ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3B1
Radio Frequency Electronics
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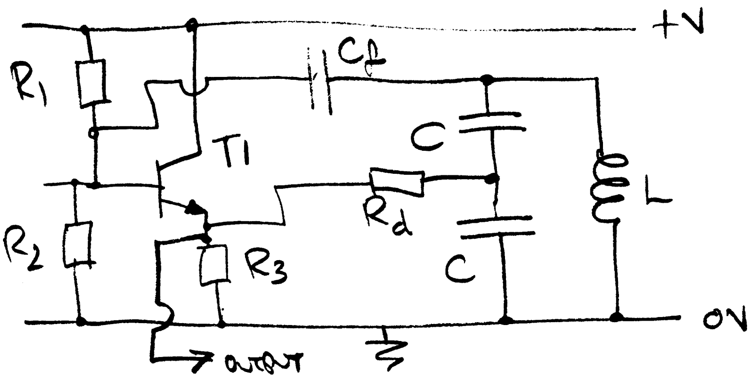
1 a)



R_1 & R_2 base bias resistors

R_3 emitter load resistor

L & C's resonant tank circuit

R_d tank drive resistor

C_f feedback path

$$f_{osc} = \frac{1}{2\pi\sqrt{LC/2}}$$

L + C's provide a resonant parallel LC circuit - with voltage gain, from C mid-point to top of tank, of ×2 at resonance. The transistor T1 is connected as an emitter follower of voltage gain ~1 ∴ loop gain ~2 and oscillation start-up. Amplitude is limited by the falling gain of T1 and non-linearity as the emitter swing approaches ~~the~~ voltage supply rails.

b) $3\,dBm \equiv 10\log_{10}\left(\frac{P}{10^{-3}}\right)$ $\therefore P = 2\times10^{-3}\,W = V^2/R$

across $1k\Omega$ $\therefore V = \sqrt{2}$ rms $\therefore V_{pk-pk} = 4V$ ✓ fine for 10V supply

$f_{osc} = 3\times10^{6} = \frac{1}{2\pi\sqrt{LC/2}}$ Choose $L = 10\,\mu H$ (arb.)

$\therefore C = 560\,pF$

$\therefore LC = 5.63\times10^{-15}$

1 b) cont. To bias emitter at +5V, set base at 5.7V.

$\therefore \frac{R_2}{R_1+R_2} \cdot 10 = 5.7$ Choose $R_2 = 100\,k\Omega$

$\therefore R_1 = 75\,k\Omega$

$R_3 \approx 1.5\,k\Omega$ (1.5 × load impedance)

$R_d \approx (R_1 \| R_2 \| h_{fe}R_3 \| h_{fe}R_{LOAD})/5$

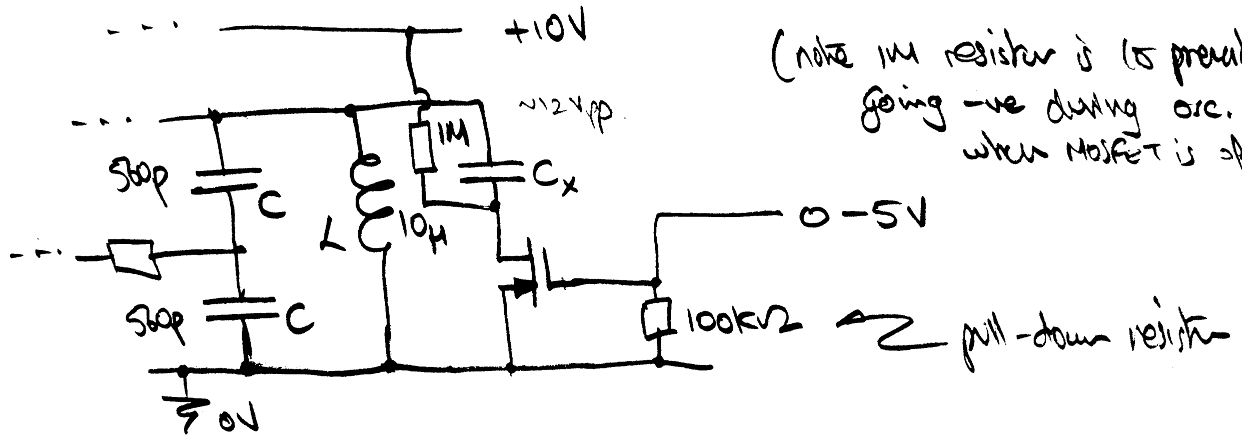
$= 6.3\,k\Omega$

(say 5.6 kΩ standard value) assume $h_{fe} = 200$

$C_f = 10\,nF$ say - large value (only 5Ω impedance @ 3MHz).

Output is likely to be 6Vpp. (60% of supply) so may need attenuator for 4Vpp.

c). Tank circuit includes switchable capacitor to drop frequency to 2.5 MHz. - use a MOSFET, check



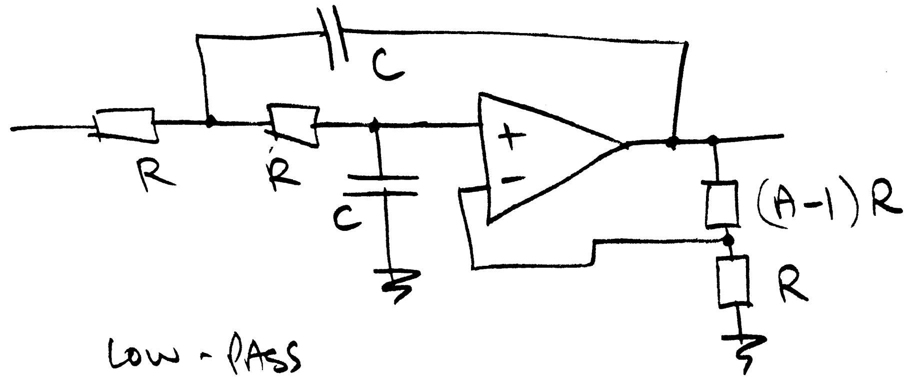
(note 1M resistor is to prevent drain going -ve during osc. cycle when MOSFET is off)

$$f_{osc}' = 2.5 \times 10^6 = \frac{1}{2\pi\sqrt{10^{-5}\cdot(280\times10^{-12}+C_x)}}$$

$\therefore C_x = 125\,pF$ (say 120pF standard value)

(note: a varactor diode could be used instead, but the large voltage swings would cause non-linear distortion in the signal.)

2a)



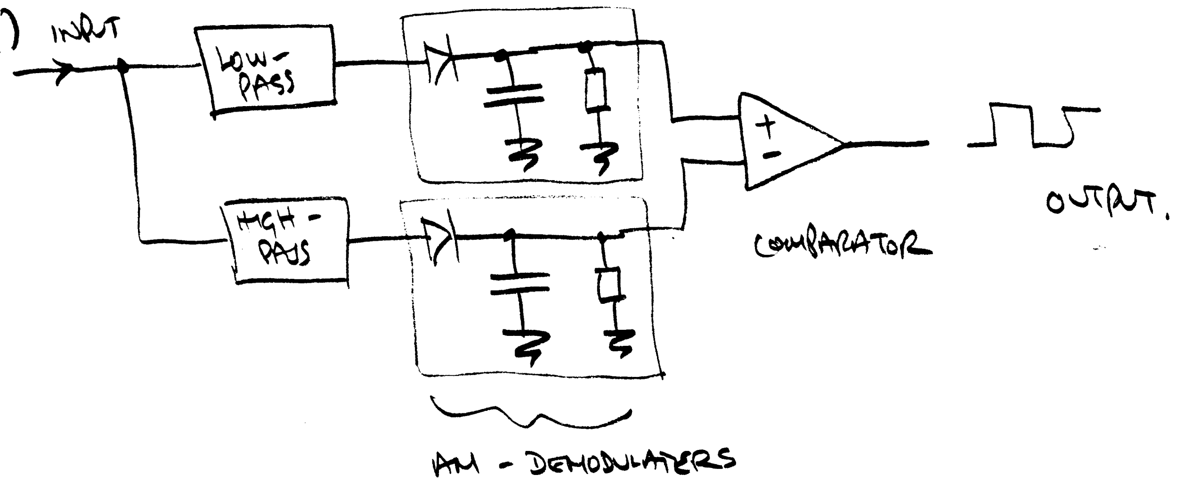
LOW-PASS

$$f_{o_{LP}} = \frac{1}{2\pi f_n CR}$$

For high-pass change over R's and C's at inputs

$$f_{o_{HP}} = \frac{f_n}{2\pi CR}$$

b)(i)



(ii) Choose Chebyshev filter for steeper response in freq. domain and set cut-off frequency at mid-point of 2 FSK frequencies ie. 2.75 MHz.

∴ For low pass. $f_n = 1.231$, $A = 1.842$

set $R = 1k\Omega$ ∴ $2.75 \times 10^6 = \frac{1}{2\pi . 1.231 . 10^3 C}$

∴ $C = 47\,nF$

$(A-1)R = 842\,\Omega$
($820\,\Omega$ std.)

For high pass. $2.75 \times 10^6 = \frac{1.231}{2\pi . 10^3 . C}$

set $R = 1k\Omega$ ∴ $C = 31\,nF$ (say $33nF$ std).

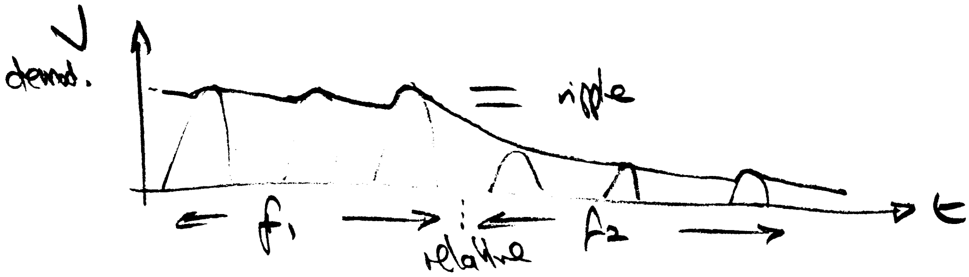
$(A-1)R = 842\,\Omega$
($820\,\Omega$ std.)

2 b) ii) cont. DEMODULATORS :-

choose Schottky diodes in demodulators for min drop.
(peak amplitude @ -3db freq. $= 0.5V \times \frac{1.842}{\sqrt{2}} = 0.65V$)

$R = 1k\Omega$ - set C for say 5 cycles equivalent time constant.

$\therefore CR = \frac{5}{2.75 \times 10^6} = 1.8\mu s \quad \therefore C = 1.8nF$



we need to check the attenuation gives greater amplitude difference than the ripple. For chebyshev, we don't know the exact rolloff but it is better than a plain CR. So for 20% freq. difference, 2-pole filter the amplitudes of HP and LP outputs will differ by ~40%, or ~0.3V.

Voltage drop with CR diode demodulator given by

$$\simeq (1 - e^{-t/CR})\, 0.65\,V \quad \text{where } t \simeq \frac{1}{2.75 \times 10^6}$$

$$= (1 - e^{-0.2})\, 0.65V$$

$$= 0.12V.$$ ✓ OK as $< 0.3V$.

b)(iii). Data rate limits are imposed by CR demodulator and depend on time-constant selected (which must be longer than carrier period). Looking at the above, the best we could do is say have the CR time constant to ~2.5 × carrier period. Then data rate max ~ 1MHz. (with CR ≈ 1μs.)

3 (a) Skin Depth: see part (d)

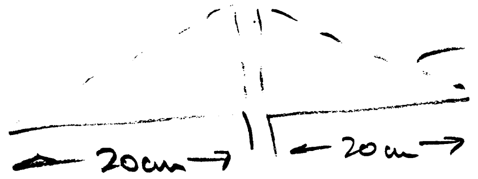
Radiation Resistance, R_r: power radiated $P_r = 1/2\, I^2 R_r$ (I is amplitude not rms)

Radiation efficiency, e: $= P_r / P_{input}$ where $P_{in} = 1/2\, I^2 (R_r + R_{ohmic})$ (5)

Gain, G: = (Max power density radiated in peak direction)/(Power density for isotropic antenna)

Effective Aperture; Ae: Power received $= A_e \cdot$ Power density

(b) 40 MHz, $\lambda = \dfrac{3\times10^8}{40\times10^6} = 7.5\,m.$



For ideal dipole (with rectangular current dist)

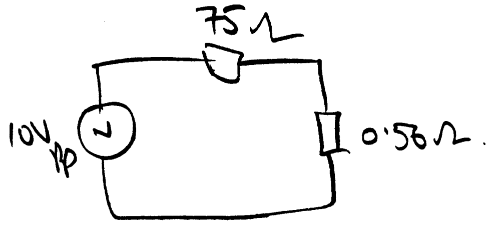
$$R_r = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

So for the linear taper case Δz is effectively halved.

$$\therefore\ R_r' = 20\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

$$R_r' = 20\pi^2 \left(\frac{0.4}{7.5}\right)^2 = 0.56\,\Omega$$

(c)



$$P_r = I_{rms}^2 R_r'$$

$10V_{pp} = 3.54 V_{rms}$, $I_{rms} = 0.047$ A $\therefore P_r = 1.23$ mW

Power density on axis $= \dfrac{P_r}{4\pi r^2} \cdot G = \dfrac{1.23\times10^{-3}}{4\pi\, 10^6} \times 1.5 = 1.47\times10^{-10}$ W/m^2

(Gain = 1.5)

Now, $G = \dfrac{4\pi A_e}{\lambda^2}$ antenna eqn.

$$\therefore\ A_e = \frac{G\lambda^2}{4\pi} = \frac{1.5\times7.5^2}{4\pi} = 6.71\ m^2$$

$\therefore$ Preceived $= 1.47\times10^{-10} \times 6.71 = 0.986$ nW

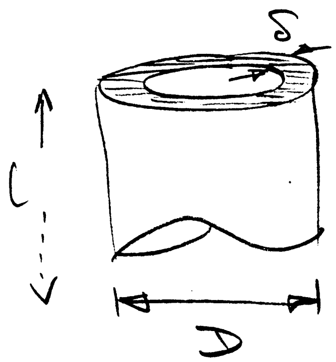
(assuming lossless antennas).

3(d) skin depth, $\delta = \sqrt{\dfrac{2}{\omega\mu\sigma}} = 7.26 \times 10^{-5}$ m

6

(current may be considered to be carried in outer skin of depth, δ)

Assume equal current over 1 arm :-



$$R_{ohmic} = \frac{L}{\pi D \delta \sigma} = \frac{0.2}{\pi . 10^{-3} . 7.26 \times 10^{-5} . 1.2 \times 10^{6}}$$

$$= 0.731 \ \Omega$$

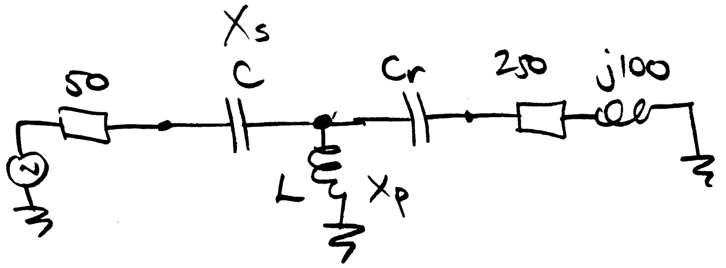
$= 0.56\Omega$ from (b)

$$\therefore \text{ Radiation efficiency} = \frac{R_r'}{(R_r' + R_{ohmic})} = 0.43$$

or 43%

(This is strictly for a rectangular current distribution over half the full length)

4 a)



@ 200MHz

- select C_r to resonate with j100 to cancel reactive part at 200 MHz

$\therefore \; -j100 = \dfrac{-j}{2\pi f C_r}$ ← 200×10^6 → $\therefore C_r = 7.96 \times 10^{-12}$ F (or 8.2pF std. value)

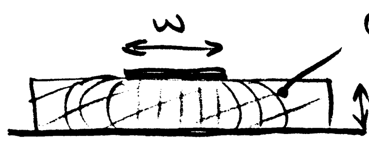
- Then match 250Ω to 50Ω using LC

$$Q = \sqrt{\frac{250}{50} - 1} = 2 = \frac{250}{X_p} = \frac{X_s}{50}$$

$$\therefore \; X_p = 125 = 2\pi f L \;\Rightarrow\; L = 99.5\text{ nH} \quad (100\text{nH std.})$$

$$X_s = 100 = \frac{1}{2\pi f C} \;\Rightarrow\; C = 7.96\text{ pF} \quad (8.2\text{pF std.})$$

b)



$$C \text{ per unit length} \simeq \frac{(w + 2t)\,\epsilon_0 \epsilon_r}{t}$$

(allowing width of t each side of w for fringing fields)

$$v = \text{speed of light in dielectric} = \sqrt{\frac{1}{LC}} = \frac{c_0}{\sqrt{\epsilon_r}}$$ (c_0 ← speed of light)

$Z_0 = \sqrt{\dfrac{L}{C}}$, characteristic impedance

$$\therefore \; 50 = \sqrt{L/C} \;\Rightarrow\; 2500\,C = L$$

$$\therefore \; \sqrt{\frac{1}{2500\,C^2}} = \frac{3 \times 10^8}{\sqrt{4.3}} \qquad \therefore C = 1.38 \times 10^{-10}\text{ F/m}$$

$$\therefore \; 1.38 \times 10^{-10} = (w + 3.2 \times 10^{-3})\,\frac{8.854 \times 10^{-12} \cdot 4.3}{1.6 \times 10^{-3}} \;\Rightarrow\; \underline{w = 2.61\text{ mm}}$$

$$\therefore \; w = 2.61 \times 10^{-3}\text{ m}$$

Q4a)

SMITH CHART ALTERNATIVE SOLN.

- $250 + j100\ \Omega$ normalise to 50Ω

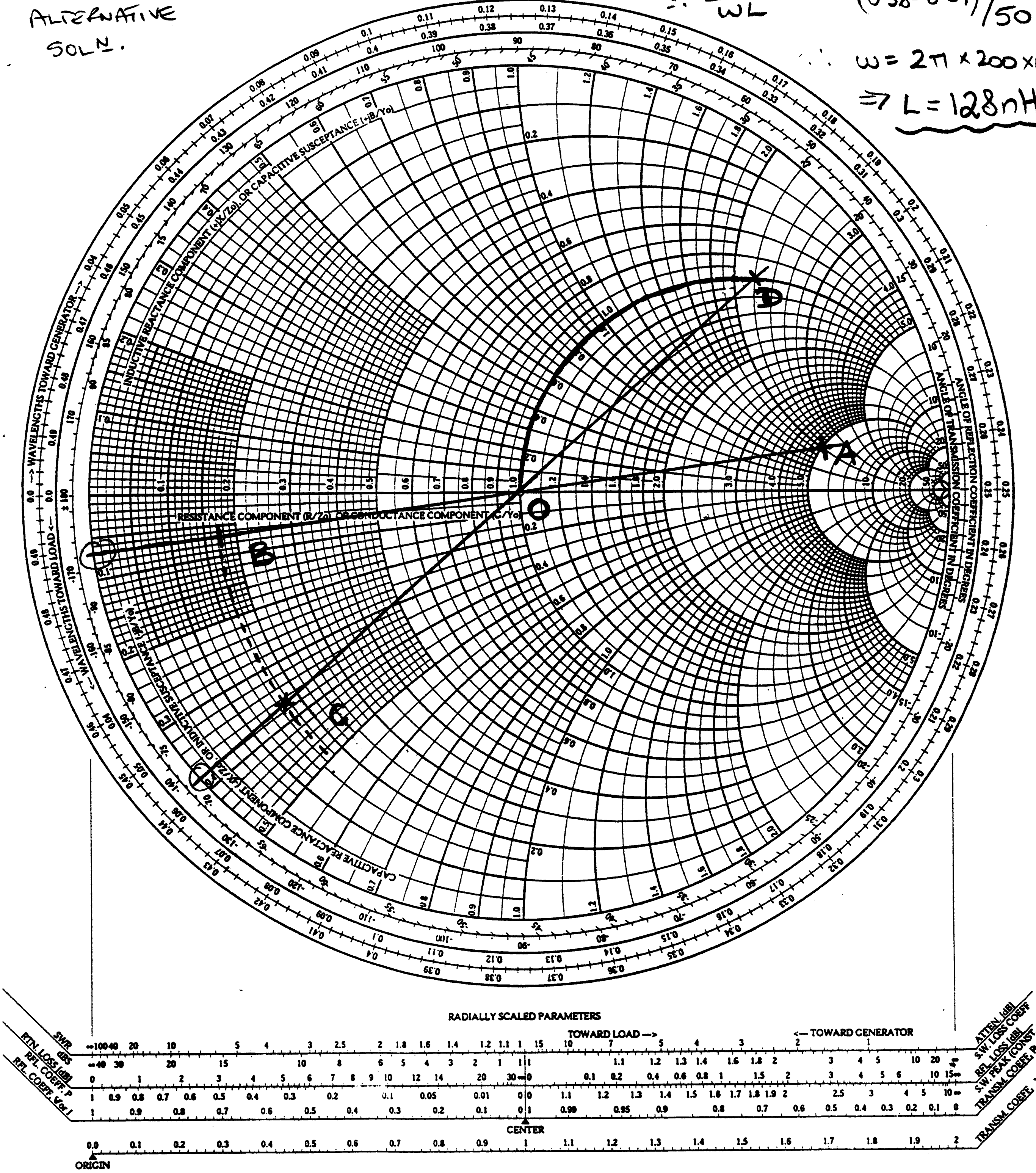
  $\Rightarrow 5 + 2j$ POINT A

- ∴ work with admittances → B
- move B→C with parallel inductor ($OC = OD$)
  (conductance component unchanged)

$$\therefore \frac{1}{\omega L} = (0.38 - 0.07)/50$$

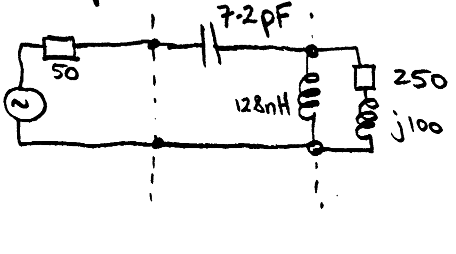
$$\therefore \omega = 2\pi \times 200 \times 10^{6}$$

$$\Rightarrow L = 128\text{nH}$$



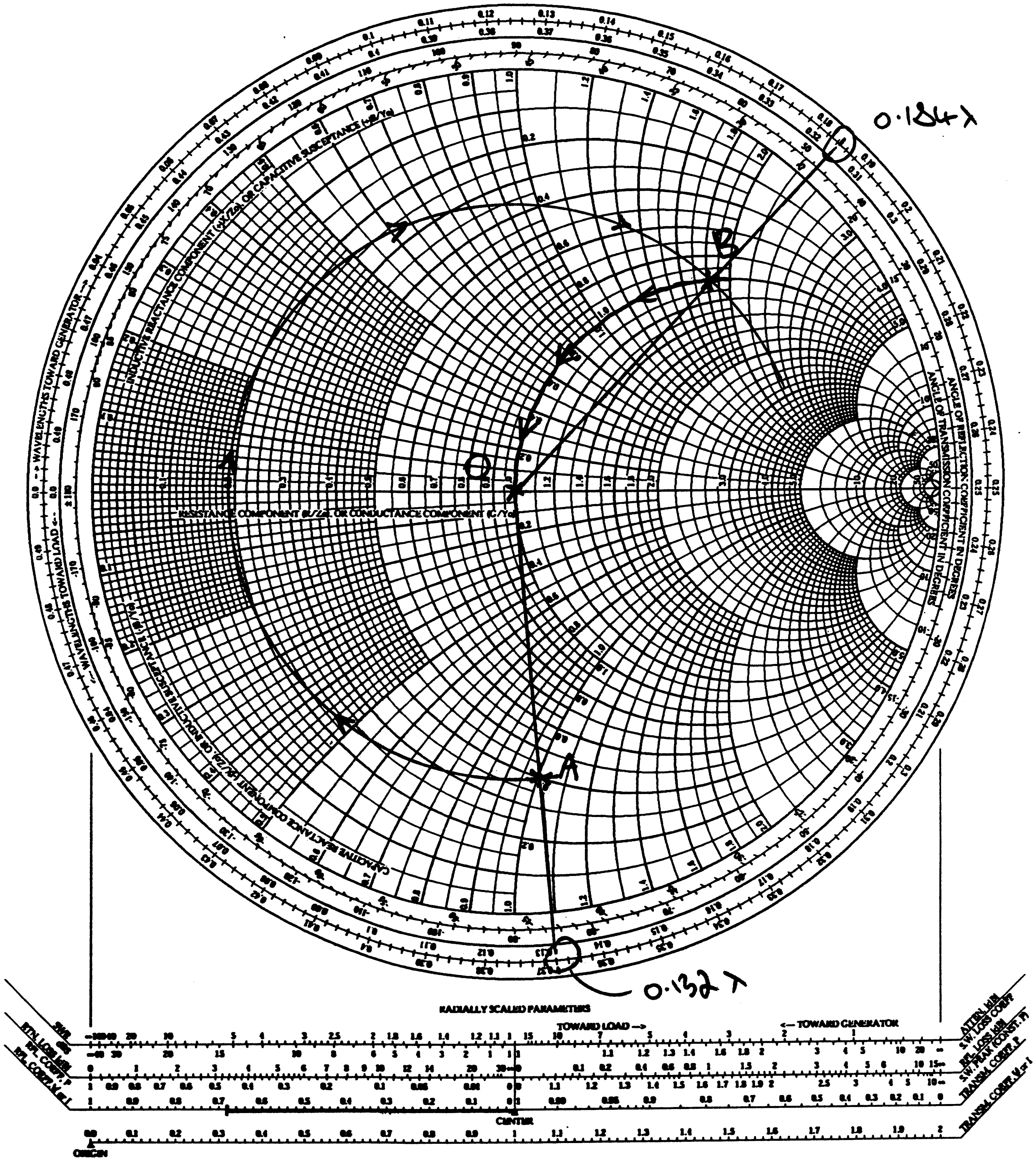
- series capacitor D→O

$$50 \times 2.2 = \frac{1}{\omega C} \qquad \therefore C = 7.2\text{ pF}$$



(parallel capacitor and series inductor soln. also possible)

Q.4c)


0.184λ
B
O
A
0.132λ
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
CENTER
ORIGIN

(10)

4c). • $0.68 \angle -85^\circ$ S_{11} plots as $(0.4 - j)$ on the Smith Chart. point A.

• To track round to point B, where the circular path centred, O, at $(1 + j0)$ cuts the Re = 1 line, we need an electrical length of $(0.132\lambda + 0.184\lambda) = 0.316\lambda$.

@ $1 + j1.82$

• To track from B to O, we need $-j1.82$ from the series capacitor.

So, $v = \dfrac{3 \times 10^8}{\sqrt{4.3}} = f\lambda$ with $f = 200 \times 10^6$ Hz

$\therefore \lambda = 0.723$ m

$\therefore 0.316\lambda \equiv$ 228 mm of microstrip

And, $-j1.82 \times 50 = \dfrac{-j}{2\pi f C_s}$

(normalising impedance for Smith chart)

$\therefore C_s = 8.7$ pF

series capacitor

(say, 3.9 pF // 4.7 pF. std. values)