ENGINEERING TRIPOS PART IIA

Saturday 24 April 2004 9 to 12

Module 3A5

ENERGY AND POWER GENERATION

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Given Maxwell's relationship for a pure substance, $(\partial s/\partial p)_T = -(\partial v/\partial T)_p$, show that

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p .$$

Hence prove that

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p .$$

[20%]

(b) The (p,v,T) equation of state of an imperfect gas is given by

$$v = \frac{RT}{p} - \frac{K}{T^3} ,$$

where K is a constant. Using the results of part (a) or otherwise, derive expressions for c_p, h and s in terms of T and p, given that at low pressure ($p \to 0$), the isobaric specific heat capacity is well represented by

$$c_p = A + BT \quad (p \to 0) ,$$

where A and B are constants. Hence show that the characteristic equation of state for the specific Gibbs function is

$$g = A(T - T\ln T) - \frac{BT^2}{2} + RT\ln p - \frac{Kp}{T^3} + h_0 - Ts_0 ,$$

where h_0 and s_0 are constants. [80%]

2 A simple cycle gas turbine may be modelled as a compressor, heater and turbine with a working fluid which behaves as a perfect gas with constant c_p and γ throughout. The turbine cooling flows and heater pressure loss may be neglected. The *isentropic* efficiencies of the compressor and turbine are η_c and η_t respectively, θ is the ratio of turbine inlet temperature T_3 to compressor inlet temperature T_1, and x is the *isentropic* temperature ratio across both the compressor and the turbine.

(a) A heat exchanger is fitted to the gas turbine to preheat the air before the combustor. It may be assumed that the effectiveness is unity and pressure losses within the exchanger may be neglected. Sketch the cycle on a (T-s) diagram and derive expressions for the dimensionless specific work output $W/(c_pT_1)$ and heat input $Q/(c_pT_1)$ in terms of η_c, η_t, θ and x. Show that $W/(c_pT_1)$ is maximised when x is given by

$$x = (\eta_c \eta_t \theta)^{1/2} \quad .$$

For $\eta_c = \eta_t = 0.90$, $\theta = 5.5$ and $\gamma = 1.4$, calculate the corresponding compressor pressure ratio and cycle efficiency. [40%]

(b) It is now decided to intercool the compressor with one stage of intercooling. After intercooling, the temperature of the working fluid is reduced to T_1 and the heat extracted is rejected to the environment. The isentropic efficiencies of the low and high pressure compressors are both η_c and it may be assumed without proof that, for maximum specific work, the isentropic temperature ratio across the low-pressure compressor x_1 is given by $x_1 = x^{1/2}$, where x is the overall isentropic temperature ratio. Sketch the cycle on a (T-s) diagram and derive an expression (in terms of η_c, η_t and θ) for the value of x which maximizes $W/(c_pT_1)$. Using the data of part (a), calculate the corresponding overall pressure ratio and cycle efficiency. [40%]

(c) Briefly discuss the advantages and disadvantages of intercooling recuperative and non-recuperative cycles. [20%]

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3 In a CCGT (combined cycle gas turbine) power plant, the exhaust from the gas turbine enters the HRSG (heat recovery steam generator) at 550 °C and leaves at 160 °C after transferring heat to the steam. Pressure losses in the HRSG can be neglected. The gas may be treated as perfect with $c_p = 1.2$ kJ kg^{-1} K^{-1} and $\gamma = 1.32$.

The steam cycle is single pressure without steam reheat or feedheating. Wet saturated water leaves the condenser and enters a feed pump where it is compressed isentropically to 60 bar before entering the HRSG. There is a 50 °C temperature difference between the gas and the steam at exit from the HRSG. It may be assumed that the expansion in the steam turbine is isentropic. In the condenser, the steam is condensed at a constant pressure of 0.08 bar. The heat released is transferred to cooling water, which is at the environment temperature of 25 °C.

Find the following, per kg of exhaust gas passing through the HRSG (use the steam tables rather than the chart):

(a) the maximum work available from the turbine exhaust gas; [15%]

(b) the feed pump work input (an approximate calculation is acceptable) and the steam turbine work output; [30%]

(c) the various lost work terms, explaining how these are calculated (derivations of equations are not required). [45%]

(d) By considering your answers to parts (a), (b) and (c), perform a check on your calculations, explaining very briefly (without derivation) the principle involved. [10%]

4 (a) By isotopic abundance, natural uranium consists of 99.285% U-238 atoms and 0.715% U-235 atoms. A spherical nuclear reactor is fuelled with 100 tonnes of natural uranium. The average neutron flux in the reactor is 1.2×10^{17} neutrons $\mathrm{m^{-2}s^{-1}}$ and the fission and capture microscopic cross-sections for U-235 are 579 and 101 barns respectively (1 barn $=10^{-28}\,\mathrm{m^2}$). If the energy released per fission is $3.2\times10^{-11}\,\mathrm{J}$, calculate the thermal power of the reactor and the total rate of consumption of U-235 in kg per day. Avogadro's number is $6.022\times10^{26}\,\mathrm{kmol^{-1}}$. [30%]

(b) The neutron flux ϕ at a radial position r in a spherical nuclear reactor is given by

$$\phi = \frac{\phi_{\max} R_0}{\pi r}\sin\left(\frac{\pi r}{R_0}\right) ,$$

where $\phi_{\max}$ is the maximum neutron flux and R_0 is the extrapolated radius of the core. Show that the average flux within the reactor ϕ_{ave} is related to the maximum flux $\phi_{\max}$ by the relation

$$\phi_{\text{ave}} = 3\phi_{\max}\left[\frac{\sin(X) - X\cos(X)}{X^3}\right] ,$$

where $X = \pi R/R_0$ with R being the (physical) radius of the core. [40%]

(c) For safety reasons the ratio $\phi_{\max}/\phi_{\text{ave}}$ must not exceed 2.0. Using an appropriate iterative method, find the limiting value of the ratio R/R_0, above which this condition is not met. A suitable first guess is $R/R_0 = 0.80$. Explain how, in practice, it is possible to make R_0 significantly greater than R. [30%]

5 (a) Briefly describe the various mechanisms of nitric oxide (NO) generation from combustion. Discuss techniques used in gas turbines and spark-ignition engines to decrease the NO emission, including comments on how the emission of *other* pollutants may be *increased* while implementing some of these techniques. [60%]

(b) An approximate theory for flame extinction in a combustor is that blow-off occurs when the chemical timescale of a flammable mixture at an initial temperature T_0 becomes greater than the residence time in the combustor (by a certain factor). Mathematically, this can be written as

$$\tau_{chem} > C\tau_{res} \ ,$$

where the chemical timescale is defined as $\tau_{chem} = \lambda/(\rho_0 c_p S_L{}^2)$ and the residence time as $\tau_{res} = L/U$ with U the bulk velocity of the mixture entering the combustor and L the combustor's length. Here, c_p is the isobaric specific heat capacity, taken as constant; λ the thermal conductivity, increasing with temperature as $\lambda/\lambda_{ref} = \left(T_0/T_{ref}\right)^{1/2}$; ρ_0 the density of the mixture entering the combustor; S_L its laminar burning velocity, increasing with reactant temperature as $S_L/S_{L,ref} = \left(T_0/T_{ref}\right)^2$, and C is a constant. Find the percentage increase in the mass flow rate at extinction when T_0 increases from 300 K to 600 K and the combustor is doubled in length, with all other parameters staying constant. [40%]

6 A cement-manufacturing furnace can be taken as an adiabatic straight long cylinder. At the inlet to the cylinder, fuel and air are very quickly mixed and a flame is established. At the exit of the cylinder, raw material in the form of solids is injected and flows along the wall of the furnace in the opposite direction to that of the gas while being heated by the flame products. The solids do not participate in the combustion process, other than releasing certain gases (as we shall explore in part (b) below).

(a) The fuel is $C_{10}H_{22}$ with a mass flow rate of 0.06 kg s^{-1} and the air has a mass flow rate of 1 kg s^{-1}. Calculate the equivalence ratio of the mixture to be burned. [30%]

(b) The raw material to be processed releases 0.1 kg s^{-1} of CO_2. This CO_2 is mixed with the flame products as they flow out of the furnace and the compound gases (combustion products plus released CO_2) are in equilibrium at the exit of the duct where the temperature is 2000 K. The pressure is 1 bar.

(i) Set-up the systems of equations needed to calculate the composition of the gases exiting the furnace assuming there are only N_2, CO_2, CO, O_2, and H_2O present. [60%]

(ii) Calculate the volume fractions of the species mentioned in (i), given that the volume fraction of O_2 is 1.93%. [10%]

7 A modified compression-ignition engine operates under the ideal constant volume cycle, with the only difference being that valve timing is such that compression begins at a volume $V_1 = r\alpha V_c$, where V_c is the clearance volume, r the compression ratio equal to 20, and α =0.8. (In a conventional cycle, compression begins at rV_c, i.e. it has α =1.) This allows the mass fraction of residual gases x_r to attain the value of 0.4. The fresh charge enters at a temperature T_{in} = 300 K, the residual gases have a temperature T_r = 520 K, the air to fuel ratio (by mass) is f = 50, and the heat release per unit mass of fuel is Q = 40 MJ kg^{-1}.

(a) Derive an expression for the temperature T_1 of the mixture (charge and residual gases) before compression starts as a function of T_r, T_{in} and x_r. [10%]

(b) Show that the heat release q per unit mass of gas in the cylinder is given by

$$q = \frac{Q(1-x_r)}{1+f} \quad .$$

[20%]

(c) Derive an expression for the work (per unit mass of mixture) performed during the compression stroke. [20%]

(d) Show that the *net* work of the cycle is given by

$$w_{net} = c_v T_1 \left\{ \left[(\alpha r)^{\gamma-1} + \frac{q}{c_v T_1} \right] \left(1 - r^{1-\gamma} \right) - \left[(\alpha r)^{\gamma-1} - 1 \right] \right\} \quad ,$$

where c_v is the specific heat capacity of the mixture (taken as air) at constant volume and γ the ratio of specific heats. [40%]

(e) The efficiency of the cycle is defined as w_{net} / q. Show that, for the values given above, the efficiency of the modified cycle is higher than the efficiency of a conventional cycle with the same r. Explain the reason for this increase in efficiency. [10%]

8 At the end of the expansion stroke of a four-stroke engine, the pressure is p_4, the temperature T_4, and the volume rV_c, where V_c is the clearance volume and r the compression ratio. At that point, the exhaust valve opens and the gases exit the cylinder and expand from p_4 to the exhaust manifold pressure p_e. This expansion is taken as polytropic with an exponent n. The engine has z cylinders and is rotating at N cycles per second and a turbocharger is fitted at the exhaust manifold.

(a) By considering the amount of mass in the cylinder before and after the valve opening, derive an expression for the mass flow *rate* of exhaust gases in the exhaust manifold for this operating condition as a function of the given parameters. [50%]

(b) The exhaust gases in the manifold are expanded across a turbine to the atmospheric pressure p_{atm}. Derive an expression for the maximum power that can be delivered by the turbine. [50%]

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