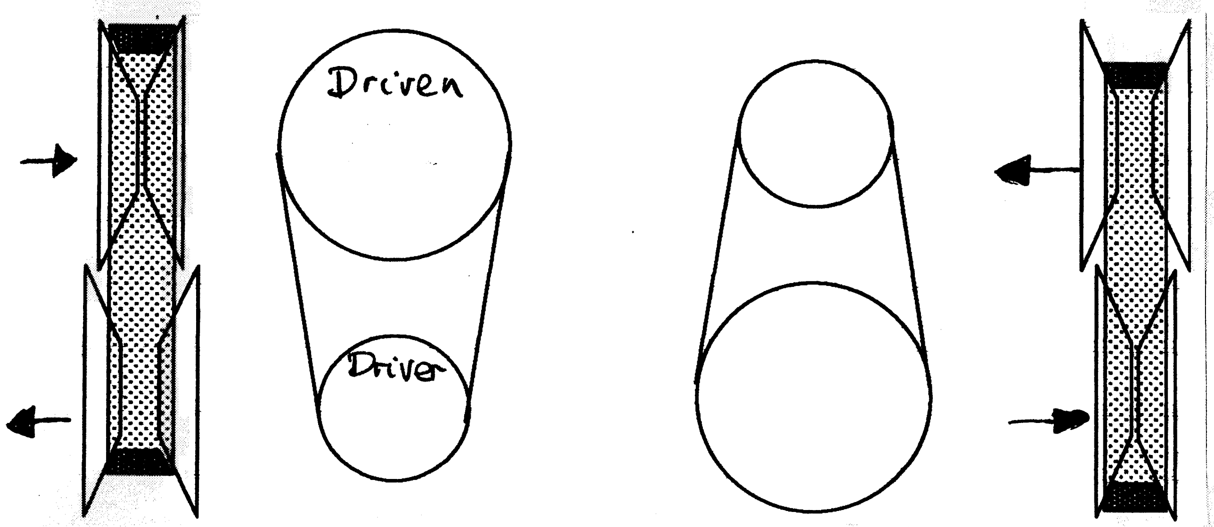
ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3C4
Machine Design - Transmissions
Principal Assessor: Dr D J Cole
Second Assessor: Dr M P F Sutcliffe

3C4 - 2004

1 a)



Generate speed change by moving pulleys apart. Need to have control system to change ratio as a function of speed (using weights, or electrically)

Belt fatigue - use belt with teeth to help curvature.

Slippage - calculate power as a function of speed, belt mass etc. Derive capstan equation including centripetal accelerations to find the change in tension from tight to slack side. Maximum power when complete slippage occurs. The effect of V angle can be dealt with using an effective μ. Could change belt tensions or materials to increase power.

Efficiency - use mass flow and belt elasticity equations to calculate η as a function of power transmitted. Could increase η by using stiffer belt, but may then have fatigue problems.

Belt Tensions - trade off between higher power possible and shorter fatigue life.

b) i) $\alpha = 0$ (C_1 fixed)

data sheet $\omega_S = (1+R)\omega_C - R\omega_A$ where $R = \frac{A}{S}$

First set: $\omega_i = \left(1 + \frac{80}{40}\right).0 - \frac{80}{40} . A_1$

$$\omega_i = -2A_1$$

Second set: $S_2 = C_1$, $C_2 = A_1$

$$S_2 = C_1 = \left(1 + \frac{90}{30}\right)A_1 - \frac{90}{30}A_2$$

$$0 = -(1+3)\frac{\omega_i}{2} - 3\omega_o$$

$$\underline{\underline{\omega_o = -\omega_i \frac{2}{3}}}$$

ii) now $C_1 = \alpha\omega_i$

First set: $\omega_i = \left(1 + \frac{80}{40}\right)\alpha\omega_i - \frac{80}{40}A_1$

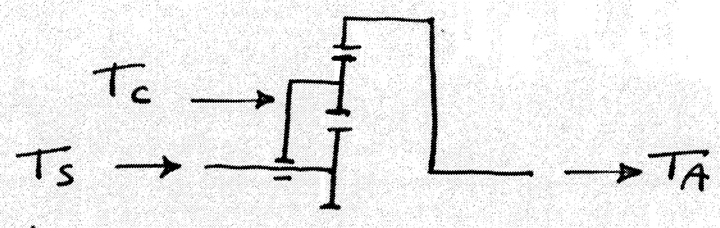
$$\omega_i(3\alpha - 1) = 2A_1$$

Second set: $C_i = \alpha\omega_i = \left(1+\frac{90}{30}\right)\frac{\omega_i}{2}(3\alpha-1) - \frac{90}{30}\omega_o$

$$\alpha\omega_i = 2\omega_i(3\alpha-1) - 3\omega_o$$

$$\omega_o = \omega_i\left(\frac{5\alpha-2}{3}\right)$$

iii) consider torques on second set:



virtual speeds, and conservation of power:

$$T_S\,\omega_S' + T_C\,\omega_C' + T_A\,\omega_A' = 0$$

speed equation

$$\omega_S' = (1+R)\omega_C' - R\omega_A' \quad \text{where } R = \frac{A}{S}$$

put $\omega_C' = 0$ $\quad \frac{T_S}{T_A} = -\frac{\omega_A'}{\omega_S'}\Big|_{\omega_C'=0} = \frac{1}{R} = \frac{1}{\frac{90}{30}} = \frac{1}{3}$

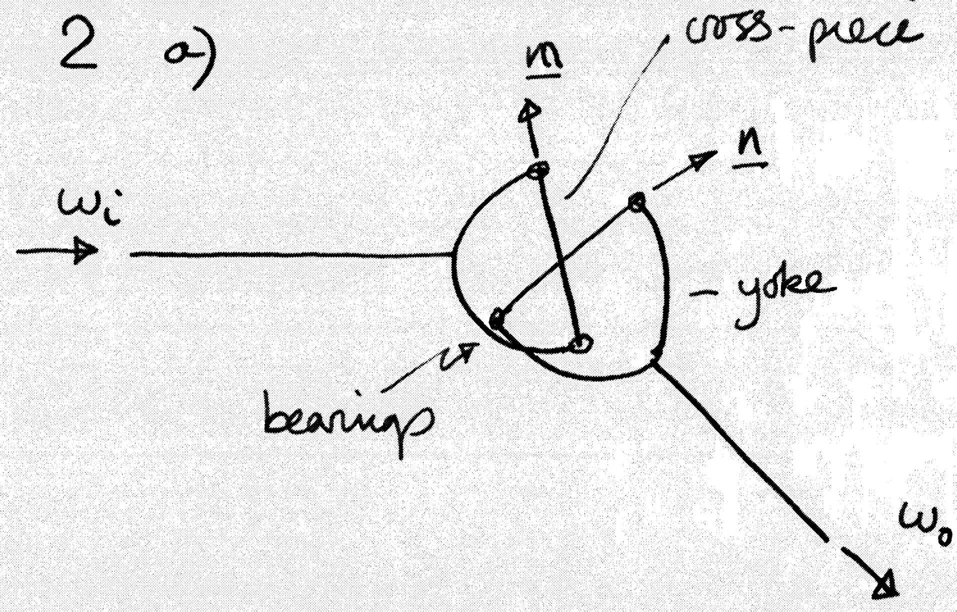
power $\quad \frac{T_{S2}\,\omega_{S2}}{T_A\,\omega_{A2}} = \frac{1}{3}\,\frac{\omega_{S2}}{\omega_{A2}}$

where $\quad \frac{\omega_{S2}}{\omega_{A2}} = \alpha\frac{\omega_i}{\omega_o} = -\frac{1}{8}\;\frac{3}{(5/8-2)} = -\frac{1}{8}\cdot\frac{24}{11}$

$\therefore \quad \frac{T_{S2}\,\omega_{S2}}{T_A\,\omega_{A2}} = -\frac{1}{3}\cdot\frac{1}{8}\cdot\frac{24}{11} = -\frac{1}{11}$ (–ve because power flows out of output shaft)

Power into S_2 is $\frac{1}{11}$ of power through output shaft.

2 a)

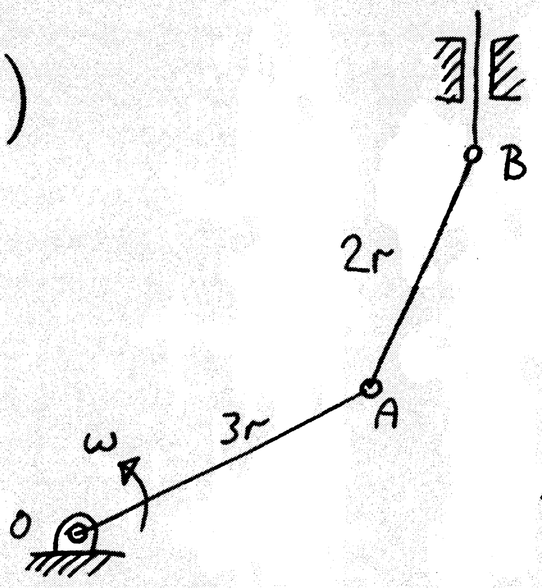


A variation in output speed can lead to vibration and torque fluctuations, and eventually premature fatigue failure.

Variation in speed ratio deduced by setting up vectors $\underline{m}$ and $\underline{n}$ for the positions of the yokes as a function of shaft orientations. Then differentiate to find the speed variation.

In practice use two Hooke's joints with the internal yoke elements in phase.
Or even neater, shrink the two joints into a single ball bearing set, as in a Birfield joint.

b) i)

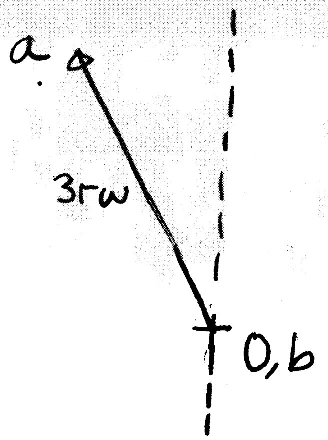


By inspection, max lift when O, A, B are collinear.

ii)

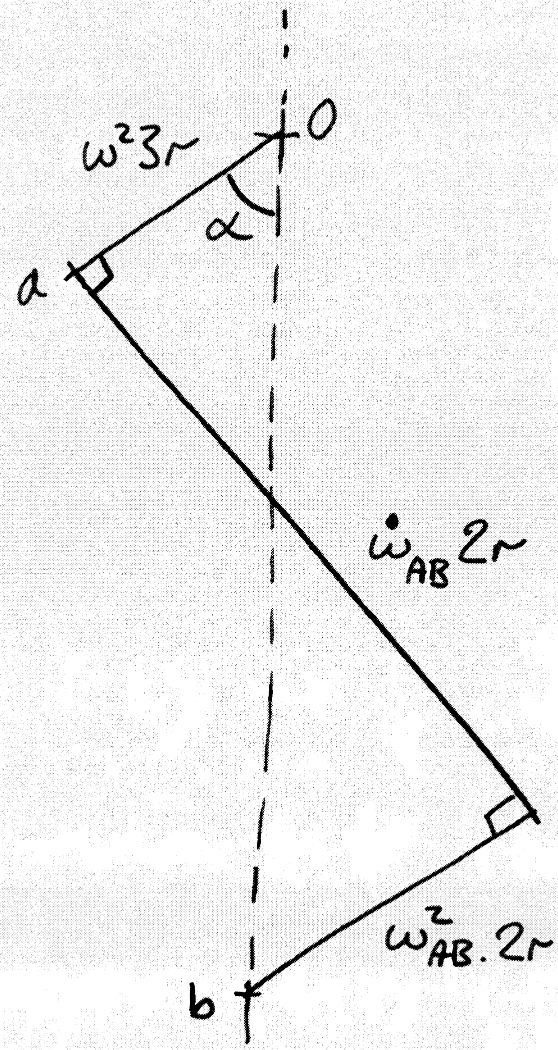
Minimum lift occurs when follower is on base circle. Radius is constant so acceleration is zero.

Velocity diagram at maximum lift:



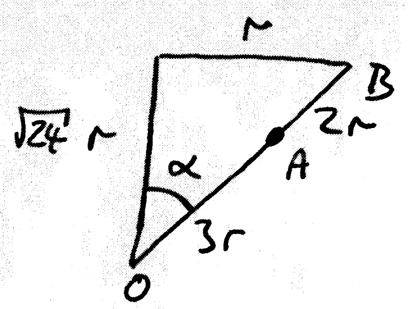
$$\omega_{AB} = \frac{3r\omega}{2r} = \frac{3}{2}\omega$$

Acceleration diagram at max lift:



Hence acceleration of B is $\dfrac{\omega^2 r\left(3+\frac{9}{2}\right)}{\cos\alpha}$ ↓

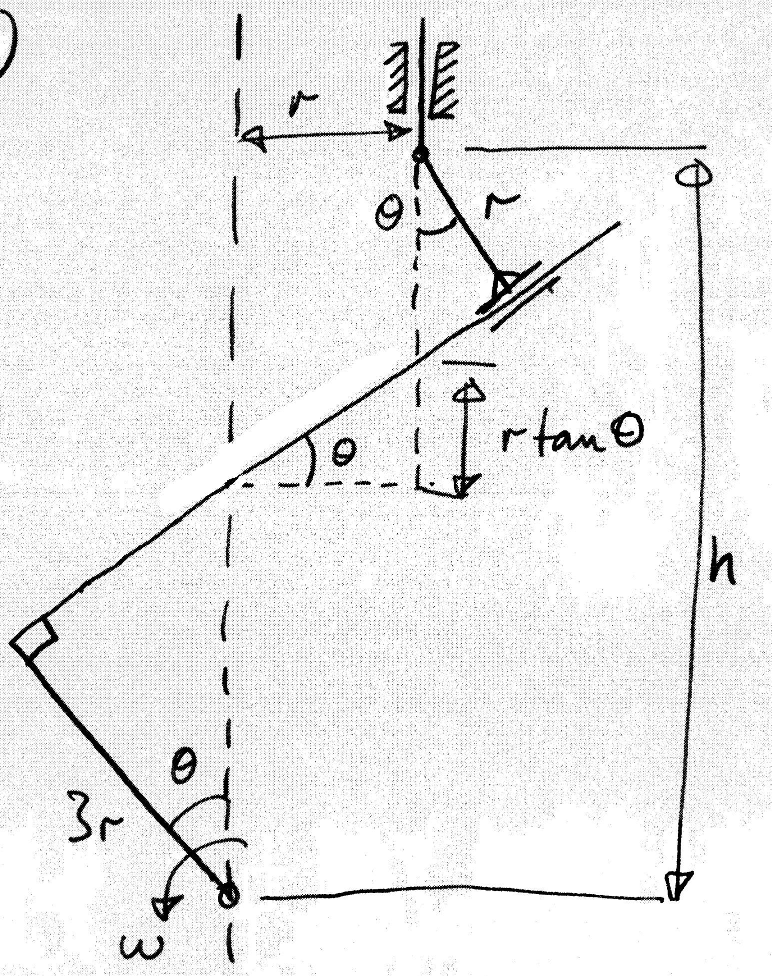
where $\cos\alpha = \dfrac{\sqrt{24}}{5}$



$\therefore$ accn B $= \omega^2 r \, \dfrac{15}{2} \cdot \dfrac{5}{\sqrt{24}}$

$= \omega^2 r \, \dfrac{75}{2\sqrt{24}}$ ↓

iii)



$$h = \frac{3r}{\cos\theta} + r\tan\theta + \frac{r}{\cos\theta}$$

$$= \frac{4r}{\cos\omega t} + r\tan\omega t$$

$$\frac{dh}{dt} = \frac{4r\omega\sin\omega t}{\cos^2\omega t} + \frac{\omega r}{\cos^2\omega t}$$

$$\frac{d^2h}{dt^2} = \omega^2 8r\frac{\sin^2\omega t}{\cos^3\omega t} + 2r\omega^2\frac{\sin\omega t}{\cos^3\omega t} + \frac{4r\omega^2}{\cos\omega t}$$

$$= 8\omega^2 r\frac{\sin^2\theta}{\cos^3\theta} + 2\omega^2 r\frac{\sin\theta}{\cos^3\theta} + \frac{4\omega^2 r}{\cos\theta}$$

3 (a)

$$\text{Torque } T = 80\left(1 - \frac{\omega}{20}\right)$$

$$\text{Power } P = T\omega = 80\omega - 4\omega^2$$

$$\frac{dP}{d\omega} = 80 - 8\omega$$

max P when $\frac{dP}{d\omega} = 0$, $\omega = 10$ rad/s

then $P_{max} = 80.10 - 4.10^2 = 400$ W

Find speeds when $P = 0.94\, P_{max}$

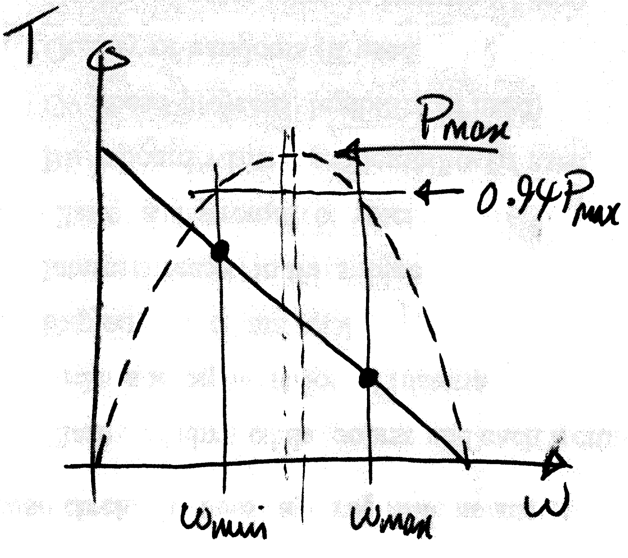
$$P = 400 \times 0.94 = 80\omega - 4\omega^2$$

$$4\omega^2 - 80\omega + 376 = 0$$

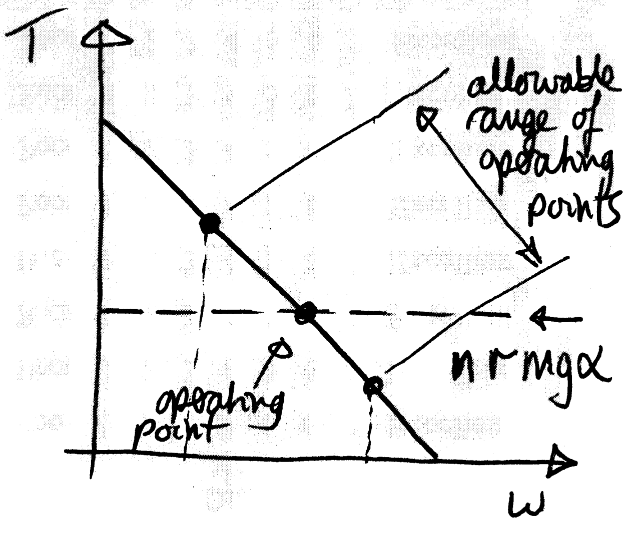
$$\omega = \frac{80 \pm \sqrt{80^2 - 4.4.376}}{2.4}$$

$\omega_{max} = 12.45$ rad/s, $\omega_{min} = 7.55$ rad/s

(b)



Torque and Power output characteristic



Load characteristic ----
superimposed on output characteristic ——

(c) Operating point is where load characteristic intersects output characteristic:

$$T = T_{wheel} \cdot n \quad \text{where} \quad n = \frac{\omega_{wheel}}{\omega}$$

$$80 - 4\omega = r.m.g\alpha \cdot n$$

$$\therefore \alpha = \frac{80-4\omega}{rmgn} \quad \text{———} \quad (1)$$

Need to allow for α between 0.05 and 0.2

What range of α can be accommodated between ω_{min} and ω_{max} for a given n?

Let $$\frac{1}{\beta} = \frac{\alpha_{\omega MAX}}{\alpha_{\omega MIN}} = \frac{\frac{80-4\omega_{max}}{rmgn}}{\frac{80-4\omega_{min}}{rmgn}} = \frac{80-4\omega_{max}}{80-4\omega_{min}}$$

$$\frac{1}{\beta} = \frac{80-4.12.45}{80-4.7.55} = \frac{1}{1.649}$$

So number of ratios N needed to get from $\alpha = 0.05$ to $\alpha = 0.2$ is given by:

$$0.05 \times \beta^N = 0.2$$

$$N = \frac{\log \frac{0.2}{0.05}}{\log 1.649} = \underline{\underline{2.77}}$$

Round up to nearest integer to give $\underline{\underline{3 \text{ ratios}}}$

Let first ratio n_1 cover $\alpha = 0.05$ (at $\omega = \omega_{max}$) to $0.05 . \beta$ (at $\omega = \omega_{min}$)

From (1) $n_1 = \dfrac{80 - 4\omega_{max}}{r m g \alpha} = \dfrac{80 - 4 . 12.45}{0.36 . 90 . 9.81 . 0.05}$

$\underline{\underline{n_1 = 1.90}}$

Let n_2 cover $\alpha = 0.05\beta$ (at $\omega = \omega_{max}$) to $0.05\beta^2$ (at $\omega = \omega_{min}$)

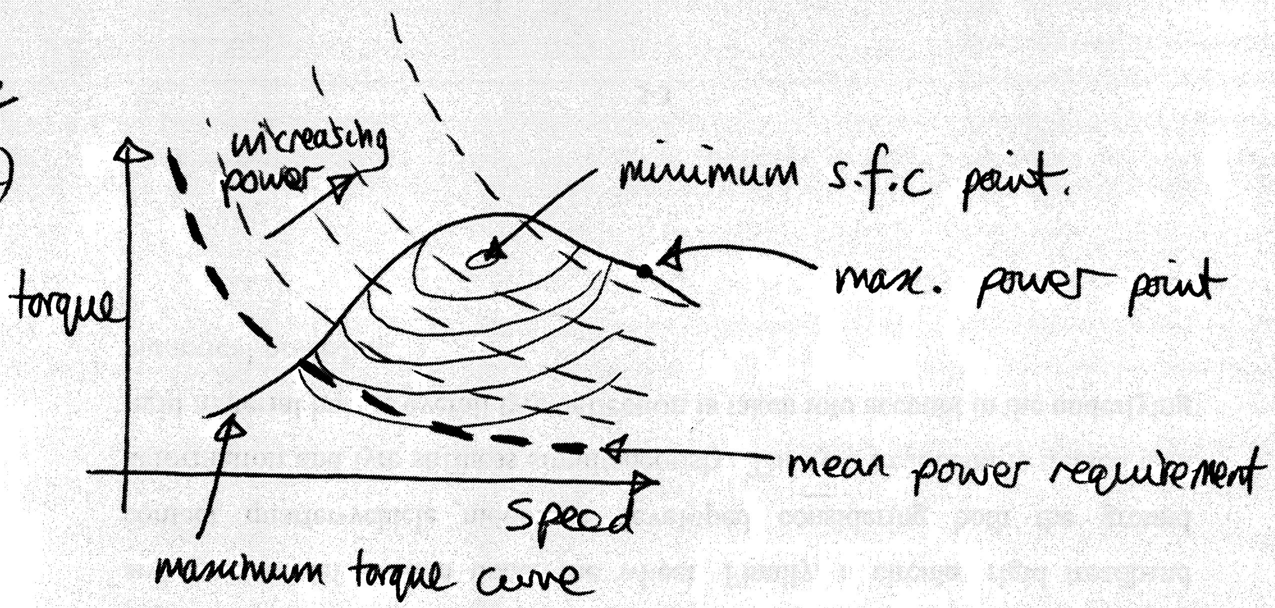
From (1) $n\alpha = $ const. for given ω

so $n_2 = \dfrac{n_1}{\beta} = \underline{\underline{1.15}}$

and $n_3 = \dfrac{n_2}{\beta} = \dfrac{n_1}{\beta^2} = \underline{\underline{0.699}}$

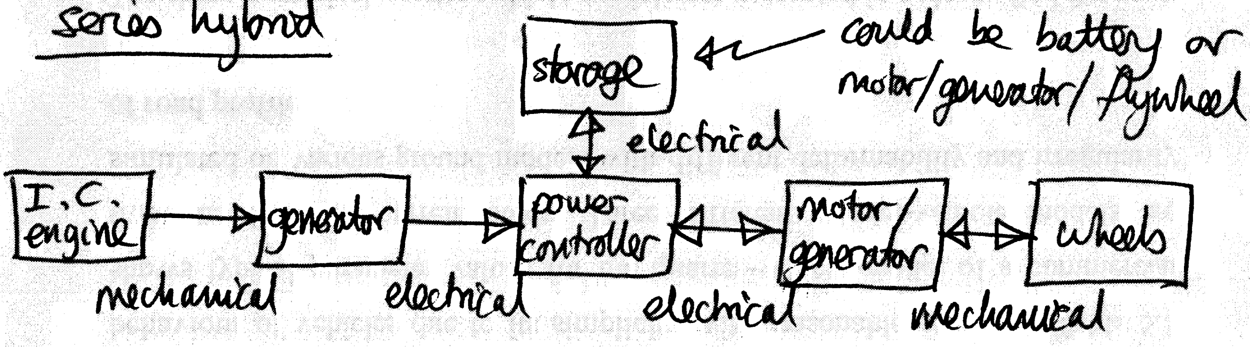
4

(a)



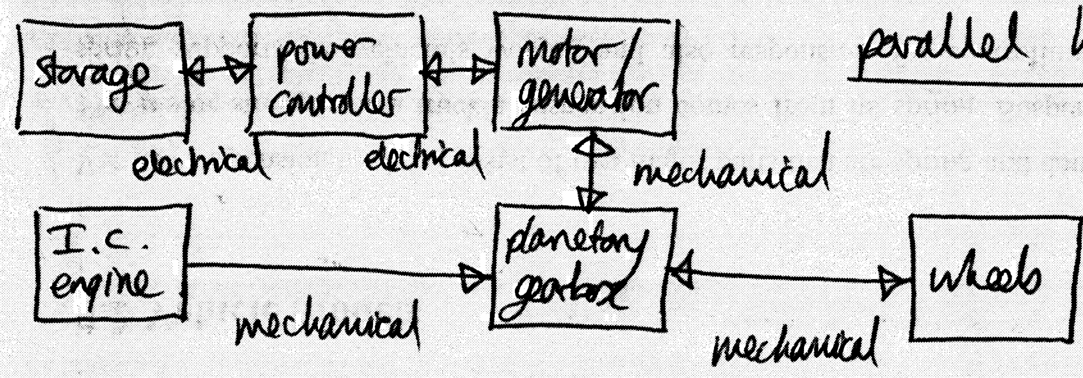
Minimum s.f.c tends to occur at a power output ($\sim \frac{3}{4}$ of maximum) much greater than mean power requirement ($\sim \frac{1}{10}$ of maximum) of car. Hence average fuel consumption is higher that derived from minimum s.f.c.

(b) <u>series hybrid</u>



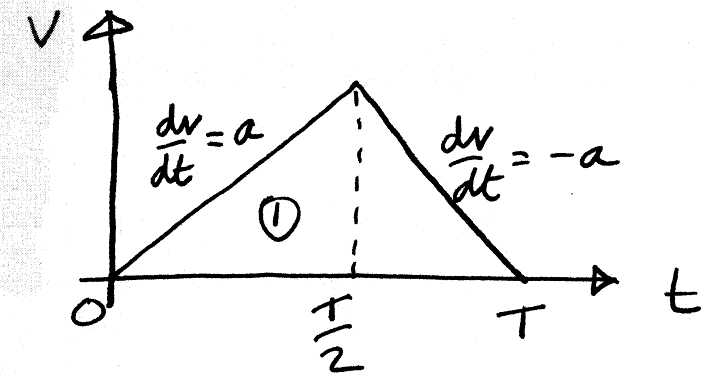
- I.C. engine is sized to provide mean power requirement and therefore can run continuously at minimum s.f.c.
- Excess energy flows into storage – during braking KE from vehicle is recovered.
- Energy flows out of storage when power demand is high e.g. acceleration.
- Losses occur in mechanical/electrical conversion – all energy from I.C. engine is converted twice
- Overall, better efficiency than conventional arrangement.

parallel hybrid



- As in series system, I.C. engine runs continuously at minimum s.f.c. to provide mean power.
- Only stored energy is converted – energy from IC engine is transmitted to wheels without conversion
- Efficiency potentially better than series hybrid.

c) i)



$$f(t) = m\frac{dv(t)}{dt} + c\,v(t)$$

power $$p(t) = f(t)\cdot v(t)$$

$$= m\frac{dv}{dt}\cdot v + cv^2$$

energy recovered during braking so zero energy required for $m\frac{dv}{dt}\cdot v$ term over period T

energy consumed $E = \int_0^{\frac{T}{2}} cv^2\,dt + \int_{\frac{T}{2}}^{T} cv^2\,dt$

where $v = at \quad 0 \leq t \leq \frac{T}{2}$

by inspection, energy consumed by viscous loss over 0 to $\frac{T}{2}$ is same as that consumed between $\frac{T}{2}$ and T

so

$$E = 2\int_0^{\frac{T}{2}} ca^2t^2\,dt$$

$$= 2\left[ca^2\frac{t^3}{3}\right]_0^{\frac{T}{2}}$$

$$= \frac{ca^2T^3}{12}$$

mean power $\bar{P} = \frac{E}{T} = \frac{ca^2T^2}{12}$

maximum power is during acceleration phase at $t = \frac{T}{2}$

$$p_{max} = ma^2\frac{T}{2} + ca^2\frac{T^2}{4}$$

$$\therefore \frac{p_{max}}{\bar{P}} = \frac{ma^2\frac{T}{2} + ca^2\frac{T^2}{4}}{\frac{ca^2T^2}{12}}$$

$$\frac{p_{max}}{\bar{P}} = \frac{6m}{cT} + 3$$

ii) The ratio is never less than 3, suggesting a hybrid drive is likely to be beneficial whatever the values of m, c and T. The ratio is minimised by minimising m and maximising c, but overall energy consumption increases with c, so minimising the ratio is not necessarily the sole objective.