ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3D2
Geotechnical Engineering
Principal Assessor: Dr K Soga
Second Assessor: Professor R J Mair

Q.1

(a) For undrained conditions, constant volume continuity of mass means that

$$2\pi r \rho = \text{constant} \quad \text{(for axisymmetric condition)}$$

where ρ is the radial displacement at radius r.

Hence $r_1 \rho_1 = r_2 \rho_2$

$$\therefore \quad \rho_2 = \frac{r_1 \rho_1}{r_2}$$

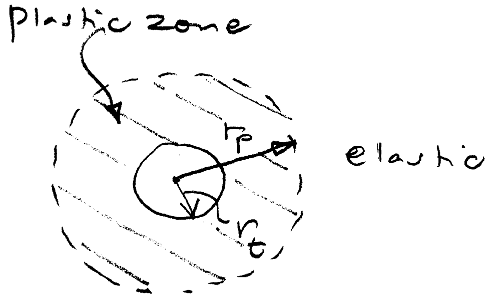
At tunnel boundary $r_1 = 2.5\text{m}$ $\rho_1 = 25\text{mm}$

At pile toe $r_2 = 7.5\text{m}$

$$\therefore \quad \rho_2 = \frac{2.5}{7.5} \times 25\text{ mm} = \underline{\underline{8.3\text{ mm}}}$$

[20%]

(b)
$$\left.\begin{aligned} \sigma_r &= \sigma_0 - \frac{G\,\delta A}{\pi r^2} \\ \sigma_\theta &= \sigma_0 + \frac{G\,\delta A}{\pi r^2} \end{aligned}\right\} \text{ for } \underline{\text{elastic}} \text{ soil}$$



$\therefore$ In elastic zone

$$\sigma_\theta - \sigma_r = 2G\frac{\delta A}{\pi r^2}$$

In plastic zone $\sigma_\theta - \sigma_r = 2c_u$

$$\therefore \text{ at boundary } (r = r_p) \quad 2c_u = 2G\frac{\delta A}{\pi r_p^2}$$

$$\therefore \quad \frac{G\,\delta A}{\pi c_u} = r_p^2$$

A is current area of cavity (tunnel) $= \pi r_t^2$

$$\therefore \quad \frac{G}{c_u}\frac{\delta A}{A} = \left(\frac{r_p}{r_t}\right)^2$$

[30%]

(c) From the Soil Mechanics Data Book

$$\delta\sigma_c = \sigma_c - \sigma_o = c_u\left(1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right)$$

$$\therefore \quad \frac{\delta A}{A} = \frac{c_u}{G}\exp\left(\frac{\delta\sigma_c}{c_u} - 1\right)$$

$$\left(\frac{r_p}{r_t}\right)^2 = \frac{G}{c_u}\frac{\delta A}{A} \quad \text{from part (b)}$$

$$\therefore \quad \left(\frac{r_p}{r_t}\right)^2 = \exp\left(\frac{\delta\sigma_c}{c_u} - 1\right)$$

$$\therefore \quad \frac{r_p}{r_t} = \left[\exp\left(\frac{\delta\sigma_c}{c_u} - 1\right)\right]^{1/2}$$

[30%]

(d) $r_p = 7.5\,m$, $r_t = 2.5 \Rightarrow r_p/r_t = 3$

$$\therefore \quad 9 = \exp\left(\frac{\delta\sigma_c}{c_u} - 1\right)$$

$$\therefore \quad \frac{\delta\sigma_c}{c_u} = 3.2$$

$c_u = 100\ kN/m^2 \Rightarrow \delta\sigma_c = 320\ kN/m^2$

$\delta\sigma_c = \sigma_o - \sigma_c$

$\sigma_o = \gamma z = 20 \times 30 = 600\ kN/m^2$

$\therefore \quad \sigma_c = \sigma_o - \delta\sigma_c = 600 - 320 = \underline{\underline{280\ kN/m^2}}$

Q.2

(a) $\sigma_{ho} = 125 \text{ kN/m}^2$

$u_o = 5 \times 10 = 50 \text{ kN/m}^2$

$\therefore \sigma_{ho}' = \sigma_{ho} - u_o = 75 \text{ kN/m}^2$

$\sigma_{vo} = 20 \times 5 = 100 \text{ kN/m}^2$

$\sigma_{vo}' = \sigma_{vo} - u_o = 100 - 50 = 50 \text{ kN/m}^2$

$$K_o = \frac{\sigma_{ho}'}{\sigma_{vo}'} = \frac{75}{50} = \underline{\underline{1.5}}$$

This means that the clay is overconsolidated and probably there would have been a considerable overburden that has been eroded.

[20%]

(b) insitu $p' = \frac{1}{3}(\sigma_{vo}' + 2\sigma_{ho}')$

$= \frac{1}{3}(50 + 2 \times 75) \text{ kN/m}^2$

$= 66.7 \text{ kN/m}^2$

cell pressure applied, $p = 100 \text{ kN/m}^2$

$\therefore$ measured pore pressure $= p - p'$

$= 100 - 66.7$

$= \underline{\underline{33.3 \text{ kN/m}^2}}$ [20%]

(c) Original stress states:

Total stress A : $\sigma_v = \sigma_{vo} = 100 \text{ kN/m}^2$

$\sigma_h = \sigma_{ho} = 125 \text{ kN/m}^2$

Pore pressure $u_o = 50 \text{ kN/m}^2$

Effective stress state A' : $\sigma_v' = \sigma_{vo}' = 50$ kN/m²

$\sigma_h' = \sigma_{ho}' = 75$ kN/m²

Undrained excavation : A → B (σ_{vo}, p_s)

Pore pressure $u_o \to u_1$ A' → B'

elastic, undrained ∴ mean normal effective stress remains constant

plane strain : $s' = \frac{1}{2}(\sigma_v' + \sigma_h') = $ constant

$$\therefore \Delta s' = \frac{1}{2}(\Delta\sigma_v' + \Delta\sigma_h') = 0$$

$$\therefore \Delta\sigma_h' = -\Delta\sigma_v'$$

$$\Delta\sigma_v' = \sigma_{vB}' - \sigma_{vA}' = (\sigma_{vo} - u_1) - (\sigma_{vo} - u_o)$$
$$= u_o - u_1$$

(vertical total stress remains constant $= \sigma_{vo}$)

$$\Delta\sigma_h' = \sigma_{hB}' - \sigma_{hA}' = (p_s - u_1) - (\sigma_{ho} - u_o)$$

$$\therefore p_s - u_1 - \sigma_{ho} + u_o = -(u_o - u_1)$$

$$\therefore u_1 = u_o - \frac{1}{2}(\sigma_{ho} - p_s)$$
$$= 50 - \frac{1}{2}(125 - 75) = \underline{\underline{25 \text{ kPa}}}$$

B : $\sigma_v = \sigma_{vo} = 100$; $\sigma_h = p_s = 75$

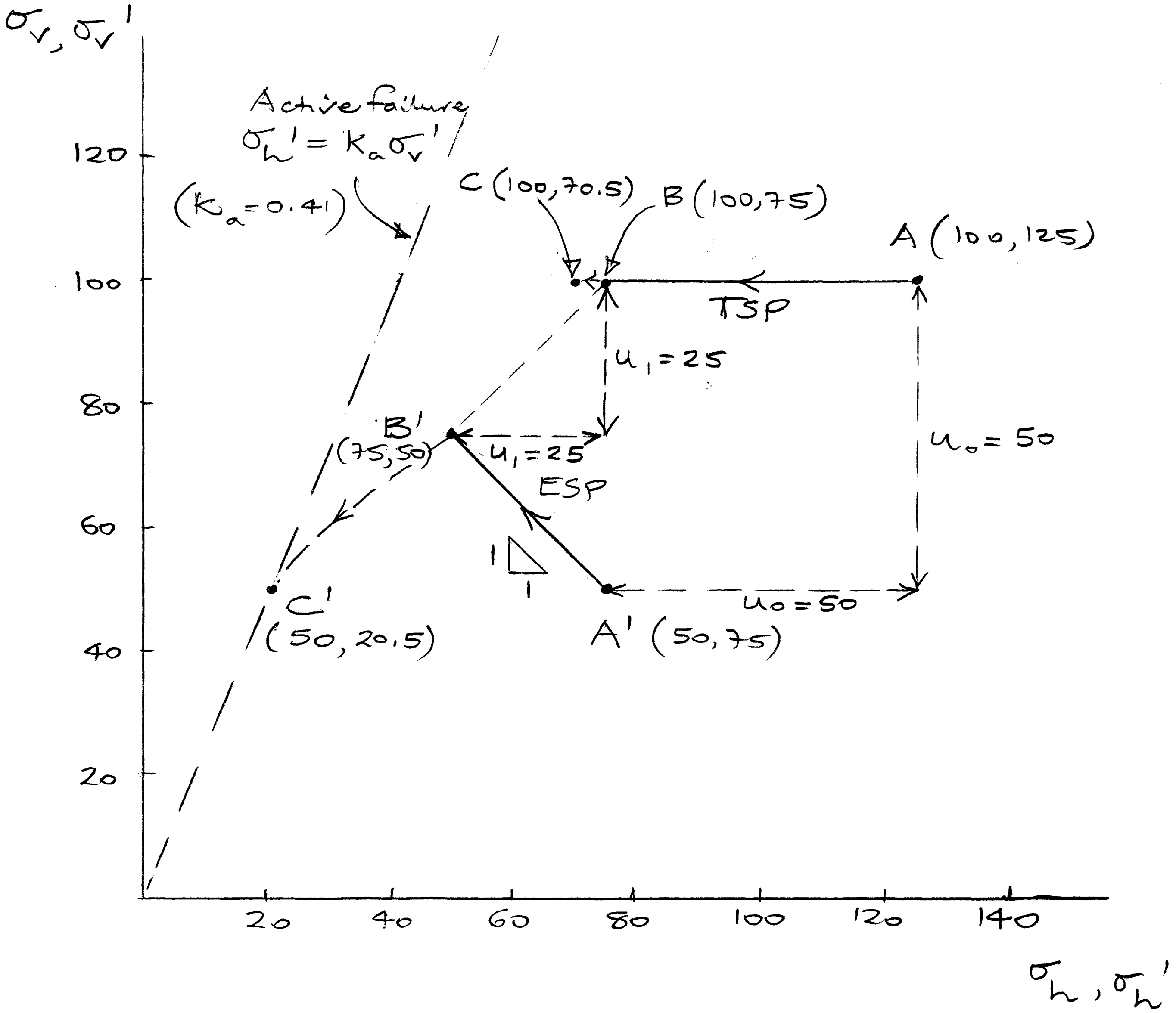
B' : $\sigma_v' = \sigma_{vo} - u_1 = 100 - 25 = 75$

$\sigma_h' = p_s - u_1 = 75 - 25 = 50$

ESP and TSP shown on sketch [40%]

(d) Final long term case, at failure $\sigma_h' = K_a \sigma_v'$

$K_a = \dfrac{1 - \sin 25°}{1 + \sin 25°} = 0.41$ $\phi' = 25°$



A, A'	before construction
B, B'	after excavation, $P_s = 75$ kN/m^2
C, C'	long term, minimum possible P_s

Final state C', C

$\sigma_v = 100 \text{ kN/m}^2 , \quad u = u_0 = 50 \text{ kN/m}^2$

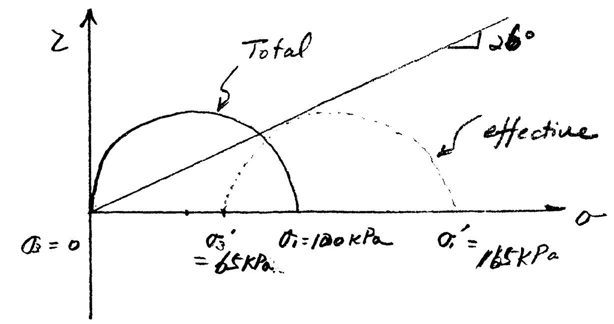
$\therefore \sigma_v' = 100 - 50 = 50 \text{ kN/m}^2$

$\sigma_h' = k_a \sigma_v' = 0.41 \times 50 = 20.5 \text{ kN/m}^2$

$\therefore \sigma_h = P_{s(min)} = \sigma_h' + u = 20.5 + 50 = 70.5 \text{ kN/m}^2$

[20%]

3. (a)



undrained shear strength $c_u = \frac{100}{2} = \underline{50 \text{ kPa}}$ //

At failure

$\sigma_1 = 100$ kPa $\quad \sigma_3 = 0$ kPa $\quad u = -65$ kPa

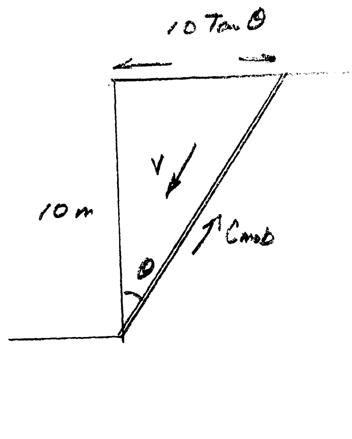
$\therefore \sigma_1' = 165$ kPa $\quad \sigma_3' = 65$ kPa

$$\sigma_1' - \sigma_3' = (\sigma_1' + \sigma_3') \sin\phi'$$

$$\sin\phi' = 0.435$$

$$\underline{\phi' = 26^\circ} \text{ //}$$

(b) (i)



Self weight $= 10^2 \tan\theta \times 18 \times \frac{1}{2}$

$= 900 \tan\theta$

External work

$= 900 \tan\theta \cdot v \cos\theta$

$= \underline{900 \sin\theta \cdot v}$ //

(ii) Internal work

$= C_{mob} \frac{10}{\cos\theta} \cdot v$

External = Internal

$$900 \sin\theta \, v = C_{mob} \frac{10}{\cos\theta} \cdot v$$

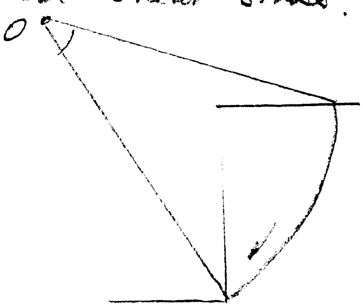
$$C_{mob} = 90 \sin\theta \cos\theta$$

$$= 45 \sin 2\theta$$

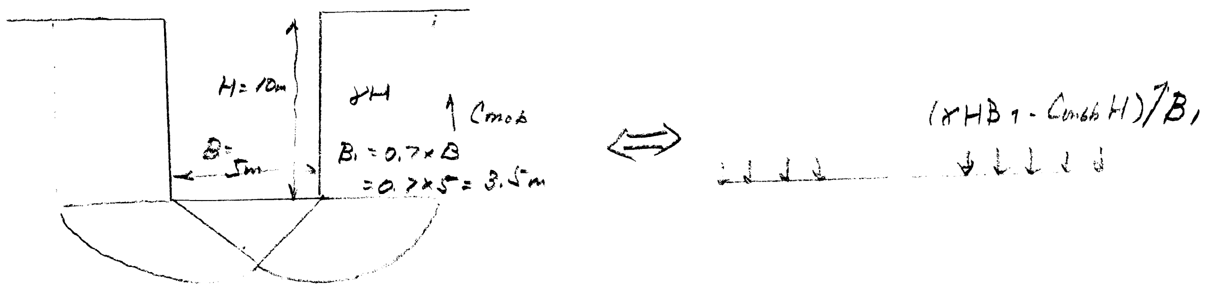
Maximum C_{mob} = 45 KPa

at $\theta = 45°$ //

iii) C_u = 50 KPa, is very close to failure, A circular slip mechanism will gives more critical mechanism. This will result in larger mobilised shear stress. Hence the clay may fail.



iv)



$$q_f = 5.14 \, C_{mob}$$

$$\gamma H - C_{mob} H / B_1 = 5.14 \, C_{mob}$$

$$18 \cdot 10 - C_{mob} \cdot 10/3.5 = 5.14 \, C_{mob}$$

$$8.0 \, C_{mob} = 180$$

$$C_{mob} = 22.5 \text{ KPa}$$ //

4

(a)

$$q_s = \gamma_{dry} D_{water} = 16 \times 1.5 = 24 \text{ kPa}$$

Taking moment around O

$$P_f \times 1 = (q_s \times 2) \times 1 + C_u \times 2\pi \times \underbrace{\frac{2}{\cos 23^\circ} \times \frac{134^\circ}{360^\circ}}_{\text{arc length}} \times \underbrace{\frac{2}{\cos 23^\circ}}_{R}$$

$$= 24 \times 2 \times 1 + 75 \times 2\pi \times 2.17 \times 0.372 \times 2.17$$

$$P_f = 48 + 825$$

$$= \underline{873 \text{ kN/m}}$$

(b)

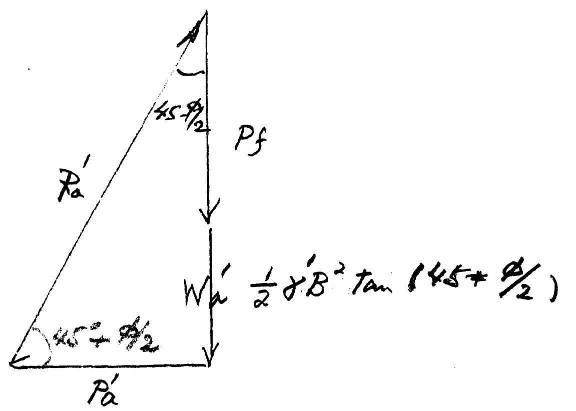
$$q_f = q_s + 2C_u + 2C_u \cdot \pi/2$$

$$= q_s + 5.14\, C_u$$

$$P_f = q_f \times 2 = (24 + 5.14 \times 75) \times 2$$

$$= \underline{819 \text{ kN/m}}$$

(c) Effective stress



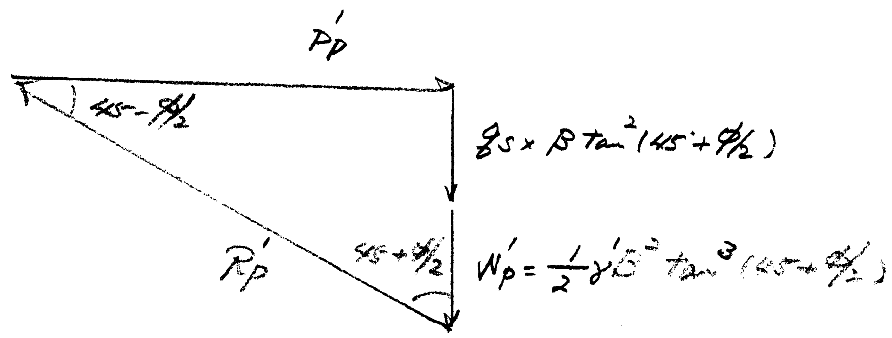
$$P_a' = \{P_f + \tfrac{1}{2}\gamma' B^2 \tan(45^\circ + \phi/2)\} \tan(45^\circ - \phi/2)$$

$$= \{P_f + \tfrac{1}{2} \times (18 - 10) \times 2^2 \tan(45 + 25/2)\} \tan(45 - 25/2)$$

$$= \{P_f + 25.1\} 0.64$$

$$= \underline{0.64 P_f + 16.1}$$

(ii)



$$P_p' = \{q_s \times B \tan^2(45+\phi/2) + \frac{1}{2}\gamma' B^2 \tan^3(45+\phi/2)\} \tan(45+\phi/2)$$

$$= \{16 \times 1.5 \times 2 \times \tan^2(45+25/2) + \frac{1}{2} \times 8 \times 2^2 \times \tan^3(45+25/2)\} \tan(45+25/2)$$

$$= \{118.3 + 61.9\} \times 1.57$$

$$= 282.9 \text{ kN/m}$$

(iii) $0.64 P_f + 16.2 = 282.9$

$P_f = 417$ kN/m //

(iv) $q_s = (18-10) \times 1.5$

$= 12$

$P_p = (59.1 + 61.9) \times 1.57$

$= 190.0$ kN/m

$0.64 P_f + 16.2 = 190.0$

$P_f = 272$ kN/m

The water pressure cancels out

The difference is $417 - 272 = 145$ kN/m