ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3D3
Structural Materials and Design
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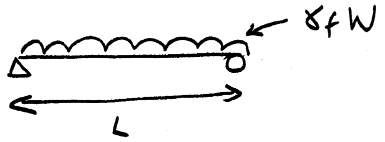
3D3 - Structural Materials and Design
Part IIA 2004

Q1a)



Plastic behaviour

$M_p = \sigma_f Z_p$

strength $\quad \dfrac{\gamma_f W L}{8} = \sigma_f Z_p \qquad (1)$

Stiffness / deflection $\quad \dfrac{5 c W L^3}{384 E I} \leq \dfrac{L}{F} \qquad (2)$

(2) / (1) $\quad \dfrac{5 c \cancel{W} L^{\cancel{3}\,2}}{384 E I} \cdot \dfrac{8}{\gamma_f \cancel{W} \cancel{L}} \leq \dfrac{L}{F} \cdot \dfrac{1}{\sigma_f Z_p}$

$$\frac{5}{48} \frac{c L^2}{E I \gamma_f} \leq \frac{L}{F \sigma_f Z_p}$$

$$L \leq \frac{48}{5} \frac{\gamma_f}{c F} \cdot \frac{E}{\sigma_f} \cdot \frac{I}{Z_p}$$

divide both sides by d

$$\frac{L}{d} \leq \frac{48}{5} \frac{\gamma_f}{c F} \cdot \underbrace{\frac{E}{\sigma_f}}_{\text{material}} \cdot \underbrace{\frac{I}{Z_p . d}}_{\text{shape}}$$

common mistakes → use of Z_e instead of Z_p
incorrect sign in eqn (2)

b) $L = 15\,m$, $c = 0.9$, $E = 210\,000$ MPa, $F = 300$, $\gamma_f = 1.5$
$\gamma_m = 1.05$ S355 steel $\sigma_y = 355$ MPa
$\therefore \sigma_f = 355 / 1.05 = 338$ MPa

for 762 x 267 x 197 UB (from Structures Databook)
$Z_e = 6234\ cm^3$
$Z_p = 7167\ cm^3$
$I_{xx} = 240000\ cm^4$
$d = 769.8$ mm
$A = 251\ cm^2$

Q1 (b) continued

designer chooses not rely on any ductility
$\therefore$ use Z_e

$$\frac{L}{d} \le \frac{48}{5} \frac{\gamma_f}{cF} \cdot \frac{E}{\sigma_f} \cdot \frac{I}{Z_e d} \qquad \frac{I}{Z_e d} = 0.5 \text{ since } Z_e = \frac{I}{d/2}$$

$$\le \frac{48}{5} \cdot \frac{1.5}{0.9 \times 300} \cdot \frac{210000}{338} \cdot 0.5$$

$$\frac{L}{d} \le 16.6 \quad \therefore \left(\frac{L}{d}\right)_{crit} = 16.6$$

compare with actual $\frac{L}{d} = \frac{15000}{769.8} = 19.5 > \left(\frac{L}{d}\right)_{crit}$

$\therefore$ likely to be stiffness controlled $\rightarrow$ deflection limit

equation (2) when deflection limit reached

$$\frac{5cWL^3}{384EI} = \frac{L}{F}$$

$$W = \frac{384EI}{5cL^2F} = \frac{384 \cdot 210000 \cdot 240000 \times 10^4}{5 \times 0.9 \times (15000)^2 \times 300}$$

$$= 637156 \text{ N}$$
$$= 637 \text{ kN}$$

optional Check - strength limit - equation (1)

$$W = \frac{8}{L\gamma_f} \cdot \sigma_f Z_e \quad \text{(for elastic behaviour)}$$

$$= \frac{8}{15000 \times 1.5} \cdot 338 \cdot 6234 \times 10^3$$

$$= 749188 \text{ N}$$
$$= 749 \text{ kN} > 637 \text{ kN}$$

$\therefore$ deflection controls (as expected) ✓
maximum load $W = 637$ kN

Q1 (b) continued

common mistakes → use of Z_p instead of Z_e
→ incorrect conclusion regarding whether strength or deflection controls

c)

$$\phi_e = \frac{I}{I_o}$$

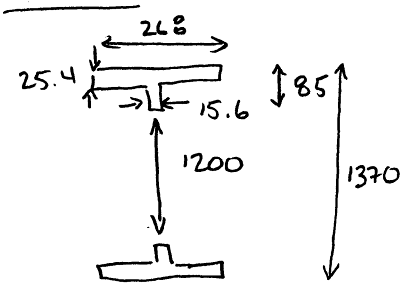
for rectangular section $b = d$ ∴ $I_o = b^4/12$
$A = b^2$ ⇒ $I_o = A^2/12$

case 1 - before cutting - 762 × 267 × 197 UB

$$\phi_{e1} = \frac{I}{I_o} = \frac{I \cdot 12}{A^2} = \frac{240000 \times 10^4 \cdot 12}{(251 \times 10^2)^2}$$

$$= 45.7$$

case 2 - after cutting



$$I = \frac{268 \times 1370^3}{12} - \frac{(1370 - 2 \times 25.4)^3 \times (268 - 15.6)}{12} - \frac{15.6 \times 1200^3}{12}$$

$$= 5.74 \times 10^{10} - 4.83 \times 10^{10} - 2.25 \times 10^9 = 6.892 \times 10^9 \text{ mm}^4$$

$$\phi_{e2} = \frac{I \cdot 12}{A^2}$$

$$= \frac{6.892 \times 10^9 \times 12}{(25100)^2}$$

$$= 131$$

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Q1c) continued

ratio $\frac{\phi_{e2}}{\phi_{e1}} = \frac{131}{45.7} = 2.9$ times greater

for castellated beam $d = 1370$

$\therefore \frac{L}{d} = \frac{15000}{1370} = 10.9 \leq 16.6$ limit

will now be strength controlled

$$W = \frac{8}{L\gamma_f} \cdot \sigma_f Z_e$$

$$= \frac{8}{15000 \times 1.5} \cdot 338 \cdot \frac{6.892 \times 10^9}{(1370/2)}$$

$$= 1.209 \times 10^3 \text{ N} = 1209 \text{ kN}$$

Common mistakes → careless errors in calculating I for the castellated beam
→ incorrectly assuming castellated beam will be controlled by deflection limit

- Will get extra deformation around holes due to shear near the beam ends. Even if bending strength is correct, the extra deflection due to shear is likely to mean deflection still governs (so 1209 kN load cannot be reached whatever we do re: stiffening etc)

Checks - shear on web plate near ends (stiffeners?)
- shear transfer across holes
- bearing stresses at reaction points (stiffeners?)
- longitudinal shear stress in welds (check $SA_c\bar{y}/I$)
- lateral torsional buckling (flange quite slender)

2a) Book work

A 'load path' is an equilibrium system of stresses, bending moments etc. carrying a load applied to a structure down to the foundations. A structure actually sustaining a load must have at least one load path.

An engineer in design may arbitrarily select a possible load path and clothe that path with adequate material (without knowing the actual path); provided all of the conditions for application of plasticity theory apply - as the engineer would be relying on the lower bound theorem of plasticity. So adequate ductility (ruling out brittle materials, over-reinforced concrete) needed, small deflection: no buckling (so beware thin-walled structures even if in ductile material): no unexpected failure (so adequate joints) etc.

An example of a load path: consider a concrete slab supported on steel beams with a path load on the concrete slab. One possible load path is from the load → slab → secondary beams → primary beams → colums → foundations.

b) i) Otherwise the primary beam would have to sustain a couple about a longitudinal axis throug the web, transmitted through the bolts, possibly twisting the web out of plane - awkward to design for

ii) Secondary beam essentially supported at AA'. Reaction spread through six bolts (3 each side) in pure single shear. Viewing connection as a whole, "force" transmitted through the secondary bolts is the reaction combined with a couple of 200kN times 50 mm (10kN.m) giving varying forces in the bolts.

b) ii) continued

Primary web : $\frac{200\text{kN}}{6} = 33.3$ kN / bolt

bolt shear stress : $\frac{33.3 \times 10^3}{\pi \times 20^2/4} = 106$ MPa

bolt ϕ

bearing stress : $\frac{33.3 \times 10^3}{20 \times 8} = 208$ MPa

cleat since $t_{cleat} < t_{web}$

Secondary web : shear force 50 kN/bolt but double shear

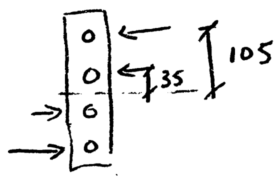
also have additional force due to moment

$\Sigma d^2 = 2 \times (35^2 + 105^2) = 24500\,\text{mm}^2$ (elastic theory)

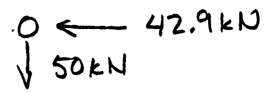
horizontal force on outer bolt $= \frac{C d_i}{\Sigma d_i^2}$ applied couple

$= \frac{\overbrace{200000 \times 50}^{} \times 105}{24500}$

$= 42.9$ kN



forces on outer bolt



resultant $= \sqrt{50^2 + 42.9^2}$

$= 65.9$ kN

bolt shear stress : $\frac{65.9}{2} \frac{\times 10^3}{\pi \times 20^2/4} = 105$ MPa

double shear

bearing stress : $\frac{65.9 \times 10^3}{20 \times \underbrace{9.5}_{t_{web}}} = 347$ MPa

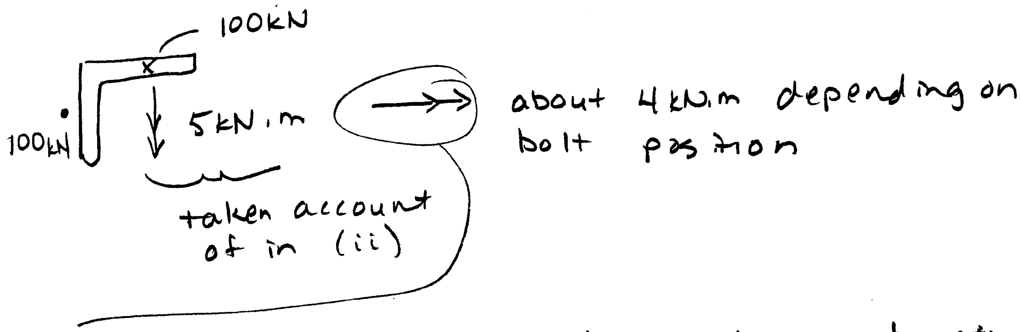
$\therefore$ design satisfactory : factor $\frac{450}{347} = 1.3$ * critical

vs factor $\frac{160}{106} = 1.5$

common mistake - forgetting additional force due to the couple in the secondary connection
- not recognising that bolt through secondary web is in double shear

2 b) continued

iii)



This moment would be sustained through a combination of lateral shear forces on the bolts into the primary web and compression / tension forces on bolts in secondary - in theory, this should be allowed for in design.

Further checks include average shear stress through cleat, stresses due to moment on cleat (at bolt position) etc.

iv) This looks OK as a connection but a long beam would need lateral stability and top flange should be properly anchored against out-of-plane movement. Either provide adequate lateral bracing near midspan or perhaps a connection of flanges? (this would be tricky due to practical constraints).

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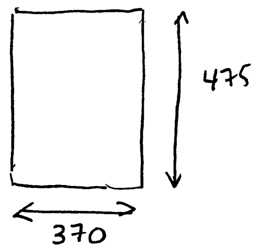
Q3



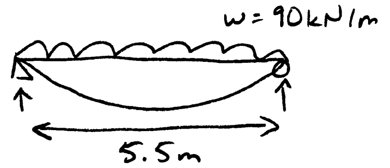
cover = 40 mm

f_{cu} = 25 MPa $\gamma_c = 1.5$

f_y = 460 MPa $\gamma_s = 1.15$

w = 90kN/m

a)



$$M_{applied} = \frac{wL^2}{8} = \frac{90 \times 5.5^2}{8} = 340 \text{ kN.m}$$

Check if compression steel is required

use $d = \underbrace{475}_{\text{beam depth}} - \underbrace{40}_{\text{cover}} - \underbrace{\sim 10}_{\text{½ bar } \phi} = 425 \text{ mm}$

$M_u = 0.225 f_{cu} bd^2/\gamma_c$ $(x = 0.5d)$

$= 0.225 \times 25 \times 370 \times 425^2 / 1.5 = 250.6 \times 10^6$ N.mm

$= 251$ kN.m

$M_{applied} > M_u$ $\therefore$ need compression steel use $d' = 50$mm

take moments about bottom reinforcement

$$M_{TOT} = 0.225 f_{cu} bd^2/\gamma_c + A_s'(d-d')f_y/\gamma_s$$

$$A_s' = \left(M_{TOT} - 0.225 f_{cu} bd^2/\gamma_c\right)\frac{\gamma_s}{(d-d')f_y}$$

$$= \left((340-251)\times 10^6\right)\frac{1.15}{(425-50)\times 460} = 593 \text{ mm}^2$$

1 No 12 → $A_s = 113$ mm² → 5 No 12 $A_s' = 565$ mm²

1 No 16 → $A_s = 201$ mm² → 3 No 16 $A_s' = 603$ mm²

use 3 No 16 mm bars

Q3 a) continued

for longitudinal equilibrium

$$\frac{A_s' f_y}{\gamma_s} + \frac{0.6 f_{cu} b d}{\gamma_c . 2} = \frac{A_s f_y}{\gamma_s}$$

$$\therefore \quad A_s = A_s' + \frac{0.6 f_{cu} b d}{\gamma_c . 2} \cdot \frac{\gamma_s}{f_y}$$

$$= 603 + \frac{0.6 \times 25 \times 370 \times 425}{1.5 \times 2} \times \frac{1.15}{460}$$

$$= 603 + 1966 = 2569 \text{ mm}^2$$

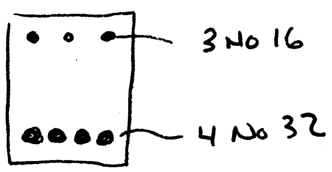
1 No 32 $\rightarrow$ $A_s = 804 \text{ mm}^2$ $\rightarrow$ 4 No 32 $A_s = 3217 \text{ mm}^2$
1 No 25 $\rightarrow$ $A_s = 490 \text{ mm}^2$ $\rightarrow$ 6 No 25 $A_s = 2945 \text{ mm}^2$

check spacing for 25mm Ø bars

$$(\underset{\uparrow \text{ width}}{370} - \underset{\uparrow \text{ cover}}{2 \times 40} - \underset{\uparrow \text{ bar diameters}}{6 \times 25} - \underset{\uparrow \text{ shear link}}{2 \times \sim 10 \text{ mm}}) / \underset{\uparrow \text{ spaces}}{5} = 25 \text{ mm}$$

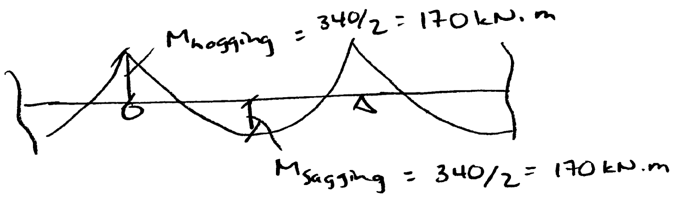
might be tight

$\therefore$ use 4 No 32 mm Ø bars



common mistake - not recognising need for compression steel

b) continuous beam



Mhogging = Msagging

$M_{applied} = 170$ kN.m $<$ 251 kN.m so compression steel is not required
singly reinforced

$A_s = 4 \times \pi \times 20^2/4 = 1257 \text{ mm}^2$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} b d} = \frac{1.5 \times 1257 \times 460}{1.15 \times 0.6 \times 25 \times 370 \times 425} = 0.320$$

$$M_u = A_s f_y d (1 - 0.5 x/d) / \gamma_s$$

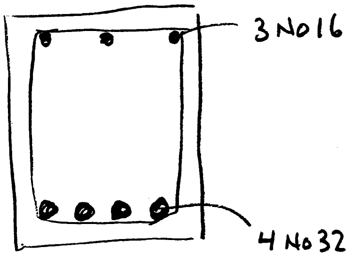
$$= 1257 \times 460 \times 425 (1 - 0.5 \times 0.320) / 1.15 = 179.5 \times 10^6 \text{ N.mm}$$

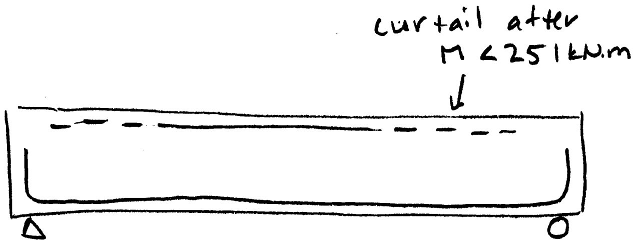
$= 180$ kN.m $>$ 170 kN.m $\therefore$ design is OK

Q3 c)

<u>simply supported beam</u>

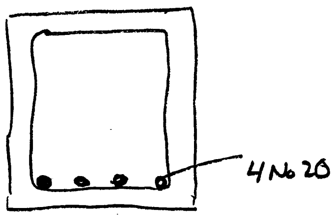
- midspan

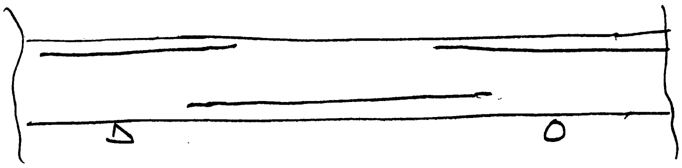




<u>continuous</u>

- mid span

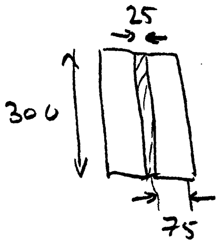




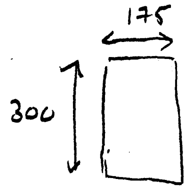
The advantages would be that simply supported beam could be pre-fabricated and dropped inplace as there is no continuity of the reinforcement. The disadvantages would be that a large amount of reinforcement steel is required and the robustness of the structure would need to be considered. The continuous beam requires less reinforcement but may be more complicated to construct. When detailing a designer need to think about the anchorage / curtailment of the reinforcement steel. Further factors include maximum & minimum percentages of reinforcement, bar congestion, crack width requirements & bar spacing.

common mistake - not answering one or more parts of the question!

Q4



flitch



timber

flitch steel ; $\sigma_y = 245\,MPa$, $E = 210\,GPa$, $\gamma_s = 1.15$

C24 timber ; $f_{m,k} = 24\,MPa$, $E_{0,mean} = 11\,GPa$, $k_h = 1$

$k_{ls} = 1$, $k_{crit} = 1$, $k_{mod} = 0.7$, $\gamma_m = 1.3$

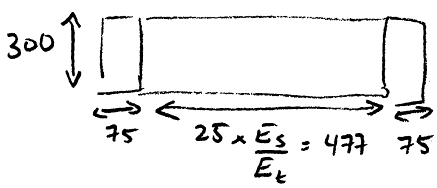
(class 3, short-term)

$$f_{m,d} = k_{mod} \times k_h \times k_{crit} \times k_{ls} f_{m,k} / \gamma_m$$

$$= (0.7 \times 1 \times 1 \times 1 \times 24) / 1.3 = 12.9\,MPa$$

a) <u>flitch beam</u>

- EI_{xx} - transform to timber

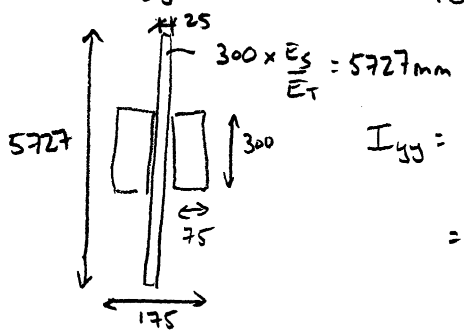


$$I_{xx} = \frac{bd^3}{12} = \frac{(477 + 2 \times 75)(300)^3}{12}$$

$$= 1.411 \times 10^9\,mm^4$$

$$EI_{xx} = 11000 \times 1.411 \times 10^9 = 15.5 \times 10^{12}\,N.mm^2$$

- EI_{yy} - transform to timber



$$I_{yy} = \frac{300 \times 175^3}{12} + \frac{(5727 - 300)\,25^3}{12}$$

$$= 141 \times 10^6\,mm^4$$

$$EI_{yy} = 11000 \times 141 \times 10^6 = 1.55 \times 10^{12}\,N.mm^2$$

<u>solid timber</u>

- $EI_{xx} = 11000 \times 175 \times 300^3/12 = 4.33 \times 10^{12}\,N.mm^2$
- $EI_{yy} = 11000 \times 300 \times 175^3/12 = 1.47 \times 10^{12}\,N.mm^2$

common mistakes - errors when transforming steel to timber

Q4 a) continued

$$\frac{EI_{xx}\ \text{flitch}}{EI_{xx}\ \text{timber}} = 3.5 \qquad \frac{EI_{yy}\ \text{flitch}}{EI_{yy}\ \text{timber}} = 1.05$$

flitch much stiffer — much the same

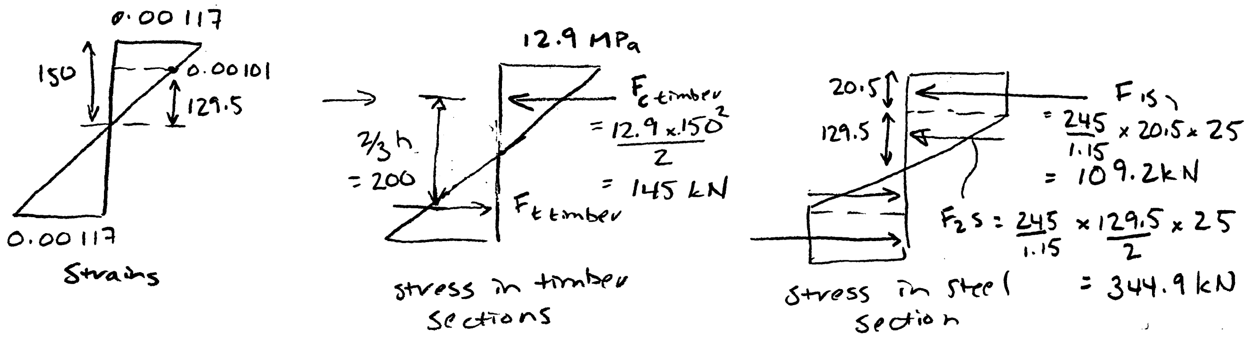
b) check if steel yields before timber fails

ε_u timber : $\frac{f_{m,d}}{E} = \frac{12.9}{11000} = 0.00117$

ε_y steel : $\frac{\sigma_y}{E} = \frac{245}{1.15 \times 210000} = 0.00101 \rightarrow$ yields first (with safety factors)

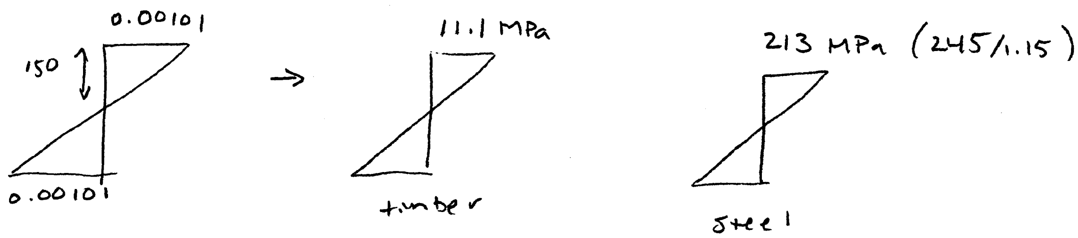
(without material safety factors
ε_u timber $= 16.8 / 11000 = 0.00153$
ε_y steel $= 245 / 210\,000 = 0.00117 \rightarrow$ yields first)

<u>flitch</u> - assume composite action - point at which timber fails (steel already started to yield)



$$M_{TOT} = 145 \times (200) + 109.2\left(150 - 20.5/2\right) + 344.9\left(\tfrac{2}{3} \times 129.5\right) \times 2$$
$$= 103.8 \times 10^3\ \text{kN.mm}$$
$$= 103.8\ \text{kN.m}$$

<u>OR</u> since ε_u timber $\approx \varepsilon_y$ steel, consider capacity when steel first yields



$$M_{TOT} = (11.1 \times 150 \times 150/2)(200) + (213 \times 25 \times 150/2)(200)$$
$$= 104.9\ \text{kN.m} \quad \text{(almost the same)}$$

Q4 b) continued

Solid timber

$$M_{max} = \frac{f_{m,d} I_{xx}}{y} = \frac{12.9 \times 175 \times 300^3/12}{150} = 33.9 \text{ kN.m}$$

∴ flitch is stronger but would need to check if assumption of perfect bond holds

Common mistake - difficulty in identifying it steel yields before timber fails

c) lateral torsional buckling → bookwork

from CTM notes: section on lateral torsional buckling (designing in ductile metal: steel) includes description of lateral-torsional buckling

$$M_{crit} = \frac{\pi}{L_{ef}} \sqrt{\underbrace{E I_{yy}}_{\text{much the same for flitch \& solid timber}} \underbrace{GJ}_{\text{steel will contribute to higher GJ for flitch but probably not a large difference?}}}$$

As the flitch beam has a higher moment capacity lateral torsional buckling may well dictate. The likelihood of lateral torsional buckling is reduced by reducing the distance between restraints and/or restraining the compression flange. In timber guidance is given on max height to breadth ratios in order to avoid lateral stability problems

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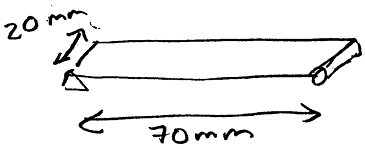
Q5 a)



0.125mm GFRP plies
balanced, symmetric

i) E_x required = 29000 MPa, 33% plies in $\pm 45°$ directions

From E-glass / epoxy chart for Young's Modulus
x-axis co-ordinate = 33%
y-axis co-ordinate = 29 GPa
→ need 50% 0° plies
for a total of 100% this leaves 17% 90° plies

33% ± 45°, 50% 0°, 17% 90°

ii) from E-glass / epoxy chart for shear Modulus
x-axis co-ordinate = 33%
y-axis co-ordinate (from graph) = 7.4 GPa
→ G_{xy} = 7400 MPa

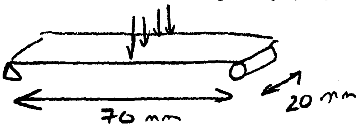
from E-glass / epoxy chart for Poisson's ratio
x-axis co-ordinate = 33%
follow curve for 50% 0° plies to find
y-axis co-ordinate = 0.25
→ ν_{xy} = 0.25

from E-glass / epoxy chart for Young's Modulus
x-axis co-ordinate = 33%
follow curve for 17% 0° plies (for E_y direction)
y-axis co-ordinate = 17 GPa
→ E_y = 17000 MPa

<u>Note</u> small variations in answers due to small differences when interpolating curves on graphs were not penalised

Q5 a) continued

iii)



$e_T = 0.3\%$, $e_c = 0.7\%$ } e_T is more critical

$$M = \frac{V.L}{4}$$

$$N_x = \frac{Myt}{I} = \frac{VL}{4} \cdot \frac{t/2 \cdot t}{bt^3} \cdot 12 = \frac{3}{2}\frac{VL}{bt}$$

Check strain in x-direction

$$\varepsilon_x = \frac{1}{E_x t}\left(N_x - N_y \cancel{\nu_{xy}}^{\,0}\right)$$

at failure $\varepsilon_x = e_T = 0.3\%$

$$e_T = \frac{1}{E_x t}\left(\frac{3}{2}\frac{VL}{bt}\right) \quad \therefore \quad t = \sqrt{\frac{3VL}{2E_x b e_T}}$$

$$t_{min} = \sqrt{\frac{3 \times 145 \times 70}{2 \times 29000 \times 20 \times 0.3/100}} = 2.96\text{ mm}$$

check strain in y-direction

$$\varepsilon_y = \frac{1}{E_y t}\left(\cancel{N_y}^{\,0} - N_x \nu_{xy}\right)$$

$$e_T = \frac{1}{E_y t}\left(\frac{3}{2}\frac{VL}{bt}\right)\nu_{xy} \quad \therefore \quad t = \sqrt{\frac{3VL\nu_{xy}}{2E_y b e_T}}$$

$$t_{min} = \sqrt{\frac{3 \times 145 \times 70 \times 0.25}{2 \times 17000 \times 20 \times 0.3/100}} = 1.93\text{ mm}$$

$\therefore$ ε_x controls, need a minimum thickness of 3 mm = 24 × 0.125 mm plies

lay up should be symmetric & balanced with

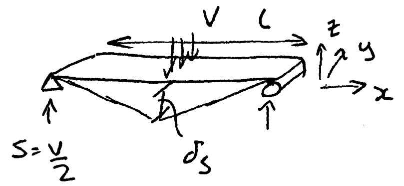
50% 0° → 12 plies

17% 90° → 4 plies

33% ±45° → 8 plies

common mistakes → not checking (or commenting upon) ε_y

Q5 b)



at neutral axis $\tau = \frac{3}{2}\frac{S}{bh}$

constant shear force in shear span

Combine equations to give

$$\left.\begin{aligned} \tau &= G\gamma \\ S &= V/2 \\ \gamma &= \frac{\delta_s}{L/2} \\ \tau &= \frac{3}{2}\frac{S}{bh} \end{aligned}\right\}$$

$$\delta_s = \frac{\tau}{G} \times \frac{L}{2} = \frac{1}{G}\left(\frac{3}{2}\frac{V}{2bh}\right)\frac{L}{2}$$

$$= \frac{3VL}{8bhG}$$

Common mistakes → forgetting that the shear force $S = V/2$

Need to consider G_{yz}, there are no vertical fibers in this direction so the stiffness will be dominated by the properties of the matrix which tends to have a low shear stiffness ∴ shear deflections may be an issue need to consider ways of designing the system to increase resistance to transverse shear.