ENGINEERING TRIPOS PART IIA 2004

Solutions to Module 3D5
Environmental Engineering I
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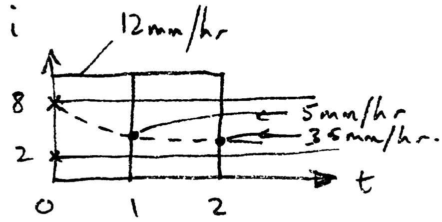
Part IIA. Module 3D5. Environmental Engineering I

Solutions. 2004

1

3D5. 2003-04. Q1.

a). $f_o = 8$ mm/hr
$f_c = 2$ mm/hr
$K_f = 0.7$/hr.



$$f = f_c + (f_o - f_c)e^{-K_f t} = 2 + 6e^{-0.7t}.$$

at $t=0$ $f = 8$ mm/hr
$t=1$ $f = 5$ mm/hr
$t=2$ $f = 3.5$ mm/hr

Crude estimate: 1st hour, infiltration ~ 6.5mm → 5.5mm ~~hr~~ runs off.
2nd hour, infiltration ~ 4 mm → 8 mm ~~/~~ ~ runs off.

$\therefore$ Total proportion $= \frac{8+5.5}{24} = 56\%$ runs off

[By calc. $F = \int_0^{t=2} f\,dt = 2t \cancel{+} - \frac{6}{0.7}\left[e^{-0.7t} - 1\right]$

$t=2$ $= 4 - \frac{6}{0.7}\left[e^{-0.7(2)} - 1\right] = \cancel{4?}\ 10.46$ mm. infiltrates.

25%

$\therefore$ Runoff $= 24 - 10.46 = 13.54 \Rightarrow \frac{13.54}{24} = 56\%$

(i.e. no real need for such accuracy)]

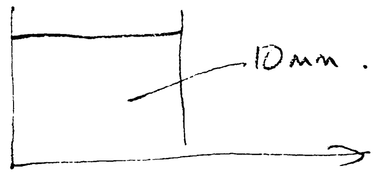
b) – Lysimetry, and actual soil moisture measurements.
– Detailed evapotranspiration formulae, such as Penman eqn ~~~~ using either measured meteorological data for event (as per MORECS) or typical historical values.
– Crude empirical methods, eg Thornthwaite Crowe.

25%

c).

1 (Contd)

3D5 (c). 30 mm of rain ≈ 1 day.

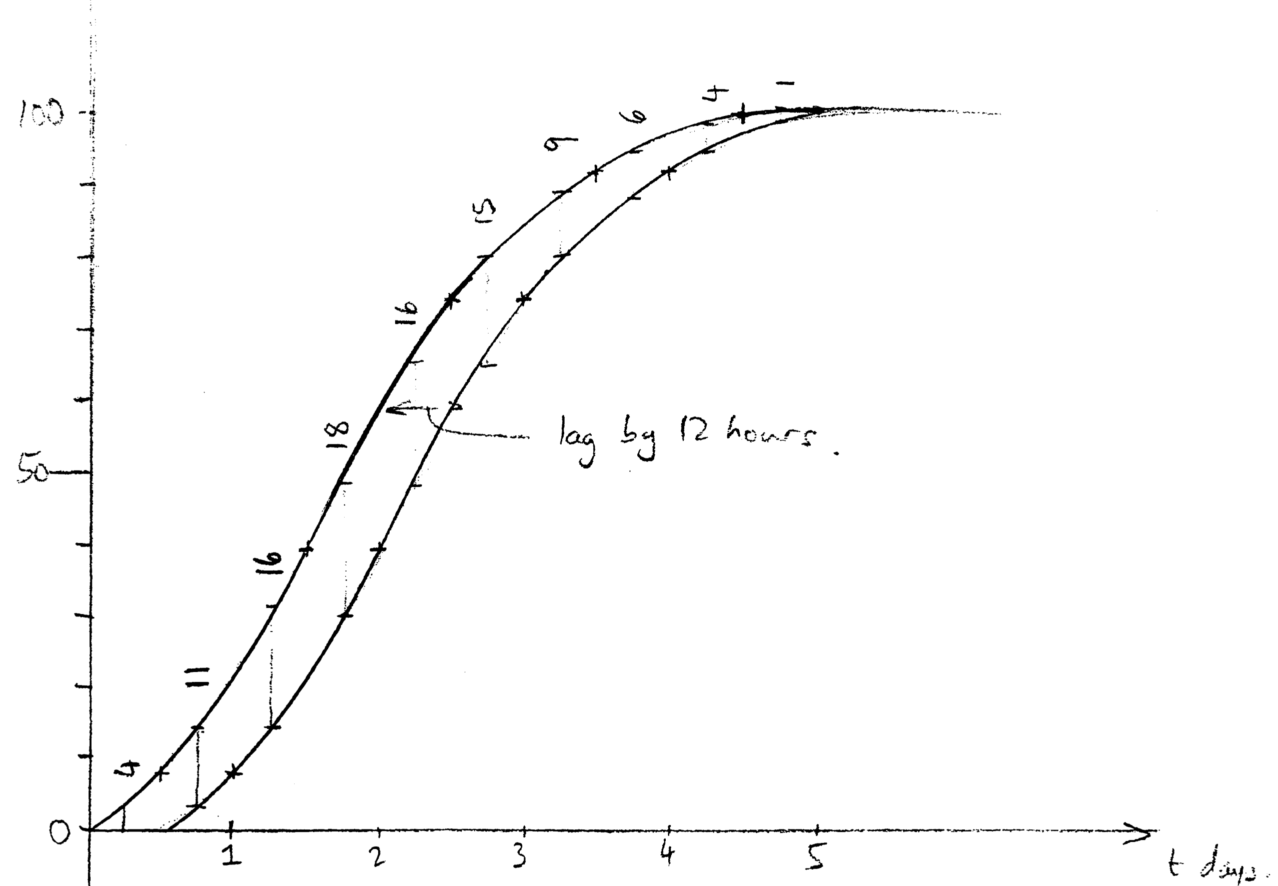


Distribution %'s : —

8, 31, 35, 18, 8 $\Sigma = 100$.

Cumulative 8, 39, 74, 92, 100.

Plot at half points. → graph paper.



so, distribution percentages for storm of 12hrs duration, (on a 12hr time basis) are

4, 11, 16, 18, 16, 15, 9, 6, 4, 1.

1(c) (Contd)

Conclude that 18% runs off in peak 12 hour period.

Total run off is 8 mm (given)

Area = 600 Km^2

Total volume run off = $600 \times 10^6 \times 0.008$

$= 4.8 \times 10^6 \ m^3$

18% of this is $0.864 \times 10^6 \ m^3$

Thus average flow rate during 12 hour period

$$= \frac{0.864 \times 10^6}{12 \times 3600} = 20 \ m^3/s$$

(i) For peak flow rate we add on baseflow. Thus

peak flow rate $= 23 \ m^3/s$

(ii) Area of cross section is $20 m^2$ so mean vel

$$= \frac{23}{20} = 1.15 \ m/s$$

(iii) Total precipitation run-off $= 4.8 \times 10^6 \ m^3$ 50%

Baseflow $= 3 \times 3600 \times 24 \times 5$

$= 1.3 \times 10^6 \ m^3$ in 5 day period

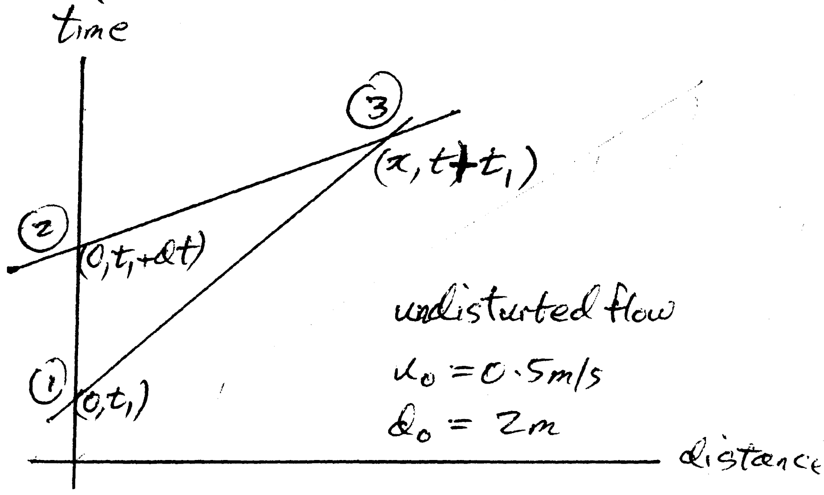
$\therefore$ Total flow $= 6.1 \times 10^6 \ m^3$

2/

At $x = 0$

$$d = d_o + 0.02t$$

$$= d_o(1 + 0.01t)$$



(a) Clearly $\left(\frac{dx}{dt}\right)_1 = u_1 + c_1 = \frac{x}{t}$

but $u_1 - 2c_1 = u_o - 2c_o$

$$\therefore \quad \frac{x}{t} = 3c_1 + A \qquad \text{—— (1)}$$

where $A = u_o - 2c_o$

Similarly, for the other +ve characteristic

$$\frac{x}{t - dt} = 3c_2 + A \qquad \text{—— (2)}$$

But $c_1 = (gd_1)^{\frac{1}{2}} = (gd_o)^{\frac{1}{2}}(1 + 0.01t)^{\frac{1}{2}} = c_o(1 + 0.01t)^{\frac{1}{2}}$

$c_2 = (gd_2)^{\frac{1}{2}} = (gd_o)^{\frac{1}{2}}(1 + 0.01t + 0.01dt)^{\frac{1}{2}} = c_o(1+0.01t)^{\frac{1}{2}}\left(1 + \frac{0.01dt}{2(1+0.01t)} + \right.$

From (1) and (2)

$$(t - dt)(3c_2 + A) = t(3c_1 + A)$$

Substituting for c_1 and c_2 and neglecting terms in $(dt)^2$ and above

$$\left(1 - \frac{dt}{t}\right)\left[3c_o(1+0.01t)^{\frac{1}{2}} + A + \frac{3c_o \times 0.01dt}{2(1+0.01t)^{\frac{1}{2}}}\right] = 3c_o(1+0.01t)^{\frac{1}{2}} + A$$

2 (Contd) $\therefore$ $$\frac{3C_0 \times 0.01}{2(1+0.01t)^{\frac{1}{2}}} = \frac{3C_0(1+0.01t)^{\frac{1}{2}}+A}{t} \qquad -(3)$$

Minimum value of t occurs when $t_1 = 0$

$\therefore$ From (3) $$t_{min} = \frac{3C_0 + A}{0.015C_0} = \frac{U_0 + C_0}{0.015C_0} = 74.19 \text{ s}$$

This is also the minimum value of $t_1 + t$

50%

i.e Min time until surge $= \underline{\underline{74.19\text{s}}}$

(b) From (1) $x_{min} = (3C_0 + A)t_{min} = (U_0 + C_0)74.19 = \underline{\underline{365.7m}}$

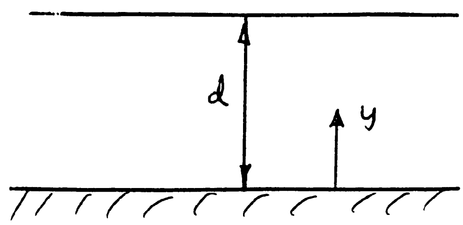
20%

(c) When +ve characteristics cross we have a ~~bore~~

The Method of Characteristics no longer applies to this situation. However, we can use continuity and momentum equations (as for a hydraulic jump) with velocity superimposed to bring the bore to rest. The only problem is that the profile of the bore changes with time so the calculation would be complex and it would only be approximate (because time varying).

30%

3 (a)



The overlap layer is the region of overlap between the defect layer and the wall layer.

In the defect layer the velocity defect is unaffected by viscosity

$\therefore \quad u_{max} - u = f(y, d, u_*)$

$$\therefore \quad \frac{u_{max} - u}{u_*} = f\left(\frac{y}{d}\right)$$

$$\therefore \quad \frac{u}{u_*} = f\left(\frac{u_* d}{\nu}\right) - f\left(\frac{y}{d}\right) \quad \text{because } \frac{u_{max}}{u_*} = f\left(\frac{u_* d}{\nu}\right)$$

—— (1)

In the wall layer the velocity is unaffected by the depth of flow

$\therefore \quad u = f(u_*, y, \nu)$

$$\therefore \quad \frac{u}{u_*} = f\left(\frac{u_* y}{\nu}\right) \quad \text{—— (2)}$$

If both Eqns (1) and (2) are correct the functions $f()$ must be logarithms. Thus

$$\frac{u}{u_*} = \frac{1}{K} \ln \frac{y}{y_0}$$

Where K is a constant and $y_0 = \text{constant} \times \frac{\nu}{u_*}$

40%

3(b) Bearing in mind that

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{gRS}$$

the Chezy formula may be written as

$$\frac{\bar{u}}{u_*} = \frac{C}{\sqrt{g}}$$

Making use of the Prandtl–Karman formula for $\bar{u}$ in the Data Sheet for smooth beds

$$C = 7.83 \log_e \frac{11.5}{C}\left(\frac{\bar{u}R}{\nu}\right)$$

but $C = \frac{1}{n} R^{1/6}$ in S.I units

$$\therefore \frac{1}{n} = \frac{7.83}{R^{1/6}} \log_e\left[\frac{11.5n}{R^{1/6}}\left(\frac{\bar{u}R}{\nu}\right)\right]$$

· Similarly for rough beds

$$\frac{1}{n} = \frac{7.83}{R^{1/6}} \log_e\left[12.1 \frac{R}{K_s}\right]$$

60%

Reasons for caution in real rivers:

(1) Not easy to estimate roughness length (e.g for rippled beds)

(2) Roughness may vary across bed (e.g boulders, bushes, etc)

(3) Cross-section may be irregular so how valid is log law?

(4) Most channels are not straight.

(5) Secondary flows significantly affect velocity profiles

(6) What happens when there is sediment movement?

(7) Log law does not apply over whole section (only the overlap layer).

4(a) $u_* = (gds)^{\frac{1}{2}} = (9.81 \times 2 \times 0.001)^{\frac{1}{2}} = 0.1401$ m/s

From Data Sheet

$$\frac{10}{0.3} = \left[\left(\frac{2-0.006}{0.006}\right)\left(\frac{0.06}{2-0.06}\right)\right]^{w/\kappa u_*}$$

35%

$$\therefore \frac{w}{\kappa u_*} = 1.505$$

$$\therefore w = 56 \times 10^4 \times D^2 \times 1.65 = 0.08434 \text{ m/s}$$

$$\therefore D = \underline{\underline{0.302 \text{ mm}}}$$

(b) From Data Sheet $\left.\begin{array}{l} I_1 = 0.379 \\ I_2 = 1.647 \end{array}\right\}$ assuming $\frac{w}{\kappa u_*} = 1.5$ and $\epsilon/d = 0.003$

Also, $\frac{u_* k_s}{\nu} = \frac{0.1401 \times 0.006}{10^{-6}} = 840.6$

i.e hydraulically rough

$$\therefore \int cu\,dy = 11.6 \times 0.1401 \times 10 \times 0.006 \times \left[0.379 \log_e\left(\frac{30.2 \times 2}{0.006}\right) + 1.647\right]$$

$$= \underline{\underline{0.501}} \text{ kg/m.s}$$

35%

4 (c) Reasons for caution :

(1) W affected by C

(2) K possibly affected by C

(3) Momentum and mass exchange coefficients may not be equal

(4) The overlap layer (log layer) does not extend to the surface

(5) Grain size may vary with height above bed

(6) $\frac{W}{\kappa u_*}$ is not exactly 1.5 as assumed for I_1 and I_2

(7) Secondary currents (e.g at bends) may significantly change concentration profile

30%

(8) Bed roughness may be difficult to estimate (ripples?)