ENGINEERING TRIPOS PART IIA

Thursday 22 April 2004 9 - 12

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (4 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 An elliptical strut as shown in Fig. 1 is immersed in a uniform flow of water.

(a) Sketch the streamlines and the surface pressure coefficient distribution in the case of :

(i) inviscid flow;
(ii) viscous flow. [25%]

(b) Assuming that the wake in viscous flow can be idealised as shown in Fig. 1, calculate the drag force per unit length and the drag coefficient. The oncoming velocity $U_0 = 4\ \mathrm{m\ s^{-1}}$, $L = 0.80\ \mathrm{m}$ and the fluid density $\rho = 1000\ \mathrm{kg\ m^{-3}}$. You may assume the pressure to be uniform and equal at inflow and outflow. [50%]

(c) Explain the different contributions to the drag of the strut and comment on how the axes ratio (D/L) of the ellipse might affect the relative importance of these contributions. Suggest approximate values for the drag coefficient for high and low values of (D/L). [25%]

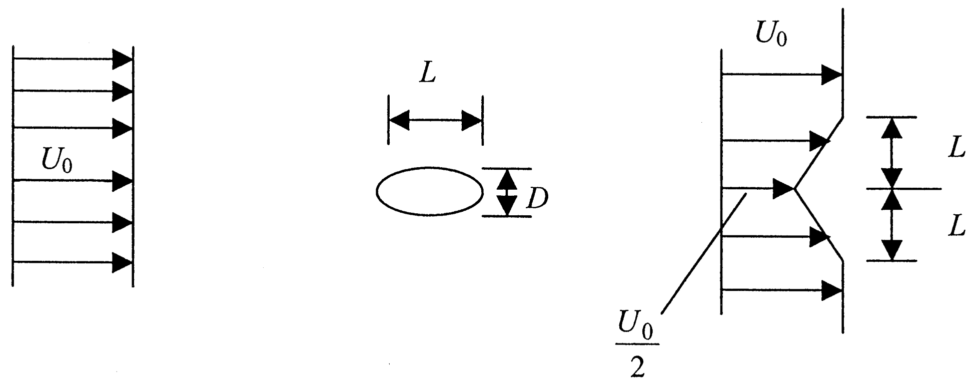

Fig. 1

2 The transformation $\zeta = z + \frac{b^2}{z}$ transforms the circle of radius a with centre located at $(x,y) = (0,c)$ in the z - plane into a curved circular-arc aerofoil. The circle intersects the x-axis ($y = 0$) at the points $x = -b,\ x = +b$. (Recall that $z = x + iy$ and $\zeta = \xi + i\eta$).

(a) What is the chord length of the "aerofoil" in the ζ- plane. (Chord length is defined as the distance along the $\eta = 0$ axis measured from leading to trailing edge.) [25%]

(b) The complex potential for flow incident at an angle α on a circular cylinder with radius a with circulation Γ and centered at $(0,c)$ is

$$F_c(z) = Ue^{-i\alpha}(z - ic) + \frac{Ua^2}{(z - ic)}e^{i\alpha} - \frac{i\Gamma}{2\pi}ln(z - ic).$$

Determine the circulation required to satisfy the Kutta-condition when this flow in the z-plane is mapped into the ζ- plane. [25%]

(c) Determine the lift coefficient C_L as a function of the angle of attack α. [25%]

(d) (i) Comment on the difference between the C_L vs α behaviour and that of a symmetric aerofoil.

(ii) Explain why you would expect the C_L vs α curve for this aerofoil in a real flow to be different from your calculated curve. [25%]

3 A strut with a bulbous upstream end, Fig. 2, is to be modelled by a source of strength 2π located at the origin and a sink of strength $-\dfrac{4\pi}{3}$ located a distance of 1 m downstream immersed in a uniform flow (left to right) of 1 m/s. The configuration is shown in Fig. 3 with angles θ and ϕ defined as shown.

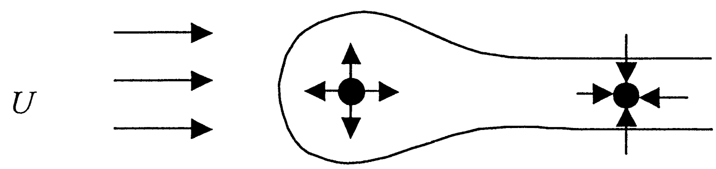

Fig. 2

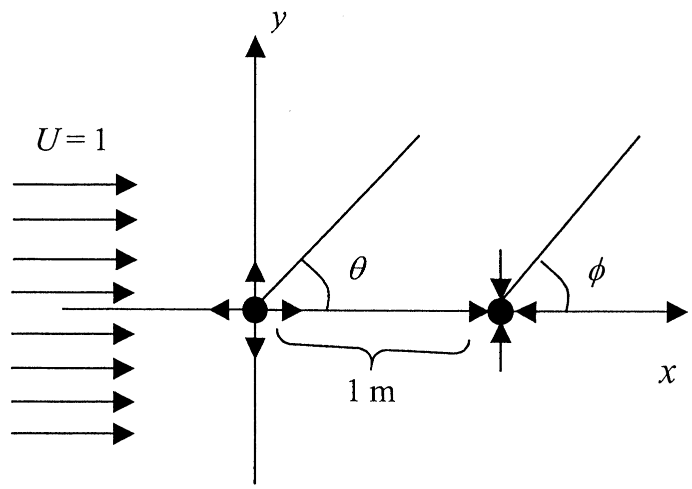

Fig. 3

(a) Write down the complex potential for this flow. [25%]

(b) Find the position of the upstream stagnation point. [25%]

(c) Find the thickness of the strut far downstream ($x \to \infty$). [25%]

(d) Write down an equation for the half thickness of the body at the position $x = 1$ m and show by substitution that this half thickness is approximately 1.21 m. [25%]

4 (a) State Kelvin's circulation theorem including any restrictions on its applicability. [25%]

(b) Use Kelvin's theorem to prove that an inviscid flow that is irrotational at some initial time will remain irrotational for all time. [25%]

(c) In the entry region of a pipe of diameter D there is a swirling flow. A probe in the pipe measures the swirl velocity at the pipe edge (just outside the very thin boundary layer) as $u_\theta = U$ – this is the velocity component along the circumference, normal to the pipe axis. The flow passes through a smooth contraction in size to a new diameter d. Estimate the swirl velocity v_θ at the edge of the smaller pipe (again, just outside the boundary layer). [25%]

(d) Explain why horseshoe shaped troughs are formed in the snow in a strong wind on the upstream side of telephone poles. Give details of how the flow develops approaching the pole and why these troughs are most distinct (or deepest) on the upstream side of the pole. [25%]

5 Compare the characteristics of a laminar boundary layer and a turbulent boundary layer over a flat plate with constant free stream velocity U_∞ in terms of

(a) the growth rate with downstream distance x; [15%]

(b) the velocity profile; [15%]

(c) the velocity gradient near the surface and hence the surface shear stress; [20%]

(d) the surface shear stress determined by applying the momentum integral equation to the growing boundary layer; [15%]

(e) the heat transfer coefficient if the surface is heated; [15%]

(f) the possible separation of the boundary layer if the external pressure gradient is made to be adverse. [20%]

6 Consider the laminar flow past a uniformly heated horizontal flat plate of length L. The free stream velocity is constant at U_∞.

(a) Using an order of magnitude argument, estimate the thickness of the velocity boundary layer at the end of the plate. [15%]

(b) If the fluid has a Prandtl number υ/α of unity, estimate the thickness of the thermal boundary layer at the end of the plate (where υ and α are the kinematic viscosity and thermal diffusivity respectively). [15%]

(c)For case (b), show that $St = C_f/2$ where St is the Stanton number $\left(h/\rho c_p U_\infty\right)$ and C_f is the shear stress coefficient $\left(\tau_0\Big/\frac{1}{2}\rho U_\infty^2\right)$. Here h is the surface heat transfer coefficient, ρ the fluid density, c_p the isobaric specific heat capacity and τ_0 the wall shear stress. [20%]

(d) Estimate how the plate temperature varies with distance x along the plate. [15%]

(e) Estimate how the plate temperature varies with distance along the plate if the fluid has a very small Prandtl number. [15%]

(f) Using physical reasoning describe how the plate temperature varies with distance along the plate for a fluid with a very large Prandtl number. Contrast this with your answer to (e). [20%]

7 (a) Explain the physical origin of the induced drag on a finite wing. State the dependence of the induced drag on wing lift and aspect ratio and explain the reasons for that dependence. [40%]

(b) The circulation of a finite wing is represented as

$$\Gamma(\phi) = 2bV\sum_{i=1}^{n} A_i \sin(i\phi),$$

where the free stream velocity is V, the wing tips are at $z = \pm\, b/2$ and the spanwise coordinate z and ϕ are related by the transformation $z = -b/2\, \cos\phi$. Show that the minimum induced drag is associated with an elliptic lift distribution. You may assume without proof the integral [60%]

$$\int_0^{\pi} \frac{\cos(n\theta)}{\cos\phi - \cos\theta} d\theta = \pi \frac{\sin(n\phi)}{\sin(\phi)}.$$

8 Describe the influence of viscosity on wing flows; in particular, emphasise the compromises that the fluid viscosity forces on the designer. Cover:

(a) lift production; [20%]
(b) 2D stalling behaviour; [40%]
(c) 3D stalling behaviour including the effect of sweep. [40%]

END OF PAPER

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Module 3A1 – Fluid Mechanics I
Incompressible Flow Data Card

Continuity equation $\nabla \cdot \boldsymbol{u} = 0$

Momentum equation (inviscid) $\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \rho \boldsymbol{g}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

Vorticity $\boldsymbol{\omega} = \text{curl}\, \boldsymbol{u}$

Vorticity equation (inviscid) $\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega} \cdot \nabla \boldsymbol{u}$

Kelvin's circulation theorem (inviscid) $\frac{D\Gamma}{Dt} = 0, \quad \Gamma = \oint \boldsymbol{u} \cdot d\boldsymbol{l} = \int \boldsymbol{\omega} \cdot d\boldsymbol{S}$

For an irrotational flow

velocity potential (ϕ) $\boldsymbol{u} = \nabla \phi$ and $\nabla^2 \phi = 0$

Bernoulli's equation for inviscid flow,

$\frac{p}{\rho} + \frac{1}{2}V^2 + gz + \frac{\partial \phi}{\partial t}$ = constant throughout flow field, $V = |\boldsymbol{u}|$.

TWO-DIMENSIONAL FLOW

Streamfunction (ψ) $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$

$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$, $u_\theta = -\frac{\partial \psi}{\partial r}$

Lift force Lift / unit length $= \rho U(-\Gamma)$

Complex potential $F(z)$ for irrotational flows, with $z = x + iy$, $F(z) = \phi + i\psi$ and $\frac{dF}{dz} = u - iv$

Examples of complex potentials

(i) uniform flow in x direction, $F(z) = Uz$

(ii) source at z_0, $F(z) = \frac{m}{2\pi}\ln(z - z_0)$

(iii) doublet at z_0, with axis in x direction, $F(z) = \frac{\mu}{2\pi(z - z_0)}$

(iv) anticlockwise vortex at z_0, $F(z) = -\frac{i\Gamma}{2\pi}\ln(z - z_0)$

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TWO-DIMENSIONAL FLOW

Summary of simple 2 - D flow fields

	ϕ	ψ	circulation	u
Uniform flow (towards $+x$)	Ux	Uy	0	$u = U,\ v = 0$
Source at origin	$\frac{m}{2\pi}\ln r$	$\frac{m}{2\pi}\theta$	0	$u_r = \frac{m}{2\pi r},\ u_\theta = 0$
Doublet at origin θ is angle from doublet axis	$\frac{\mu\cos\theta}{2\pi r}$	$-\frac{\mu\sin\theta}{2\pi r}$	0	$u_r = -\frac{\mu\cos\theta}{2\pi r^2},\ u_\theta = -\frac{\mu\sin\theta}{2\pi r^2}$
Anticlockwise vortex at origin	$\frac{\Gamma}{2\pi}\theta$	$-\frac{\Gamma}{2\pi}\ln r$	Γ around origin	$u_r = 0,\ u_\theta = \frac{\Gamma}{2\pi r}$

THREE-DIMENSIONAL FLOW

Summary of simple 3 - D flow fields

	ϕ	u
Source at origin	$-\frac{m}{4\pi r}$	$u_r = \frac{m}{4\pi r^2},\quad u_\theta = 0,\quad u_\phi = 0$
Doublet at origin θ is angle from doublet axis	$\frac{\mu\cos\theta}{4\pi r^2}$	$u_r = -\frac{\mu\cos\theta}{2\pi r^3},\quad u_\theta = -\frac{\mu\sin\theta}{4\pi r^3},\quad u_\phi = 0$

February 2003

Paper 3A1 - Applications to external flows

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Data Sheet

Coefficients:

Pressure coefficient $C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

Section (local) lift coefficient $c_l = \dfrac{l}{c \times \frac{1}{2}\rho_\infty V_\infty^2}$ (Section chord **c**)

Wing lift coefficient $C_L = \dfrac{L}{A \times \frac{1}{2}\rho_\infty V_\infty^2}$ (Wing area **A**)

Section and wing drag coefficients $c_d = \dfrac{d}{c \times \frac{1}{2}\rho_\infty V_\infty^2}$ $c_D = \dfrac{D}{A \times \frac{1}{2}\rho_\infty V_\infty^2}$

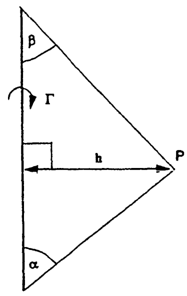
Vortices:

Biot-Savart:

$$\mathbf{dv} = \frac{\Gamma}{4\pi}\frac{\mathbf{dl} \times \mathbf{r}}{|\mathbf{r}|^3}$$

Line vortex:

$$u = \frac{\Gamma}{4\pi h}(\cos\alpha + \cos\beta)$$



Lifting Line Theory:

Lift / unit length and wing lift (span **b**) $l = \rho\, V\, \Gamma(z) \quad L = \rho V \int_{-b/2}^{b/2} \Gamma(z)\, dz$

Induced drag $D = \rho \int_{-b/2}^{b/2} \Gamma(z)\, w(z)\, dz$

Downwash $w(z_0) = \dfrac{1}{4\pi}\displaystyle\int_{-b/2}^{b/2} \frac{d\Gamma(z)}{dz}\frac{dz}{z_0 - z}$

Lift curve slope of aerofoil section $a_0 = \dfrac{dc_l}{d\alpha}$

Local circulation $\Gamma(z) = \frac{1}{2} V\, c\, a_0 \left(\alpha - \dfrac{w(z)}{V}\right)$

Elliptic Lift Distribution:

Circulation $\Gamma(z) = \Gamma_0\sqrt{1 - (2z/b)^2}$

Lift $L = \dfrac{\pi}{4}\rho\, V\, b\, \Gamma_0$

Downwash $w(z) = \dfrac{\Gamma_0}{2\, b}$

Induced drag coefficient $C_D = \dfrac{C_L^2}{\pi\, A_R}$ (Aspect ratio $A_R = b^2/A$)

March 22, 2004

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Module 3A1 – Fluid Mechanics I

VISCOUS FLOW AND BOUNDARY LAYERS DATA CARD

Navier-Stokes equation:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mathbf{f} + \mu \nabla^2 \mathbf{u}$$

where D/Dt denotes the material derivative, $\partial/\partial t + \mathbf{u}.\nabla$ and where $\mathbf{f}$ is a volume force density (gravity, electromagnetic, Coriolis, ...)

Convection-diffusion of heat:

$$\frac{DT}{Dt} = \alpha \nabla^2 T$$

Boussinesq approximation:

$$\rho \mathbf{g} = \rho_0 [1 - \beta (T - T_0)] \mathbf{g}$$

Prandtl' equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\rho^{-1} \frac{dP_\infty}{dx} + f_x + \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Displacement thickness:

$$\delta^* = \frac{\int_0^\infty (U_\infty - u) dy}{U_\infty}$$

Momentum thickness:

$$\theta = \frac{\int_0^\infty u (U_\infty - u) dy}{U_\infty^2}$$

Shape factor:

$$H = \frac{\delta^*}{\theta}$$

Integral momentum equation:

$$U_\infty^2 \frac{d\theta}{dx} + U_\infty \frac{dU_\infty}{dx} \theta (H + 2) = \nu \left(\frac{\partial u}{\partial y} \right)_0$$