ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3B4
Electric Drive Systems
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Engineering Tripos part II A.

Module 3B4 Electric Drive Systems

2004 Exam Crib

1

a) Given $B_g(\theta,t) = \sqrt{2}B_{rms}\cos(\omega t - p\theta)$

$$d\phi = B_g(\theta,t).dA = B_g(\theta,t).\frac{d}{2}.l.d\theta.$$

$$\phi = \int_0^{\pi/p} d\phi$$

$$= \left[\sqrt{2}B_{rms}\sin(\omega t - p\theta)\right]_0^{\pi/p} . l . \frac{d}{2p}$$

$$= \sqrt{2}B_{rms}\sin\omega t . l . \frac{d}{p}$$

Note: there are 2 lots of $\sin\omega t \Rightarrow$ division of 2 disappears.

$$E_{ph} = -\frac{N d\phi}{dt}.$$

$$= -\frac{Nld}{p}\sqrt{2}.\omega.\cos\omega t . B_{rms}.$$

$$= -\sqrt{2}E_{rms}\cos\omega t . B_{rms}$$

$$\Rightarrow E_{rms} = \frac{N.l.d.\omega}{p}.B_{rms} = V_{ph}.$$

b) specific electric loading for a concentrated winding:

$$\pi d \bar{J} = 3 \times 2 . N_{ph} . I_{ph}$$

$$\Rightarrow \quad \bar{J} = \frac{6 N_{ph} I_{ph}}{\pi d}$$

Specific Magnetic loading

$$\bar{B} . \frac{\pi}{p} = \int_0^{\pi/p} \sqrt{2} B_{rms} \cos(\omega t - p\theta) d\theta$$

$$\Rightarrow \quad \bar{B} = \frac{2\sqrt{2} B_{rms}}{\pi}$$

$$S = 3 V_{ph} I_{ph}$$

$$= 3 \cancel{N_{ph}} d l \frac{\omega}{p} \frac{\pi \bar{B}}{2\sqrt{2}} \frac{\pi d \bar{J}}{6 \cancel{N_{ph}}}$$

$$= \frac{\pi}{\sqrt{2}} \underline{\pi \left(\frac{d}{2}\right)^2} l \frac{\omega}{p} \underline{\underline{\bar{B}}} \, \underline{\underline{\bar{J}}}$$

c) $\hat{B} = \sqrt{2} B_{rms} = \sqrt{2} \frac{\pi}{2\sqrt{2}} \bar{B}$

$$= \frac{\pi}{2} \bar{B}$$

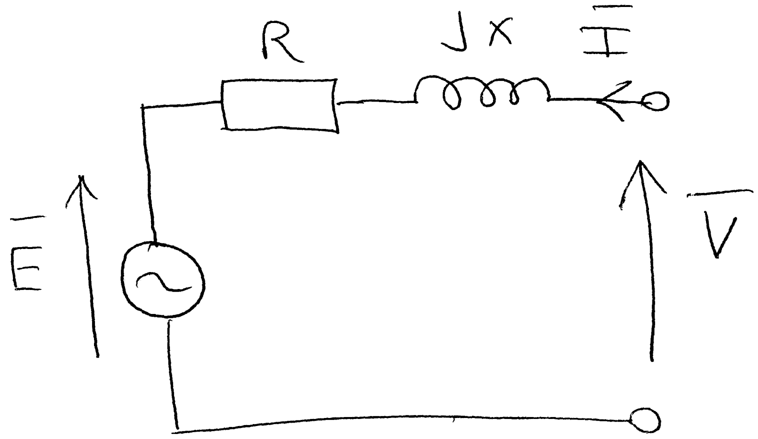
slot and teeth are equal widths

⇒ assuming that all the flux goes through the teeth we obtain

$$\hat{B}_t = 2\hat{B}$$

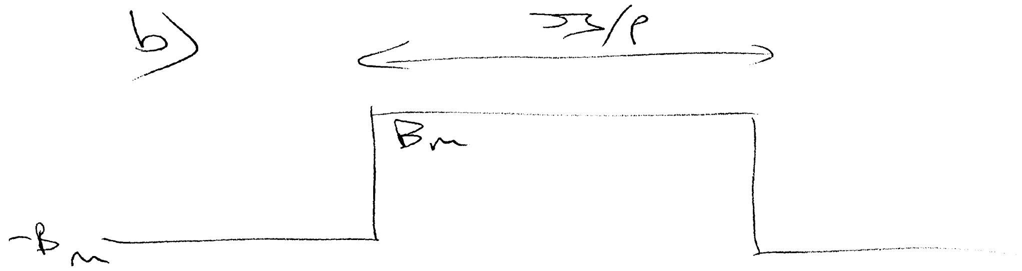
d) you can increase J by better cooling depending on the size of the machine this could be done (for small machines) by putting fins on the can or at the other extreme using hollow conductors and passing coolant down them. Another way to increase $\bar{J}$ would be to use different materials eg. oxygen free copper or possibly superconductors. could also look at increasing $\bar{B}$ by using better magnetic materials.

2 a)



$$\bar{V} = \bar{E} + JX\bar{I} + R\bar{I}$$

b)



$$\hat{\psi} = B_m \left(\frac{d}{2}\frac{\pi}{p}\right) N l$$

Flux goes $+\hat{\psi}$ to $-\hat{\psi}$ in $\pi/p\omega_r$

Hence $$\frac{d\psi}{dt} = \frac{2B_m}{\frac{\pi}{p\omega_r}} = B_m d N l \omega_r$$

c) $P = 2e_aI$ (2 phases conducting at any point in time)

$$= 2B_m d N l \omega_r I = T\omega_r$$

$$\Rightarrow T = 2B_m d N l I$$

Flux per pole $\psi = B_m \frac{d}{2} \frac{\pi}{p} l N$

$$\Rightarrow T = \frac{4p}{\pi} N \psi I$$

(since $\frac{4p}{\pi} N \psi = 2 B_m d N l$ as required)

$$\Rightarrow \quad T = k \phi I$$ where $k = \frac{4pN_{ph}}{\pi}$.

d)

Case 1 – heating up

$$\theta = \frac{P}{k}\left(1 - e^{-t/\tau}\right) + \theta_i e^{-t/\tau}$$

$$\tau = C/k$$

$$\theta_\infty = P/k.$$

Case 2 – cooling down.

$$\theta = \theta_i e^{-t/\tau}$$

Note all temperatures are expressed as temperature above ambient.

Case 1 $\Rightarrow$

$$80 = P/k\left(1 - e^{-240/\tau}\right) + 60 e^{-240/\tau}$$

Case 2 $\Rightarrow$

$$60 = 80 e^{-90/\tau}$$

$$\Rightarrow \; -\tau = \frac{90}{\ln(0.75)} \;\Rightarrow\; \tau = 312.8 \text{ seconds}$$

$$80 = \frac{200}{k}\left(1 - e^{-240/312.8}\right)$$

$$+ \; 60\, e^{-240/312.8}$$

$$\Rightarrow \; k = \frac{200\left(1 - e^{-240/312.8}\right)}{80 - 60e^{-240/312.8}}$$

$$\Rightarrow \; k = 2.06 \; \mathrm{WK^{-1}}$$

$$\tau = c/k \;\Rightarrow\; c = 2.06 \times 312.8$$

$$= 642.7$$

3. a) describe one of relative and up to three of absolute

Relative Position Measurements

1 of these

1. **Odometry** This method uses encoders to measure wheel rotation and/or steering orientation. Odometry has the advantage that it is totally self-contained, and it is always capable of providing the vehicle with an estimate of its position. The disadvantage of odometry is that the position error grows without bound unless an independent reference is used periodically to reduce the error.

2. **Inertial Navigation** This method uses gyroscopes and sometimes accelerometers to measure rate of rotation and acceleration. Measurements are integrated once (or twice) to yield position. Inertial navigation systems also have the advantage that they are self-contained. On the downside, inertial sensor data drifts with time because of the need to integrate rate data to yield position; any small constant error increases without bound after integration. Inertial sensors are thus unsuitable for accurate positioning over an extended period of time. Another problem with inertial navigation is the high equipment cost. For example, highly accurate gyros, used in airplanes, are inhibitively expensive. Very recently fiber-optic gyros (also called laser gyros), which are said to be very accurate, have fallen dramatically in price and have become a very attractive solution for mobile robot navigation.

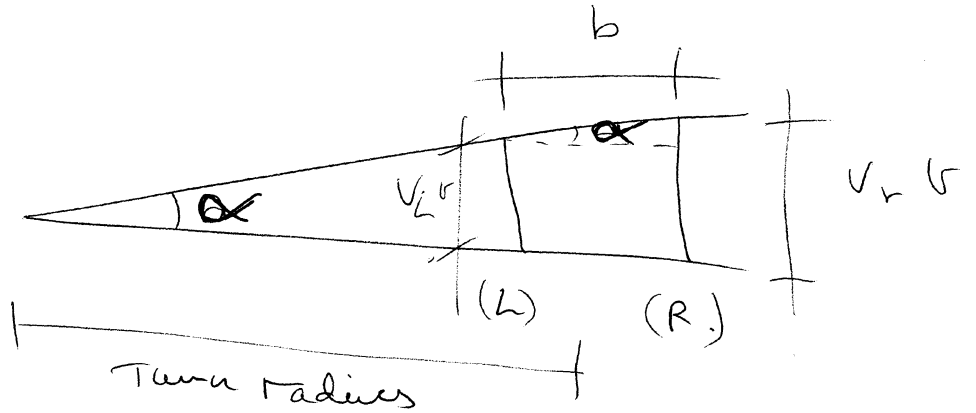
Absolute position measurement

up to 3 of these

1. **Active Beacons** This method computes the absolute position of the robot from measuring the direction of incidence of three or more actively transmitted beacons. The transmitters, usually using light or radio frequencies, must be located at known sites in the environment.
2. **Artificial Landmark Recognition** In this method distinctive artificial landmarks are placed at known locations in the environment. The advantage of artificial landmarks is that they can be designed for optimal detectability even under adverse environmental conditions. As with active beacons, three or more landmarks must be "in view" to allow position estimation. Landmark positioning has the advantage that the position errors are bounded, but detection of external landmarks and real-time position fixing may not always be possible. Unlike the usually point shaped beacons, artificial landmarks may be defined as a set of features, e.g., a shape of an area. Additional information, for example, distance, can be derived from measuring the geometric properties of the landmark, but this approach is computationally intensive and not very accurate.
3. **Natural Landmark recognition.** Here the landmarks are distinctive features in the environment. There is no need for preparation of the environment, but the environment must be known in advance. The reliability of this method is not as high as with artificial landmarks.
4. **Model Matching.** In this method information acquired from the robot's onboard sensors is compared to a map or world model of the environment. If features from the sensor-based map and the world model map match, then the vehicle's absolute location can be estimated. Map-based positioning often includes improving global maps based on new sensory observations in a dynamic environment and integrating local maps into the global map to cover previously unexplored areas. The maps used in navigation include two major types: geometric maps and topological maps. Geometric maps represent the world in a global coordinate system, while topological maps represent the world as a network of nodes and arcs.

⑦

b)



$$\frac{V_r t - V_L t}{b} = \tan\alpha$$

Also $$\frac{\frac{V_r t + V_L t}{2}}{\text{Turn radius}} = \tan\alpha$$

$$\Rightarrow \quad \frac{b(V_r + V_L)}{2(V_r - V_L)} = r.$$

wheel radius.

$$\frac{b}{2}\left(\frac{V_r + V_L}{V_r - V_L}\right) = \frac{b}{2}\left(\frac{R_R + R_L}{R_R - R_L}\right)$$

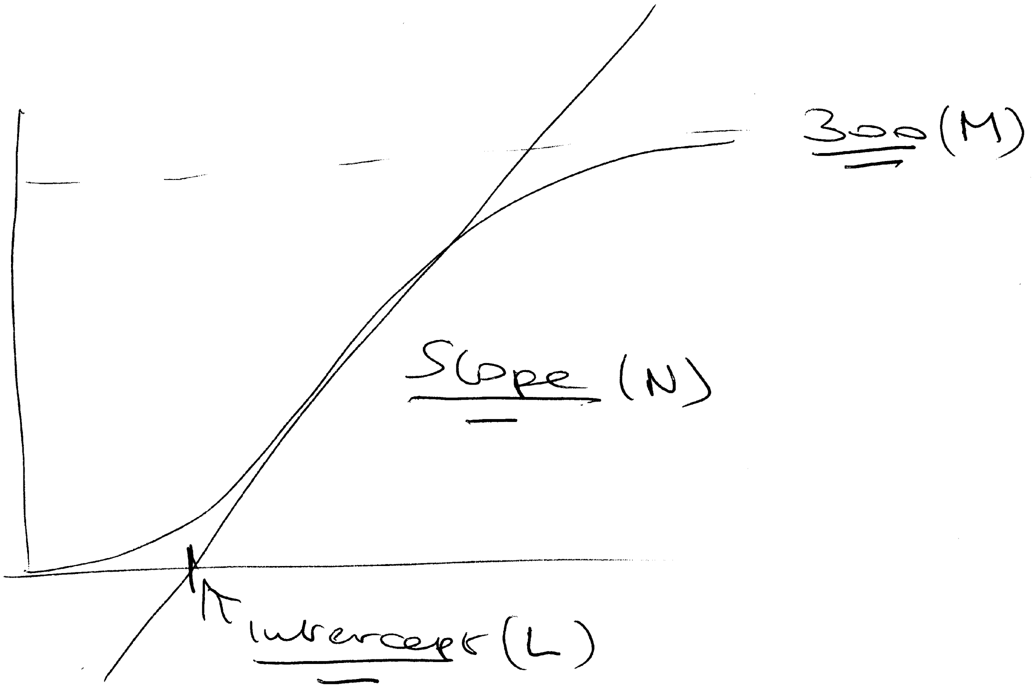
$$= \frac{b}{2}\left(\frac{2r + t\tan\theta}{t\tan\theta}\right)$$

$$= \frac{1.435}{2}\left(\frac{1.2 + 0.1\tan 2^\circ}{0.1\tan 2^\circ}\right)$$

$$= 247\text{ m.}$$

⑧

c)



$$N = \frac{245.4 - 54.6}{2} = 95.4$$

$$M = 300$$

$$L = 4$$

$$\Rightarrow K_p = \frac{1.2 \times 300}{95.4 \times 4} = 0.943$$

$$T_i = 2L = 8 \qquad T_d = 0.5L = 2.$$

d) closed loop response vs solely proportional feedback. Adjust gain until oscillations just occur. This gives K_u the so called ultimate gain and the frequency ω and period P_u of the oscillations. From K_u and P_u we can calculate the K_c T_d and T_i necessary for P/PD/PI or PID control.

⑨

where K_c is the proportional gain, and T_d and T_i are the differential and integral time constants

4

a) $T_A = -\frac{1}{2} \times 6 I_A^2 L \sin 6\theta$

$= -\frac{1}{2} I^2 6 L \frac{1}{\left(\frac{11\pi}{6} - \frac{7\pi}{6}\right)} \int_{\frac{7\pi}{6}}^{11\pi/6} \sin(6\theta)\, d(6$

$= -\frac{1}{2} I^2 6 \times \frac{6}{4\pi} \Big[-\cos(6\theta)\Big]_{7\pi/6}^{11\pi/6}$

$= -\frac{1}{2} I^2 6 L \frac{6}{4\pi} \left[-\cos\frac{11\pi}{6} + \cos\frac{7\pi}{6}\right]$

$= \frac{1}{2} I^2 6L \frac{3\sqrt{3}}{2\pi}$

$= \frac{3\sqrt{3}}{2\pi} \hat{T}$

b) $T = -\hat{T} \sin N_s\theta.$

$\simeq -\hat{T} N_s\theta$

$-\hat{T} N_s \theta = J\ddot{\theta}$

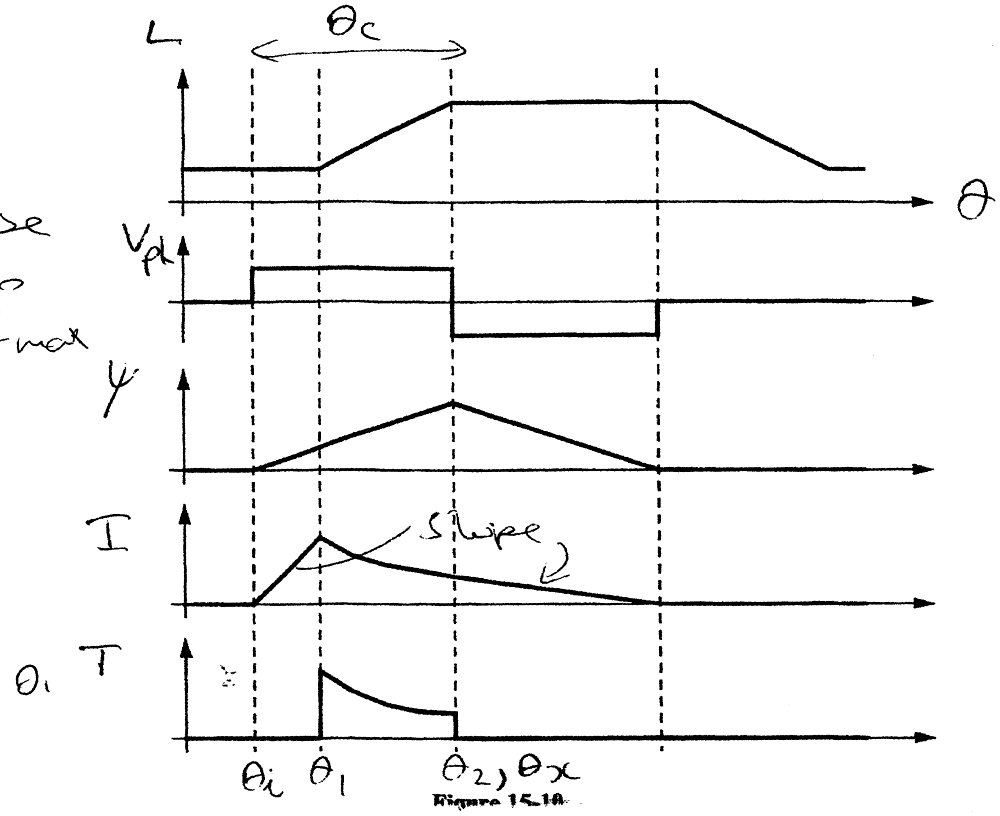
$\ddot{\theta} + q^2\theta = 0$ where $q = \sqrt{\frac{\hat{T} N_s}{J}}$

$\Rightarrow \theta = \hat{\theta} \sin q t$

wher q = the natural frequency.

⑩

c) This allows the current to reach a maximum before overlap starts to occur and the inductance starts rising. Thus generating maximum Torque. Note since the current is switched on when the inductance is a minimum $\frac{di}{dt} = \frac{v}{L}$ is a maximum and the current builds quickly.



Choose θi to get Imax at θ1

Figure 15-10

d) $$V = \frac{d}{dt}\left(LI\right) = L\frac{dI}{dt} + I\frac{dL}{dt}$$

Input power

$$VI = IL\frac{dI}{dt} + I^2\frac{dL}{d\theta}\frac{d\theta}{dt}.$$

$$= \underbrace{\frac{d}{dt}\left(\tfrac{1}{2}LI^2\right)}_{\text{rate of change of stored magnetic energy}} + \underbrace{I^2\frac{dL}{d\theta}\omega}_{\text{output Mechanical power}}$$