ENGINEERING TRIPOS PART IIA

Friday 23 April 2004 9 to 12

Module 3A3

FLUID MECHANICS II

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: Special data sheet 3 pages.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Describe, with the help of sketches, what is meant by an overexpanded and an underexpanded convergent-divergent nozzle. [10%]

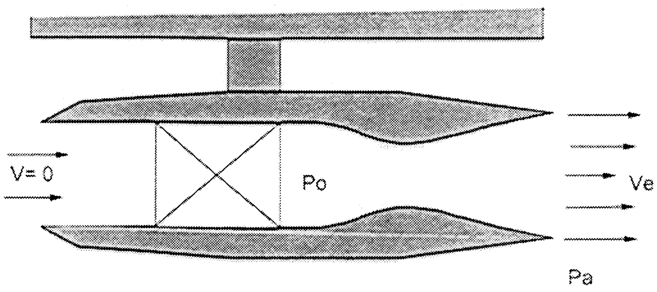


Fig. 1

(b) The thrust of a turbojet engine is measured in a stationary test facility as illustrated in Fig.1. The propulsive nozzle of the engine is a convergent-divergent nozzle with a throat diameter of 0.3 m and an exit area to throat area ratio A_e/A^* of 1.25. The stagnation pressure at inlet to the nozzle is 4.25 bar and the ambient static pressure in the test facility is 1 bar. The flow velocity at inlet to the engine is negligible. Show that these conditions are compatible with isentropic flow within and downstream of the nozzle and calculate the thrust that the engine produces at this condition. [35%]

(c) The turbojet is tested with two different propulsive nozzles each with the same throat area as the original nozzle but with area ratios A_e/A^* of 1.555 and 1.066. The nozzle inlet stagnation pressure and ambient static pressure remain unchanged. Determine whether these nozzles are underexpanded or overexpanded and calculate the thrust produced by the engine with each of them. [35%]

(d) The thrust developed by a nozzle is given by

$$T = \dot{m} V_e + (p_e - p_a) A_e$$

where p_e is the exit static pressure, p_a the ambient pressure, V_e the exit velocity and $\dot{m}$ the mass flow rate. The nozzle exit area A_e may be varied whilst keeping the inlet conditions and the throat area constant, and the throat choked. By combining the above equation with the differential form of the Euler equation prove that, for *isentropic flow* in the nozzle, the maximum thrust is obtained when $p_e = p_a$. [20%]

2 (a) Show that, for a stationary hydraulic jump in an incompressible fluid, the Froude number upstream of the jump F_{r1} is given by

$$F_{r1}^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right) ,$$

where the depths of the uniform flow of water upstream and downstream of the jump are h_1 and h_2. [30%]

(b) Water flows in a channel under a sluice gate. The velocity and depth of the flow downstream of the sluice gate are $1.5\ \mathrm{ms^{-1}}$ and $2\ \mathrm{m}$ respectively. The sluice gate is slowly raised to an unknown height. Some distance downstream of the sluice gate a fully-developed hydraulic jump is observed travelling at a velocity of $8\ \mathrm{ms^{-1}}$. What is the new depth and new velocity of the flow downstream of the sluice gate? [40%]

(c) Raising the sluice gate in part (b) takes 10 seconds. Draw a space-time diagram showing the motion of both water surface waves and of a selection of water particles downstream of the sluice gate. Calculate the distance downstream of the gate at which the travelling hydraulic jump becomes fully developed. [30%]

3 (a) Figure 2 shows a straight duct connected to the end of a convergent-divergent nozzle. The inlet of the nozzle is connected to a reservoir of constant stagnation pressure and temperature. The exit of the duct is open to the atmosphere. The nozzle may be assumed to be frictionless and the flow in both the nozzle and the duct is adiabatic. Describe, with the aid of sketches, all the various flow regimes which may occur in the nozzle and the duct when the friction factor in the duct is altered. Sketch the different flow regimes on a single temperature-entropy diagram. It may be assumed that the throat of the nozzle remains choked. [30%]

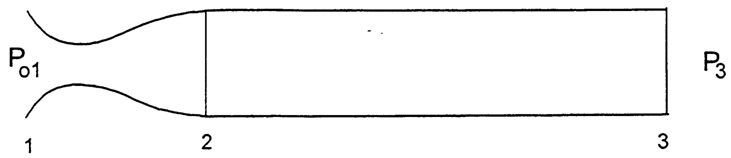


Fig. 2.

(b) The stagnation pressure at the inlet of the nozzle P_{01} is 2.725 bar and there are no shocks in the nozzle. The static pressure at exit of the duct P_3 is 1 bar. The diameter and length of the duct respectively are 0.5 m and 6.05 m. If the Mach number at the exit of the duct M_3 is 1, calculate two possible values of M_2 the Mach number at the inlet of the duct. Calculate the coefficient of friction C_f in the duct for each case. Take $\gamma = 1.4$. [30%]

(c) Over a number of years the inside of the duct is found to have rusted raising its coefficient of friction. Static pressure measurements along the duct wall show that a normal shock occurs 2.03 m from the inlet of the duct and that the Mach number at the duct exit remains 1. The static pressure immediately downstream of the shock is measured as 1.49 bar. Calculate the Mach number immediately downstream of the shock. Calculate the average coefficient of friction between the shock and the end of the duct. Show that the average coefficient of friction between the inlet of the duct and the shock is compatible with that downstream of the shock. [40%]

4 Figure 3 shows a two-dimensional supersonic diffusing part of a wind tunnel, designed to produce a uniform flow of air in the section downstream of FD when the upstream Mach number is 2.385 . The upper wall and the sections AB, CD and DE of the lower wall are straight, while the section BC of the lower wall is shaped so as to generate an isentropic compression, with Mach lines all converging on the point F on the upper wall. A shock wave FD is reflected from that point. The section CD is at an angle of $10°$ to the upstream and downstream flow directions.

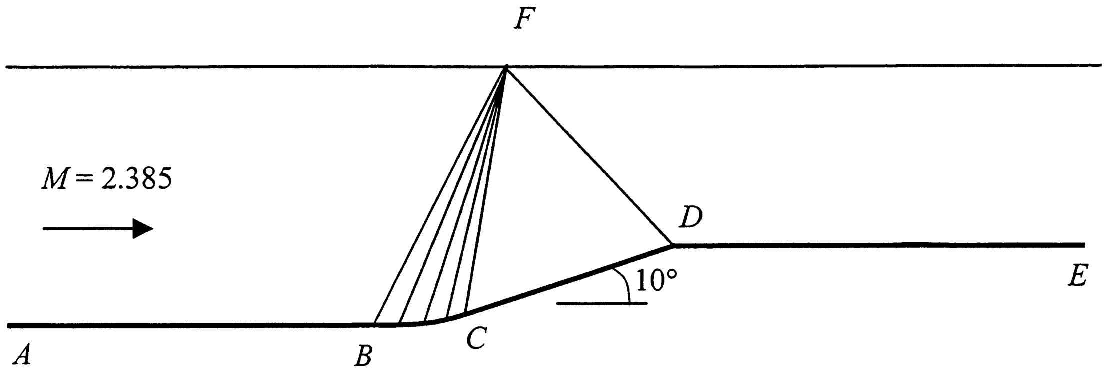


Fig. 3.

(a) Determine the Mach number of the flow downstream of the shock wave FD and the overall area ratio across the diffuser. The value of $\frac{\dot{m}\sqrt{c_p T_0}}{A P_0}$ at $M = 2.385$ is 0.5405. [40%]

(b) If the width of the tunnel at A is 10 cm, find the position of D relative to B. [40%]

(c) Sketch the wave pattern in the tunnel when the upstream Mach number is:

(i) higher than 2.385;

(ii) lower than 2.385. [20%]

5 A thin, two-dimensional, wedge-shaped aerofoil, with chord length c , and included angles at the leading and trailing edges of β, is placed at an angle of incidence α in a uniform supersonic stream of Mach number M_∞ , as shown in Fig. 4. The incidence α is defined as the angle between the oncoming stream and the *lower surface* of the aerofoil.

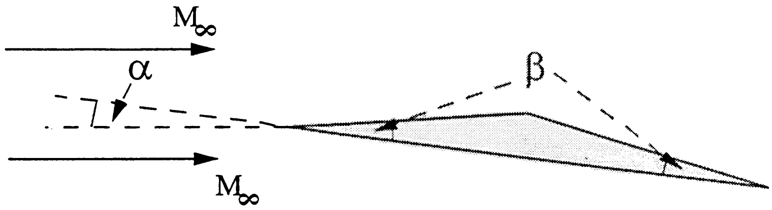


Fig. 4.

(a) Sketch the wave pattern around the aerofoil and show that, using linearised theory, the lift and drag coefficients are given by

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_D = \frac{2}{\sqrt{M_\infty^2 - 1}}\left(2\alpha^2 + \beta^2\right),$$

where $C_L = \dfrac{Lift}{\frac{1}{2}\rho_\infty V_\infty^2 c}$ and $C_D = \dfrac{Drag}{\frac{1}{2}\rho_\infty V_\infty^2 c}$. [40%]

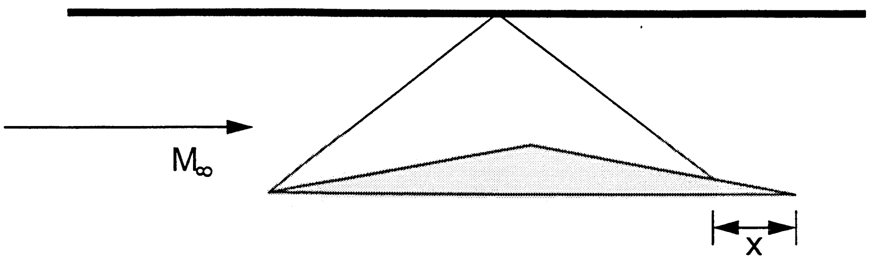


Fig. 5.

(b) The aerofoil is placed at zero incidence in a wind-tunnel, as shown in Fig.5, so that the characteristic emanating from the leading edge, after reflection by the tunnel wall, meets the aerofoil at an axial distance x from the trailing edge. Using linearised theory find an expression for the drag coefficient for the aerofoil, as a function of x for $0 \leq x \leq c/2$. [45%]

What happens to the drag as x becomes greater than $c/2$? [15%]

6 (a) Describe the use of Gauss's theorem to calculate the derivatives of flow properties within a computational cell when values are stored at the 4 corners of the cell. [10%]

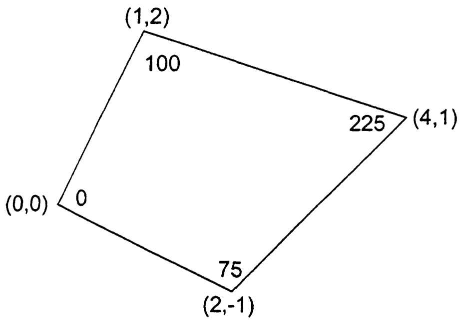


Fig. 6 .

Fig. 6 shows values of velocity potential and coordinates at the 4 corners of a computational cell. The (x, y) coordinates of the corners are shown outside the cell and values of the velocity potential ϕ are shown within it. All quantities are in SI units. Calculate the average velocity components in the x and y directions within the cell. If the velocity potential at the right hand corner of the cell, point (4,1), changes by $\Delta\phi$ derive expressions for the changes in the velocity components. [40%]

(b) An axial flow air compressor consists of eight stages: each stage has similar blades so that the velocity triangles are the same for all stages. If the machine is designed so that the mid-span radius is constant explain why the radial length of the blades will be reduced progressively through the machine from entry to exit. [10%]

In this compressor the *absolute* flow direction into each rotor row is $+10°$ from the axial direction. The axial velocity is constant through the machine and equal to 144 ms^{-1}. The blade speed is 320 ms^{-1} and the *relative* flow is turned in the rotor blades by 30°.

(i) Make careful sketches of the velocity triangles, evaluate all the flow angles and find the work input to each stage per kilogram of air flowing. [20%]

(ii) Find the degree of reaction of a stage. [20%]

7 Show that for the steady compressible flow of an inviscid fluid with uniform stagnation enthalpy and entropy flowing around a bend, as illustrated in Fig. 7, the velocity gradient perpendicular to the streamlines is given by

$$\frac{\partial V}{\partial y} = -\frac{V}{r_c},$$

where r_c is the radius of curvature of the streamlines and y is the direction normal to the streamlines. [15%]

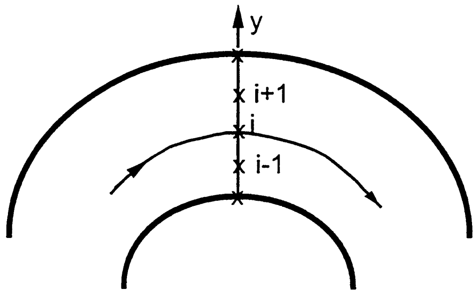


Fig. 7.

(a) A method for predicting the velocity profile in a bend solves the above equation numerically by starting from a known velocity V_i at grid point i and using previously calculated (hence fixed) values of the radii of curvature at all grid points to obtain the velocity at grid point $i+1$. A simple numerical approximation is to take

$$V_{i+1} = V_i - \frac{V_i \Delta y}{r_{c,i}},$$

where Δy is the grid spacing in the y direction. Show that this approximation is first order accurate and find an expression for its truncation error. [30%]

(b) By considering an error ΔV_i introduced at grid point i and advancing the solution to grid point $i+1$ obtain the limit on the step length Δy for which this method is stable. [25%]

(c) A different approximation is to take

$$V_{i+1} = V_i - \frac{(V_i + V_{i+1})\Delta y}{(r_{c,i} + r_{c,i+1})}$$

Assuming that r_c varies only slowly with y so that it can be treated as a constant, find the order of accuracy of this approximation and obtain an expression for its truncation error. [30%]

8 (a) A small gas turbine has a stagnation pressure ratio of 4:1 across a centrifugal compressor. The air entering the centrifugal compressor has no swirl and an inlet stagnation pressure of 1 bar. The blade speed at exit is 380 ms^{-1} and the slip factor is 0.88. The radial component of velocity at the impeller exit is 160 ms^{-1} and the impeller blades are backswept by 30°.

(i) Determine the swirl velocity at the exit from the impeller. [15%]

(ii) If the mass flow rate is 2 kgs^{-1} determine the power required to drive the compressor. [25%]

(b) The power to drive the compressor is supplied by a single-stage axial flow turbine. The air enters the turbine at a stagnation pressure of 4 bar and a stagnation temperature of 1100 K and leaves it without any swirl. The axial velocity through the turbine is constant and equal to 95 ms^{-1} and the blade speed is 310 ms^{-1}. The mass flow through the turbine is the same as that through the compressor and the properties of air may be used throughout the expansion in the turbine.

(i) Determine the stage loading coefficient, the absolute flow angle and the Mach number at exit from the turbine stator blades. [25%]

(ii) If the turbine total-total isentropic efficiency is 0.85 calculate the static and stagnation pressures at the exit from the turbine. [25%]

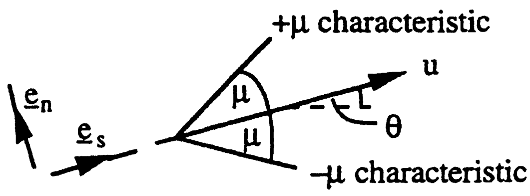
(c) Describe the shock wave pattern that develops in a turbine blade passage as the exit Mach number increases. Comment on the significance of the "limit loading" condition. [10%]

END OF PAPER

PART IIA PAPER 3A3: Two-Dimensional Compressible Flow Data Sheet

Method of Characteristics for 2-D supersonic flow

Applicable to adiabatic (h_0 = constant), isentropic flow.



Mach number $M = u/c$

Mach angle $\mu = \sin^{-1}\left(\frac{1}{M}\right)$

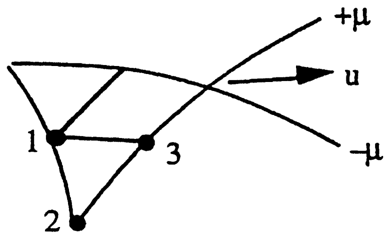
Prandtl-Meyer function $\nu = \int_{M=1}^{M} \sqrt{M^2-1}\,\frac{du}{u}$

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{(\gamma-1)}{(\gamma+1)}(M^2-1)} - \tan^{-1}\sqrt{M^2-1} \quad \text{for a perfect gas}$$

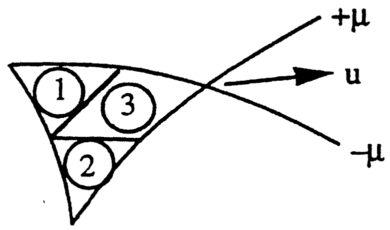
ν is tabulated under 'ω' in the CUED Gas Flow Tables

Calculations

Lattice method



Field (or wave) method



Lattice method:

$\nu_3 - \theta_3 = \nu_2 - \theta_2$ along +μ

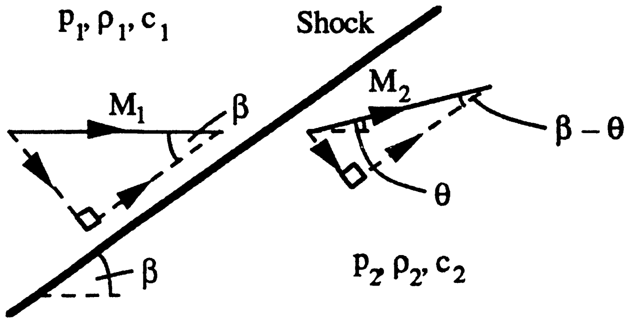
$\nu_3 + \theta_3 = \nu_1 + \theta_1$ along −μ

Field (or wave) method:

$\nu_3 + \theta_3 = \nu_1 + \theta_1$ across +μ

$\nu_3 - \theta_3 = \nu_2 - \theta_2$ across −μ

Oblique shock waves in a perfect gas



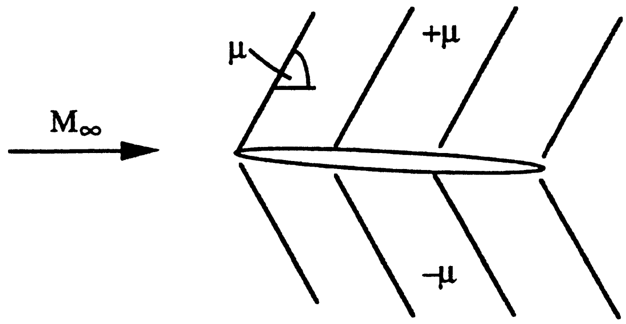
$$\tan\theta = 2\cot\beta \frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma+\cos 2\beta)+2}$$

$$M_2^2\sin^2(\beta-\theta) = \frac{1+\frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2\sin^2\beta - \frac{\gamma-1}{2}}$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}M_1^2\sin^2\beta - \frac{\gamma-1}{\gamma+1}$$

$$\frac{\rho_1}{\rho_2} = \frac{2}{\gamma+1}\frac{1}{M_1^2\sin^2\beta} + \frac{\gamma-1}{\gamma+1}$$

Linearised Method of Characteristics (Thin aerofoil theory)

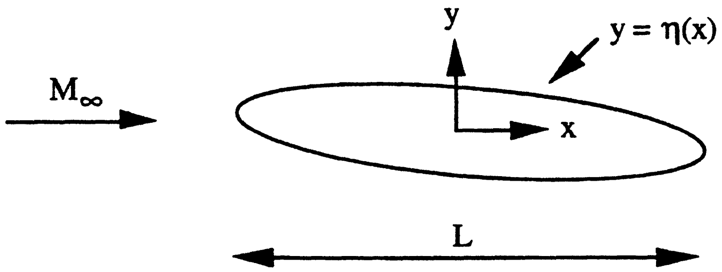


$$\mu \approx \sin^{-1}(1/M_\infty)$$

$$\Delta p \approx \pm\frac{\rho_\infty u_\infty^2\Delta\theta}{\sqrt{M_\infty^2-1}}$$ across ±μ waves

Pressure coefficient $c_p = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2} = \pm\frac{2\theta}{\sqrt{M_\infty^2-1}}$ on upper/lower surface

Prandtl-Glauert rule for linearised potential flow past geometrically similar bodies



Pressure coefficient $c_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$

For geometrically similar bodies, with $\frac{\eta}{L} = f\left(\frac{x}{L}\right)$ and $c_p(M_\infty = 0) = c_{p0}$,

$$c_p = \frac{c_{p0}}{\sqrt{1 - M_\infty^2}}$$ in subsonic flow

$$c_p \propto \frac{1}{\sqrt{M_\infty^2 - 1}}$$ in supersonic flow