ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3D1
Soil Mechanics
Principal Assessor: Professor M Bolton
Second Assessor: Professor R J Mair

Engineering Tripos Part IIA 2003-4
Module 3D1 : Solutions

1. a) For Fill A : $\rho_d = 1600\ kg/m^3$, $\underline{\gamma_d = 15.7\ kN/m^3}$

$G_s = 2.65$, so $1+e = 2.65/1.60 = 1.656$

$\therefore e = 0.656$. Then $\rho_{sat} = \dfrac{(2650 + 656)}{1.656} = 1996\ kg/m^3$

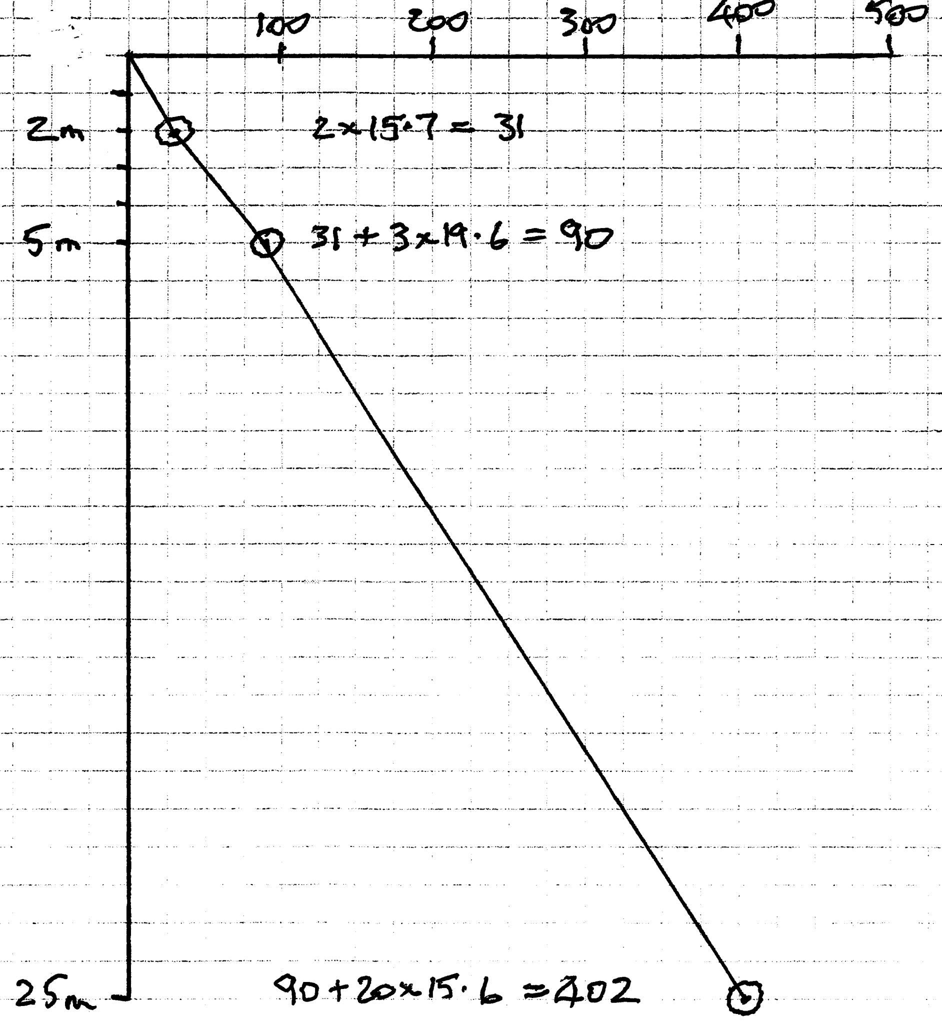
So $\underline{\gamma_{sat} = 19.6\ kN/m^3}$

For clay B : $w = 0.66$, Assume saturated

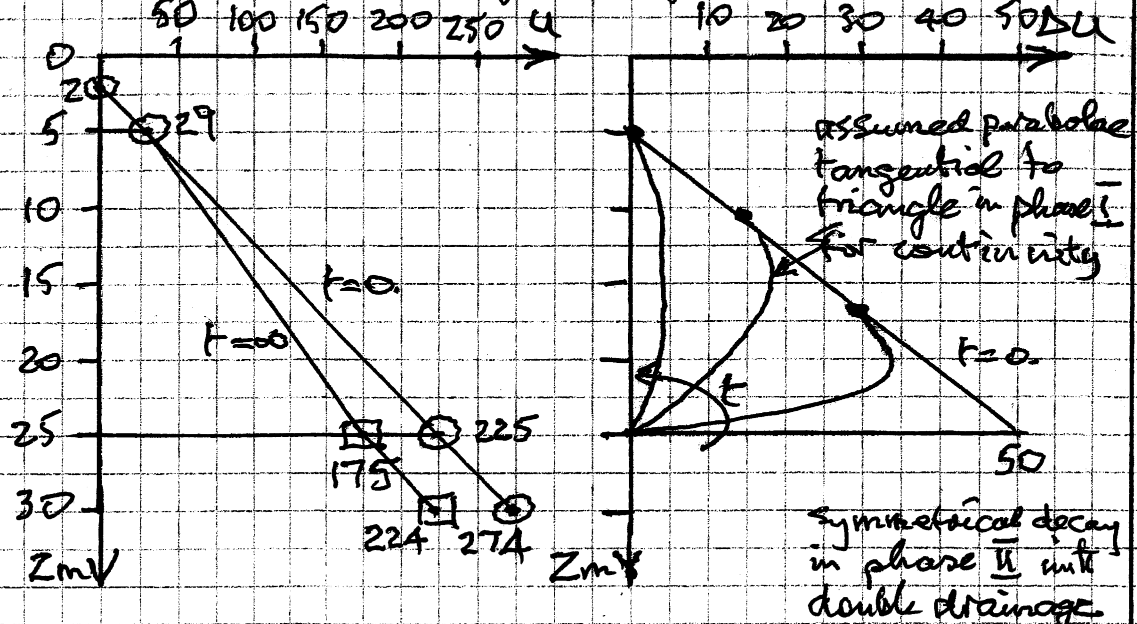
$G_s = 2.61$ (Kaolin). $\therefore e = 0.66 \times 2.61 = 1.723$

Then $\rho_{sat} = \dfrac{(2610 + 1723)}{2.723} = 1591\ kg/m^3$

So $\underline{\gamma_{sat} = 15.6\ kN/m^3}$



1. b) Take the unit weight of groundwater as 9.8 kN/m³



4

c) At $z = 15$ m, $\sigma_v = (90+402)/2 = 246$ kPa

Initially $u_0 = (29+225)/2 = 127$ kPa

Finally $u_\infty = 127 - 25 = 102$ kPa

So initial $\sigma_v' = 119$ kPa; final $\sigma_v' = 144$ kPa

Since the clay was normally compressed

$\Delta v = \lambda \ln(\sigma_v'{}_{final} / \sigma_v'{}_{initial})$

$= 0.26 \ln 1.21$

$= 0.0496$

Initially $v = 3.767 + 0.26 - 0.05 - 0.26 \ln \sigma_v'$

$= 2.734$

$\therefore \varepsilon_v = \Delta v / v = 0.0181$ at $t = \infty$

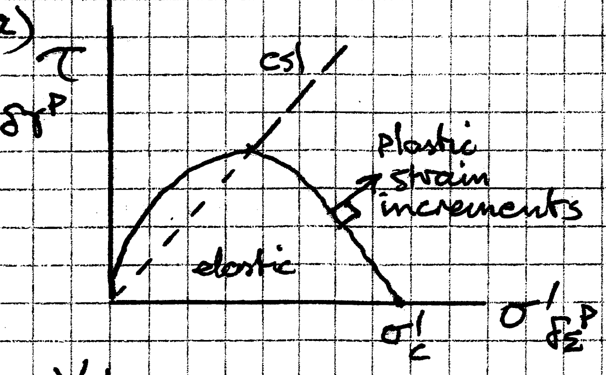
Taking this as representative of the 20 m stratum, the regional subsidence would ultimately be 0.36 m. 6

d) At $R_v = 0.10$, $T_v = 0.75 \times 10^{-2} = 10^{-7} t / 10^2$

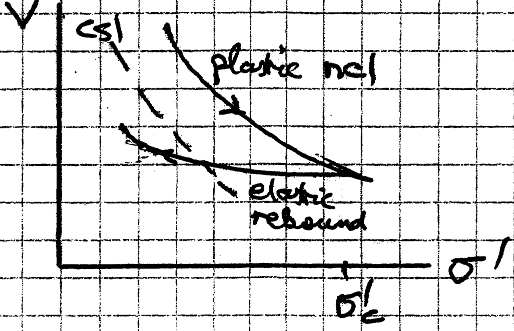
$\therefore t = 0.75 \times 10^7$ s $=$ 87 days 4

e) Only a fraction κ/λ of any subsidence would be reversed, since swelling would be "elastic". This is $0.05/0.26 = 19\%$ 2

2. a)



Plastic compression along the normal compression line expands the yield surface on a $[\tau\,\sigma']$ diagram to the current σ'_c value.



Any rebound inside the yield surface gets a non-linear elastic response, following a k-line.

Yield creates plastic components of strain normal to the yield surface. This rule gives maximum possible plastic work.

All shear tests lead the soil state towards the critical state line for large shear strains. Because no further volume change occurs at a critical state, there is no further plastic hardening or softening. 8

b) Weald Clay ncl: $V = 2.060 + 0.093 - 0.035 - 0.093 \ln\sigma'$

$= 2.118 - 0.093 \ln\sigma'$

At 100 kPa, $V_A = 1.690$

$\tau_{crit} = 100 \tan 24^\circ = 44.5\,\text{kPa}$

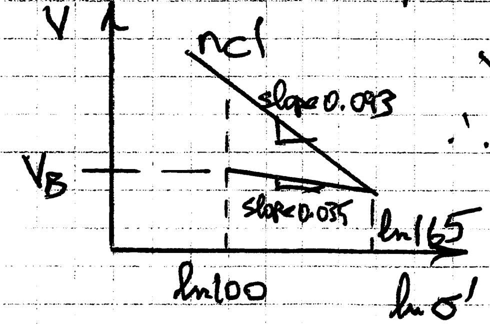
So at B, $\underline{\tau_B = 22.3\,\text{kPa}}$, $\underline{\sigma'_B = 100\,\text{kPa}}$

Find the current value of σ'_c for the soil at B.
From Data Book P.10

$\tau_B/\sigma'_B = \tan\phi_{crit} \ln(\sigma'_{cB}/\sigma'_B)$

$\therefore\ 0.223 = 0.446 \ln(\sigma'_{cB}/100)$

$\therefore\ \sigma'_{cB} = 165\,\text{kPa}$



$V_B = 2.118 - 0.093 \ln 165 + 0.035 \ln\frac{165}{100}$

$\therefore\ \underline{V_B = 1.661}$

6

c) In undrained test $\underline{V_c = V_B = 1.661}$

csl: $V_c = 2.060 - 0.093 \ln\sigma'_c$

$\therefore\ \underline{\sigma'_c = 73\,\text{kPa},\ \tau_c = 32.5\,\text{kPa}}$ 6

3. a) i) ncl: $v = 2.759 + 0.099 - 0.161 \ln \sigma'$

$= 2.858 - 0.161 \ln \sigma'$

so at $\sigma' = 500$ kPa, $\underline{V_A = 1.857}$

ii) swelling to $V_B = V_A + 0.062 \ln(500/50) = \underline{2.000}$

iii) csl: $V_C = 2.000 = 2.759 - 0.161 \ln \sigma_C'$

$\therefore \sigma_C' = 111.5$ kPa

$\therefore \tau_C = 111.5 \times \tan 23° = \underline{47.3 \text{ kPa}}$ 8

b) i) swelling from C to D, $V_D = 2.000 + 0.062 \ln \frac{115}{20}$

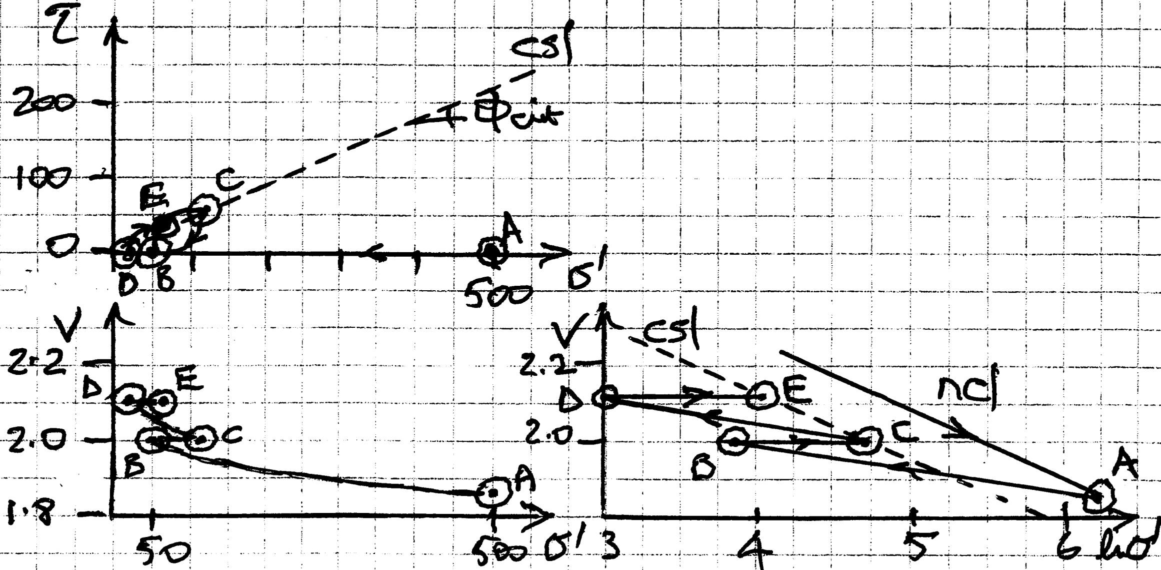
$\therefore \underline{V_D = 2.108}$

ii) csl: $V_E = 2.108$ since undrained from D

$\therefore 2.108 = 2.759 - 0.161 \ln \sigma'_E$

$\therefore \sigma'_E = 57$ kPa

$\underline{\tau_E = 24 \text{ kPa}}$ 8



c) Note that undrained strength drops by factor 2 from C to E due to intermediate swelling.
This is significant:
- eg earthwork may be unstable in short term
- eg compaction plant may get bogged down

Rules:
- avoid double handling where possible
- seal top surface of spoil heap to prevent water ingress eg recompact in stockpile
- use deep stockpiles & discard top 1m

4

4. a) Dog's Bay Sand: P10 of Data Book

loose $\sigma_c' = 500$ kPa, dense $\sigma_c' = 1500$ kPa

$\phi_{crit} = 39°$

Lower bound : loosest, $I_D \to 0$, $\phi \to 39°$
Upper bound : densest, $I_D \to 1$, $\phi_{max} > \phi_{crit}$

$$\phi_{max} = 39 + 3\left(\ln\frac{1500}{p'} - 1\right) \text{ degrees}$$

If $\sigma_1' = 300$ kPa, assume $\phi = 45°$
then $\sigma_3' = 300(1-\sin 45°)/(1+\sin 45°) = 51$ kPa
so $p' = (300 + 2\times 51)/3 = 134$ kPa

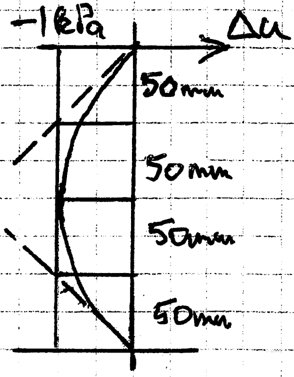
Then $\phi_{max} = 39 + 3(2.42 - 1) = 43°$
then $\sigma_3' = 57$ kPa
So $p' = 138$ kPa
This confirms $\underline{\phi_{max} = 43°}$ 10

b) Rate of dilatancy of densest soil beneath spudcan: Data Book P.13:

$$\left(\frac{-d\varepsilon_v}{d\varepsilon_1}\right)_{max} = 0.3 I_R = 0.3 \times 1.42 \simeq 0.4$$

Darcy permeability $k = 0.01(0.002)^2 = 4\times10^{-8}$ m/s as given in Data Book P4.

Assume a steady parabolic isochrone with 1kPa of excess pore suction at the centre of the sample.



exit pore pressure gradient $= 1/0.05$
$= 20$ kPa/m

$\therefore$ exit hydraulic gradient $i_0 = 2$ m/m
$\therefore$ Darcy entry velocity $V_0 = k i_0 = 8\times10^{-8}$ m/s

$$\therefore -\frac{d\varepsilon_v}{dt} = \frac{2V_0A}{(AH)} = \frac{2\times8\times10^{-8}}{0.1} = 1.6\times10^{-6}\,s^{-1}$$

But
$$\frac{d\varepsilon_1}{dt} = \left(-\frac{d\varepsilon_v}{dt}\right)\left(-\frac{d\varepsilon_1}{d\varepsilon_v}\right) = \frac{1.6\times10^{-6}}{0.4} = 4\times10^{-6}\,s^{-1}$$

So the velocity of the top platen

$$V_{top} = 200\,\text{mm} \times 4\times10^{-6}\,s^{-1}$$
$$= 0.8\times10^{-3}\,\text{mm/s}$$
$$= \underline{2.9\,\text{mm/hour}}$$

10