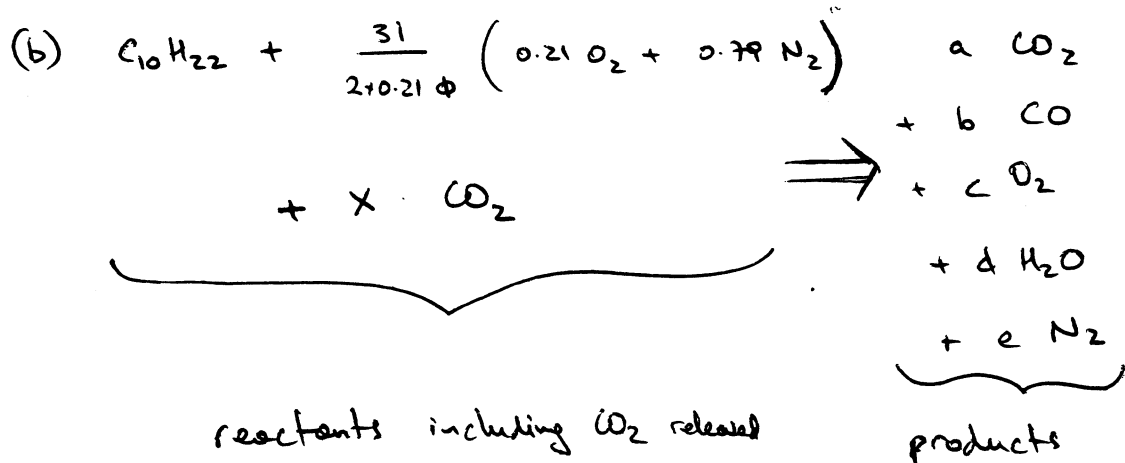
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Hence 142 kg $C_{10}H_{22}$ (1 kmol has 142 kg)
 needs $\frac{31}{2 \times 0.21} (0.21 \times 32 + 0.79 \times 28) = 2128.67$ kg air

$$\text{Hence } \phi = \frac{\dot{m}_f / \dot{m}_{air}}{(\dot{m}_f / \dot{m}_{air})_{st}} = \frac{0.06 / 1}{142 / 2128.67} = \underline{\underline{0.90}}$$



$$\frac{x \cdot 44}{142} = \frac{0.1}{0.06} \Rightarrow x = 5.379$$

Hence released CO_2 brings in 5.379 kmol of CO_2 per kmol of fuel.

To calculate a, b, c, d, e , we compile system of equations:

- Conservation of C : $10 + X = a + b$ (1)

- Conservation of H : $22 = 2d$ (2)

- Conservation of O : $\frac{31}{\phi} + 2X = 2a + b + 2c + d$ (3)

- Conservation of N : $\frac{31 + 79}{2 + 21\phi} = e$ (4)

- Equilibrium of $\text{CO} + \frac{1}{2}\text{O}_2 = \text{CO}_2$

$$\begin{aligned} \Rightarrow K_p &= \frac{(P_{\text{CO}_2}/P^0)}{(P_{\text{CO}}/P^0)(P_{\text{O}_2}/P^0)^{1/2}} = \frac{(X_{\text{CO}_2}P/P^0)}{(X_{\text{CO}}P/P^0)(X_{\text{O}_2}P/P^0)^{1/2}} \\ &= \frac{\frac{a}{n}}{\frac{b}{n}(\frac{c}{n})^{1/2}(\frac{P}{P^0})^{1/2}} \end{aligned} \quad (5)$$

At 2000 K, $\ln K_p = 6.634 \Rightarrow K_p = 760.518$

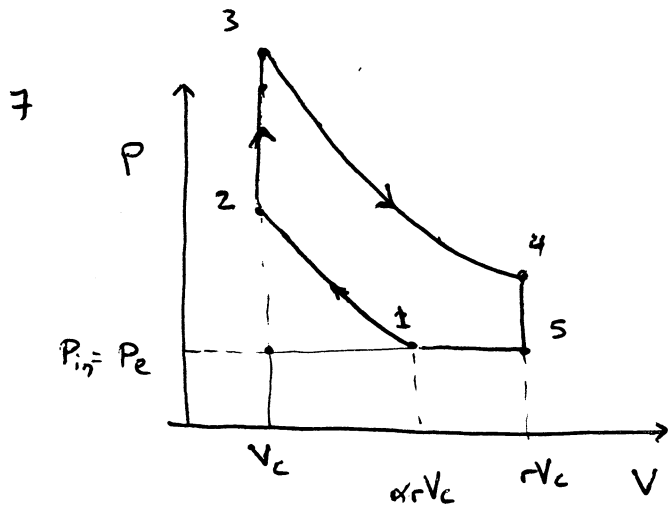
- $n = a + b + c + d + e$ (6)

This system gives us enough equations for all our unknowns.

(e) Using $X_{\text{O}_2} = 0.0193 \Rightarrow \frac{c}{n} = 0.0193 \Rightarrow \frac{a}{b} = 105.65$ (7)

\Rightarrow System (1), (2), (3), & (7) can easily be solved to

give :	$a : 15.236$	$X_{\text{CO}_2} : 16.39\%$
	$b : 0.143$	$X_{\text{CO}} : 0.15\%$
	$c : 1.794$	$X_{\text{O}_2} : 1.93\%$
	$d : 11$	$X_{\text{H}_2\text{O}} : 11.83\%$
	$e : 64.788$	$X_{\text{N}_2} : 69.69\%$
	$n = 92.961$	



m_r = residual gas mass
 m_a = air mass
 m_f = fuel mass

(a) Energy balance for mixture:

$$(m_a + m_f) c_v T_{in} + m_r c_v T_r = m c_v T_1$$

$$\Rightarrow \frac{m_a + m_f}{m} T_{in} + \frac{m_r}{m} T_r = T_1$$

But $m = m_a + m_f + m_r$

\Rightarrow

$$T_1 = (1 - x_r) T_{in} + x_r T_r$$

(b) Heat release per unit mass of gas in cylinder = q

$$q = \frac{m_f Q}{m}$$

Since $m_a + m_f + m_r = 1 \Leftrightarrow \frac{m_a}{m} + \frac{m_f}{m} + \frac{m_r}{m} = 1$

$$\Leftrightarrow \frac{m_a}{m_f} \frac{m_f}{m} + \frac{m_f}{m} + \frac{m_r}{m} = 1 \Leftrightarrow (1 + f) \frac{m_f}{m} + x_r = 1$$

$$\Leftrightarrow \frac{m_f}{m} = \frac{1 - x_r}{1 + f}$$

$$\Rightarrow q = \frac{(1 - x_r) Q}{1 + f} \quad \text{Q.E.D.}$$

$$(c) \quad w_{12} = -C_v (T_2 - T_1)$$

Isentropic compression $TV^{\gamma-1} = \text{const} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$

$$\Rightarrow w_{12} = -C_v T_1 [(r)^{\gamma-1} - 1]$$

(d) Going round the cycle, expressing everything in terms of T_1 :

$$m_f Q = m C_v (T_3 - T_2) \Rightarrow T_3 - T_2 = \frac{m_f Q}{m C_v} = \frac{q}{C_v}$$

$$\Rightarrow \frac{T_3}{T_1} - \frac{T_2}{T_1} = \frac{q}{C_v T_1} \Rightarrow \frac{T_3}{T_1} = (r)^{\gamma-1} + \frac{q}{C_v T_1} \quad (1)$$

For point 4: $\frac{T_4}{T_3} = \frac{1}{r^{\gamma-1}} \Rightarrow \frac{T_4}{T_1} \frac{T_1}{T_3} = \frac{1}{r^{\gamma-1}}$

$$\Rightarrow \frac{T_4}{T_1} = r^{1-\gamma} \left[(r)^{\gamma-1} + \frac{q}{C_v T_1} \right] \quad (2)$$

$$\begin{aligned} w_{34} &= C_v (T_3 - T_4) = C_v T_1 \frac{T_3}{T_1} \left(1 - \frac{T_4}{T_3} \right) \\ &= C_v T_1 \left[(r)^{\gamma-1} + \frac{q}{C_v T_1} \right] (1 - r^{1-\gamma}) \end{aligned}$$

Hence net work : $w_{34} + w_{12}$

$$= C_v T_1 \left\{ \left[(r)^{\gamma-1} + \frac{q}{C_v T_1} \right] (1 - r^{1-\gamma}) - [(r)^{\gamma-1} - 1] \right\}$$

QED

$$(e) \quad \eta = \frac{w_{\text{net}}}{q}$$

$$\text{For } \alpha=1, \quad \eta_{\text{conventional}} = \frac{C_v T_1}{q} \left\{ \left(r^{\gamma-1} + \frac{q}{C_v T_1} \right) \left(1 - \frac{1}{r^{\gamma-1}} \right) - r^{\gamma-1} + 1 \right\}$$

$$= \frac{C_v T_1}{q} \left[\cancel{r^{\gamma-1}} + \frac{q}{C_v T_1} - \cancel{1} - \frac{q}{C_v T_1} \frac{1}{r^{\gamma-1}} - \cancel{r^{\gamma-1}} + \cancel{1} \right]$$

$$= \left(1 - \frac{1}{r^{\gamma-1}} \right) \quad (\text{as it should be for the standard cycle})$$

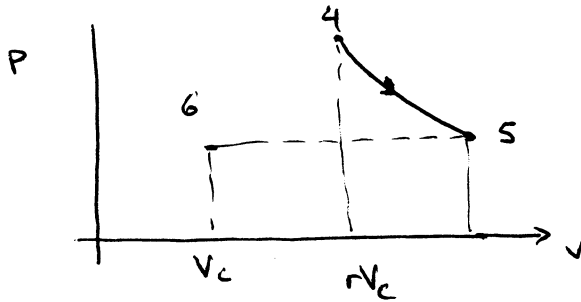
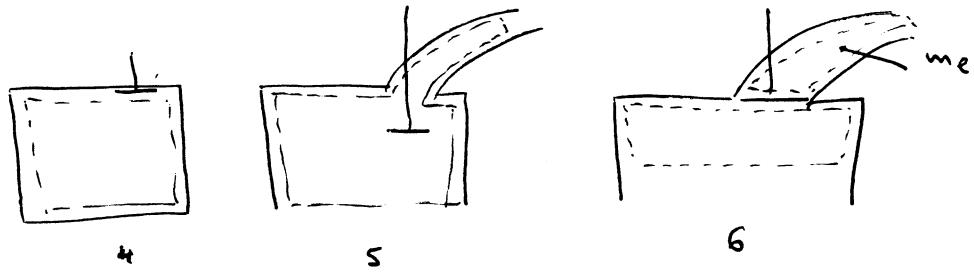
Putting numbers :

$\alpha = 0.8$	$\omega = 700 \text{ J/kgK}$
$\Gamma = 20$	$Q = 40 \text{ MJ/kg}$
$\gamma = 1.4$	$x_r = 0.4$
$f = 50$	$T_i = (1-x_r) T_{in} + x_r T_r$

$$\frac{\eta_{\text{MODIFIED}}}{\eta_{\text{CONVENTIONAL}}} = 1.07$$

The increase in efficiency is due to the reduction of compression work. The penalty is a reduction in specific work

8. (a)



$$\dot{m}_e = \frac{m_e N}{2} \cdot z$$

\uparrow cycles per second
 \uparrow # of cylinders
 because we have 4-stroke engine

$$m_e = m_5 - m_6 = m_4 - m_6$$

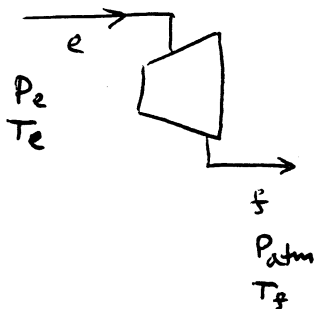
$$m_4 = \rho_4 V_4 = \rho_4 r V_c$$

$$m_6 = \rho_6 V_6 = \rho_5 V_6 = \rho_4 \left(\frac{P_5}{P_4}\right)^{1/n} V_c$$

$$= \rho_4 \left(\frac{P_e}{P_4}\right)^{1/n} V_c$$

$$\text{Therefore } \dot{m}_e = \frac{P_4}{RT_4} V_c \frac{N}{2} z \left[r - \left(\frac{P_e}{P_4}\right)^{1/n} \right]$$

(b)



$$\dot{W}_{turb} = \dot{m}_e \cdot C_p \cdot (T_e - T_f)$$

$$T_e = T_5 = T_4 \left(\frac{P_e}{P_4}\right)^{\frac{n-1}{n}}$$

T_f will be found from isentropic expansion (\Rightarrow maximum power)

$$\Rightarrow T_f = T_e \cdot \left(\frac{P_{atm}}{P_e}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\begin{aligned}
 \text{Therefore } \dot{w}_{\max} &= \dot{m}_e c_p T_e \left[1 - \left(\frac{P_{\text{atm}}}{P_e} \right)^{\frac{\gamma-1}{\gamma}} \right] \\
 &= \frac{P_4}{R T_4} V_c \frac{N}{2} \approx c_p T_e \left[r - \left(\frac{P_e}{P_4} \right)^{1/n} \right] \left[1 - \left(\frac{P_{\text{atm}}}{P_e} \right)^{\frac{\gamma-1}{\gamma}} \right] \\
 \left(\text{Using } \frac{c_p}{R} = \frac{\gamma}{\gamma-1} \right) &= P_4 V_c \frac{N}{2} \approx \frac{\gamma}{\gamma-1} \left[r - \left(\frac{P_e}{P_4} \right)^{1/n} \right] \left(\frac{P_e}{P_4} \right)^{\frac{\gamma-1}{n}} \left[1 - \left(\frac{P_{\text{atm}}}{P_e} \right)^{\frac{\gamma-1}{\gamma}} \right]
 \end{aligned}$$

Interestingly, only P_4 , P_e & P_{atm} affect \dot{w}_{\max} , not T_4 .