

**ENGINEERING TRIPOS PART IIA 2004**

Solutions to Module 3B6

Photonic Technology

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## Cribs to Photonic Technology 3B6, 2004

Q.1 (a) Answers are primarily from bookwork

- LEDs generate light by spontaneous emission, so this should be described.

The answer should include a description of a typical Surface Emitting Light Emitting Diode structure, highlighting techniques that are used to minimise thermal effects and maximise efficiency

$$(b) P_{LED} = \frac{hc}{\lambda} \cdot \frac{I}{e} \cdot \eta_{int} \cdot \eta_{ext}$$

$$\begin{aligned} \Rightarrow V_{ref} &= IR_s + V_f = \frac{P_{LED} \cdot \lambda \cdot e}{hc \eta_{int} \eta_{ext}} + \frac{hc}{e\lambda} \\ &= \underline{1.39V} \end{aligned}$$

$$(c) \eta_{int} = 1/\tau_{rr} / (1/\tau_{rr} + 1/\tau_{nr}) \Rightarrow \tau_{rr} = \frac{\tau_{nr}}{\eta_{int}} (1 - \eta_{int})$$

$$\Rightarrow \tau_{rr} = \underline{2ns.}$$

$$\tau_s = 1 / (1/\tau_{rr} + 1/\tau_{nr}) = \underline{1.2 ns.}$$

(d) The answer should describe how the drive pulse is modified with a positive spike at the start and a negative spike at the pulse end to enhance modulation response.

Q. 2(d) This is mostly a bookwork section and answers should show how mesa, ridge or buried heterostructure devices all provide improved performance.

3(a) Bookwork but to include

	Free space	Guided wave
Pros	<ul style="list-style-type: none"> <li>Unregulated EM spectrum</li> <li>High data rates cf. radio</li> <li>low physical infrastructure levels</li> </ul>	<ul style="list-style-type: none"> <li>low attenuation (&gt;100 km links possible)</li> <li>High data rates (&gt;10 Gbps)</li> <li>Compact + light cables</li> <li>Immune to EMI</li> <li>Flexible cabling architectures</li> </ul>
Cons	<ul style="list-style-type: none"> <li>Atmospheric variations in channel</li> <li>Short (&lt;1km) link lengths</li> <li>Accurate tracking or pointing required</li> </ul>	<ul style="list-style-type: none"> <li>Relatively poor source efficiency</li> <li>Fibres not mechanically robust</li> <li>Difficult to use spectrally efficient modulation formats</li> <li>COO almost universally used</li> </ul>

(b) (i) Bookwork

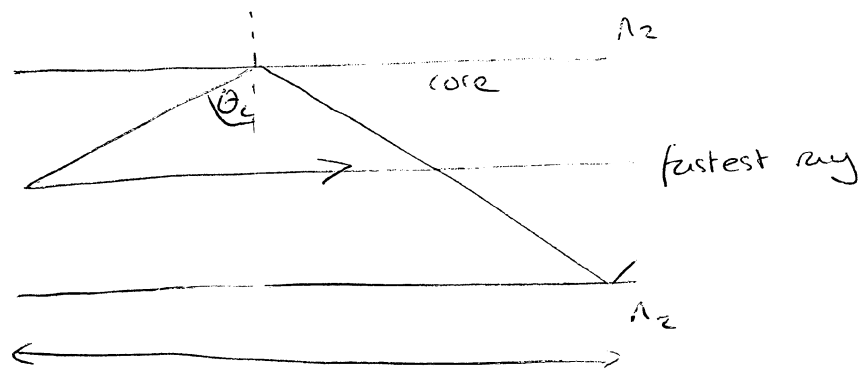
$$\begin{aligned}
 \text{(ii)} \quad V &= \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \\
 &= \frac{2\pi \cdot 50 \times 10^{-6}}{1.3 \times 10^{-6}} \sqrt{1.458^2 - 1.450^2} = 36.86
 \end{aligned}$$

Note  $V \gg 2.4 \Rightarrow$  fibre is heavily multi-moded

$$\text{No. of modes} \quad N \approx \frac{V^2}{2} = \frac{36.86^2}{2} = 679$$

- (iii) Bookwork but should include discussion of chromatic, waveguide + inter-modal dispersion. Inter-modal dispersion is by far the biggest contributor to dispersion in step index MMF. It is therefore safe to assume that chromatic + waveguide effects can be ignored.

Inter-modal dispersion



Critical angle given by

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{Snell's Law})$$

Fastest ray takes time  $t_{\min}$  to traverse  $L$

$$t_{\min} = \frac{L n_1}{c}$$

Slowest ray takes

$$t_{\max} = \frac{L n_1}{c \sin \theta_c} = \frac{L n_1^2}{c n_2}$$

Difference between fastest & slowest rays

$$\begin{aligned}\Delta t &= t_{\max} - t_{\min} \\ &= \frac{Ln_1^2}{cn_2} - \frac{Ln_1}{c}\end{aligned}$$

$$\begin{aligned}\text{Dispersion} &= \frac{\Delta t}{L} = \frac{n_1}{c} \left( \frac{n_1}{n_2} - 1 \right) = \frac{n_1}{cn_2} (n_1 - n_2) \\ &= \frac{1.458}{3 \times 10^8 \times 1.450} (1.458 - 1.450) \\ &= 26.8 \text{ ps/m or } 26.8 \text{ ns/km}\end{aligned}$$

$$(iv) \quad t_{\text{out}}^2 = t_{\text{in}}^2 + \Delta t^2$$

$$\text{So for } 1.25 \text{ Gb/s, } t_{\text{in}} = \frac{1}{1.25 \times 10^9} = 0.8 \text{ ns}$$

$$t_{\text{out}} = 1.5 t_{\text{in}} = 1.2 \text{ ns}$$

$$\Rightarrow \Delta t^2 = t_{\text{out}}^2 - t_{\text{in}}^2 = 1.2^2 - 0.8^2 = 0.8 \text{ ns}^2$$

$$\Delta t = \sqrt{0.8} = 0.894 \text{ ns}$$

$$\Delta t = \text{dispersion} \times L_{\max}$$

$$\Rightarrow L_{\max} = \frac{\Delta t}{\text{dispersion}} = \frac{0.894 \text{ ns}}{26.8 \text{ ns/km}}$$

$$= 0.0334 \text{ km or } 33.4 \text{ m}$$

(c) You could use single mode fibre but the most common technique for these distances would be to use graded index multimode fibre. Dispersion of order  $< 1 \text{ ns/km}$  are possible, allowing 550m lengths to be achieved.

4(a) Bookwork but to include descriptions of quantum, thermal and shot noise terms

(b) (i) Photocurrent  $I_{ph}$  for given input power  $P$

$$I_{ph} = \eta \frac{e \lambda}{hc} P$$

$$\text{Responsivity } R = \frac{I_{ph}}{P} = \frac{\eta e \lambda}{hc}$$

$$= \frac{0.9 \times 1.602 \times 10^{-19} \times 1535 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$$= 1.114 \text{ A/W} = 1.11 \text{ A/W (3s.f.)}$$

(ii) For given pulse energy  $E$

$$\text{No of photogenerated electrons } N = \frac{\eta \lambda E}{hc}$$

For quantum limited sensitivity, assume prob. density function for electrons is Poisson

$$P(N|s) = \exp(-s) \frac{s^N}{N!} \quad \text{where } P(N|s) \text{ is prob. of } N \text{ electrons being generated from } s \text{ photons}$$

Assume that if one electron (or more) is detected, bit is received as a "1"

$$\begin{aligned} \text{Prob Error} &= \frac{1}{2} (P(0|1) + P(1|0)) \\ &= \frac{1}{2} \exp(-s) \frac{s^0}{0!} = 10^{-12} \end{aligned}$$

$\rightarrow$  if no other noise  $\Rightarrow = 0$

$$\Rightarrow \exp(-s) = 2 \times 10^{-12}$$

$$\Rightarrow s = 27 \text{ photons/bit}$$

So quantum limited sensitivity can be calculated as follows

Assume half the bits in a stream are "1"s + half "0"s

$$\begin{aligned} \text{Average no. of photons/sec @ 27 photons/bit} \\ = 0.5 \times 10^9 \times 27 = 135 \times 10^9 \text{ photons/sec} \end{aligned}$$

$$\begin{aligned} \text{Quantum limited sensitivity} &= 135 \times 10^9 \text{ photons/s} \times \frac{hc}{\lambda} \\ &= 135 \times 10^9 \times 6.62 \times 10^{-34} \times \frac{3 \times 10^8}{1535 \times 10^{-9}} \\ &= 17.5 \text{ nW} \\ &= -47.6 \text{ dBm} \end{aligned}$$

$$\begin{aligned} \text{(iii) SNR} &= \frac{\left( \frac{qe\lambda}{hc} P_{\text{sig}} \right)^2 \leftarrow (\text{sig current})^2}{\left[ \underset{\substack{\text{shot} \rightarrow \\ \text{noise}}}{2e \left( \frac{qe\lambda}{hc} P_{\text{sig}} + I_d \right)} + \underset{\substack{\leftarrow \\ \text{thermal noise}}}{\frac{4kT}{Z}} \right] B} \\ &= \frac{(R P_{\text{sig}})^2}{\left[ 2e (R P_{\text{sig}} + I_d) + \frac{4kT}{Z} \right] B} \quad Z = \text{load impedance} \\ &= \frac{(1.114 \times 4 \times 10^{-6})^2}{(1.602 \times 10^{-19} (1.114 \times 4 \times 10^{-6} + 0.5 \times 10^{-9}) + \frac{4 \times 1.38 \times 10^{-23} \times 298}{10^3}) 10^6} \\ &= 111.1 \\ &= 20.5 \text{ dB} \end{aligned}$$

↑  
this term dominated.



cc) (i) Bookwork

$$\text{(iii) SNR for APD} = \frac{M^2 (R P_{\text{sig}})^2}{\left[ 2eM^{2+x}(R I_d) + \frac{4kT}{Z} \right] B}$$

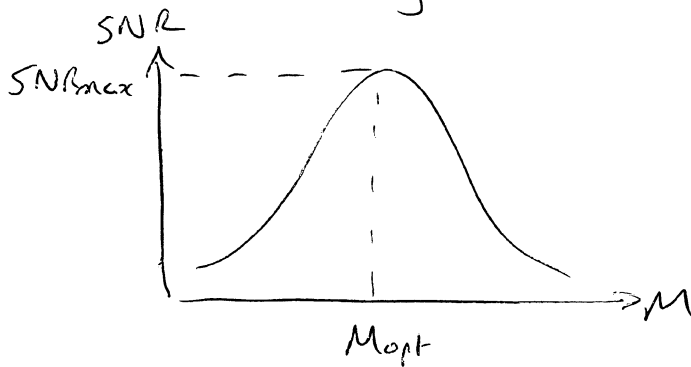
↙ signal power increased by  $M^2$

↗ shot noise increased by  $M^{2+x}$   
- dominates for high  $M$

↖ thermal noise unchanged.

By inspection, the SNR v.  $M$  curve will

look something like



Can find  $M_{\text{opt}}$  (+  $\infty$  SNR max) by

finding  $\frac{d(\text{SNR})}{dM}$  and setting  $= 0$