UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Tuesday 4th May 2004 9.00 – 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments at the end of this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 a) Find the solution to the time-independent Shroedinger Equation for a particle in the 1-dimensional potential well of Fig.1, for the case $E << V_0$ (i.e. the depth of the well can be considered infinite). State all assumptions and determine all integration constants. [30%]

b) Sketch the wavefunctions $\psi_1(x)$ and $\psi_2(x)$ corresponding to the two lowest values of the particle energy, $E_1 < E_2$. [10%]

c) Assuming that the probability of finding the particle inside the well is unity, calculate the probabilities that a particle with wavefunction $\psi_1(x)$ is in the regions

(i) $0 < x < \frac{a}{4}$;

(ii) $\frac{3}{4}a < x < a$.

Calculate also the probabilities that a particle with wavefunction $\psi_2(x)$ is in these regions. [30%]

d) Find the solution to the time-independent Shroedinger Equation for the case of the potential well of Fig.1, when the depth of the well is finite and $E < V_0$. Do not evaluate the integration constants.

Sketch the wavefunction corresponding to the lowest value of E and comment on the differences between this case and that of part (b). [30%]

Note: The time-independent Shroedinger Equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0.$$

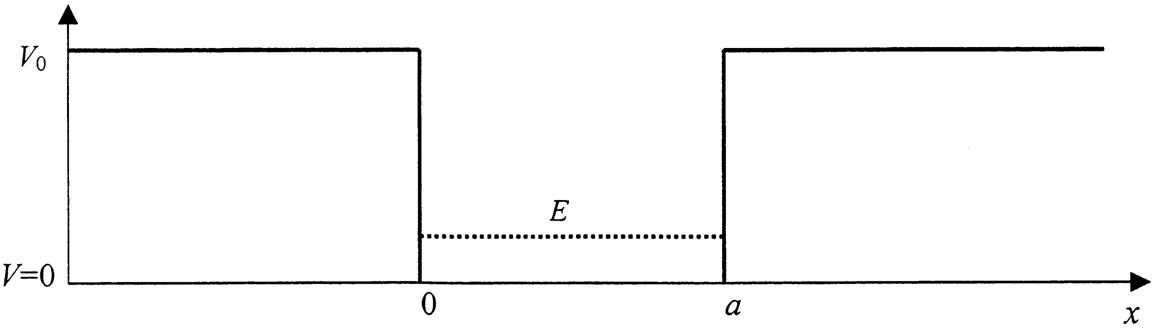

Fig. 1

2 a) Sketch the E-k curve for an electron in a 1-dimensional crystal according to the nearly-free electron approximation. Identify the 1st, 2nd and 3rd Brillouin Zones and explain the formation of the energy gaps in the framework of the nearly-free electron approximation. [30%]

b) Express the electron velocity $v = \frac{p}{m}$ in terms of E and k and hence derive the expression for the effective mass [40%]

$$m^* = \hbar^2 \left(\frac{d^2 E}{dk^2} \right)^{-1} .$$

c) For the 1st Brillouin Zone, in the nearly-free electron approximation, sketch the velocity - wavenumber (v - k) and effective mass - wavenumber (m^* - k) curves.

Explain the meaning of a negative effective mass. [30%]

Note: The E-k relationship for the case of a free electron is

$$E = \frac{\hbar^2 k^2}{2m},$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber.

3 (a) Explain how a small base current can control the collector current in a p^+np bipolar junction transistor (BJT). Draw a schematic diagram showing clearly the flow of electrons and holes in the device. [35%]

(b) (i) Starting from the Continuity Equation, derive the expression

$$\Delta p_n(x) = \Delta p_n(0)\frac{\exp((W_b - x)/L_h) - \exp((x - W_b)/L_h)}{\exp(W_b/L_h) - \exp(-W_b/L_h)}$$

for the variation in excess carrier concentration across the base as a function of distance x from the edge of the depletion region in the base at the junction with the emitter. W_b is the undepleted width of the base. L_h is the diffusion length of holes in the n-type semiconductor and $\Delta p_n(0)$ is the density of excess holes injected into the base from the emitter at $x = 0$. State any assumptions made. [35%]

(ii) Sketch how the excess hole concentration varies with position across the base in the case where $W_b > L_h$ and $W_b << L_h$. Why is the shape of the curve different in the two cases and which case will produce a BJT with high gain? [30%]

Note: The Continuity Equation for holes is

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}.$$

4 (a) With the aid of band diagrams where appropriate, explain the meaning of the following terms as applied to the Metal Oxide Semiconductor (MOS) structure incorporating an n-type semiconductor:

flat band condition;
accumulation;
depletion;
inversion;
strong inversion. [25%]

(b) An n-type MOS structure has been fabricated from silicon, which has an intrinsic carrier concentration of 1.5×10^{16} m^{-3} and has been doped with $N_D = 10^{22}$ m^{-3} of phosphorous atoms. If the device is to be operated at a temperature of 298 K , calculate the surface potential V_{si} at the silicon-silicon oxide interface at the onset of strong inversion. You may assume that the device is ideal, with the flat band condition occurring when no bias is applied to the device. State any other assumptions made. [25%]

(c) Starting from the Gauss Law of Electrostatics, show that the width of the depletion region at the onset of strong inversion is given by

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r V_{si}}{eN_D} \right)^{1/2}$$

where e is the magnitude of the electronic charge, ε_0 is the permittivity of free space and ε_r is the relative permittivity of the semiconductor, which is 11.8 for silicon. State any assumptions made. Calculate this width for the device described in part (b). [35%]

(d) Under what circumstances may the depletion width be greater than that given by the equation in part (c)? What is this regime called and why does it occur? [15%]

(Cont.

Note: For a semiconductor

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

where E_F is the Fermi energy, E_i is the Fermi energy in intrinsic material and other symbols have their usual meaning.

END OF PAPER