UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Tuesday 4 May 2004 2.30 to 4.00

Module 3D1

SOIL MECHANICS

Answer not more than 3 questions.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Soil Mechanics Data Book (19 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 Figure 1 shows the ground profile beneath an expanding city. The water table in the coarse granular fill layer A is set by a major river, and remains about 2m below ground level. The clay layer B is known to be normally compressed. Sand layer C has been used as the bearing zone for deep foundation piles beneath high-rise buildings, and it is now proposed to abstract water from it.

Trial pits reveal that the dry density of the fill A is about 1600 kg/m^3, and that the density of its grains relative to water is 2.65. A tube sample from the mid-depth of clay B has a natural water content of about 66%. Oedometer tests reveal that the clay has mechanical properties similar to Kaolin, and a coefficient of consolidation of about 10^{-7} m/s^2.

(a) Estimate the dry and saturated unit weights of fill A, and the saturated unit weight of clay B. Plot a diagram showing the variation of total vertical stress through layers A and B. [20%]

(b) Abstraction of water from sand C has recently reduced the water pressures in that layer by 50 kPa. Plot a profile of pore water pressure with depth below ground level from 0 to 30 m, both for the historic past and for the far future, assuming abstraction continues indefinitely. Deduce the profile of excess pore pressures in the clay layer due to pumping, and sketch a family of isochrones that display the progress of consolidation, explaining their shape. [20%]

(c) Using the properties of Kaolin given in the attached Data Book, estimate the ultimate change in specific volume of the clay at a depth of 15 m below ground level. Estimate the ultimate regional subsidence due to clay consolidation resulting from the pumping. [30%]

(d) One tenth of that magnitude of subsidence would begin to cause obvious damage to roads and services around high-rise buildings. Estimate the time that would elapse from the commencement of pumping prior to this manifesting itself, assuming that the Data Book relation between R_v and T_v still applies in the case. [20%]

(e) Discuss the degree to which the situation would be remedied if pumping was halted and historic groundwater levels were reasserted. [10%]

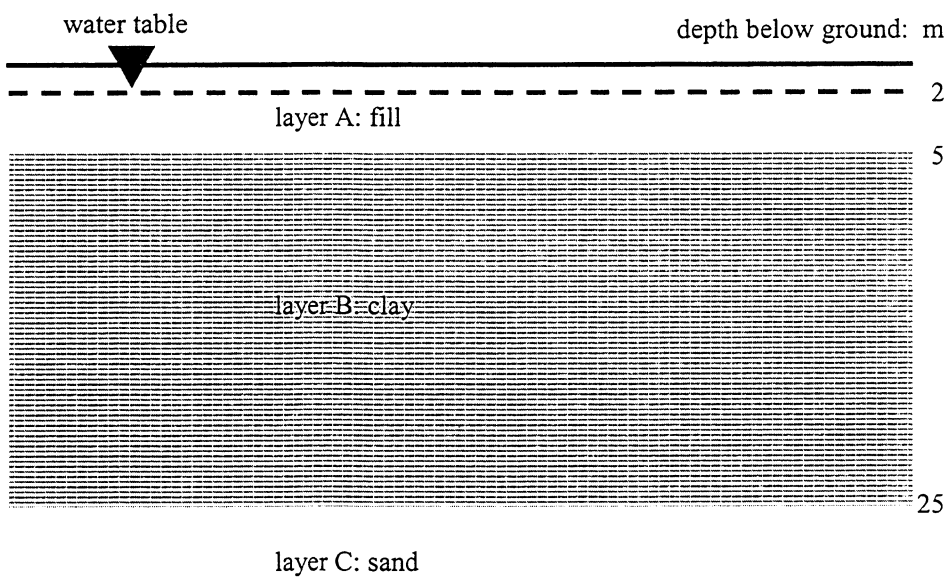

Fig. 1

2 (a) Use appropriate sketches to explain how Cam Clay distinguishes between elastic behaviour and plastic yielding, and the significance of the normality principle and of critical states. [40%]

(b) A silty clay with mechanical properties similar to Weald Clay (as listed in the attached Data Book) is first brought into a state A of normal compression under a vertical effective stress of 100 kPa in a Simple Shear Apparatus. It is then sheared very slowly at constant vertical stress, with drainage permitted from top and bottom platens, until it is mobilising half its ultimate shear strength. Calculate its current state B (τ_B, σ'_B, v_B). [30%]

(c) If the drains were then closed, and shearing were recommenced at constant vertical stress until the sample reached a plateau of constant shear stress in state C, predict that state. [30%]

3 (a) Calculate key points and sketch state paths, on (τ, σ'), (v, σ') and (v, ln σ') diagrams appropriate to a Simple Shear Apparatus, corresponding to the following sequence of processes carried out on London Clay (whose properties are given in the attached Data Book). Assume that the Cam Clay model of soil behaviour is valid.

(i) Normal compression from a mud to a stiff clay in state A carrying its maximum historic effective vertical stress $\sigma' = 500$ kPa. [10%]

(ii) Swelling back to state B with an effective vertical stress of 50 kPa. [10%]

(iii) Undrained shearing to state C developing its critical state strength. [20%]

(b) Suppose that London Clay in a shallow borrow pit is in state B prior to being dug by an excavator during which its state goes to C. Extend the state paths from part (a) to illustrate the following additional processes.

(i) The clay is stored in a deep stockpile and is subject to percolating rainwater which permits all pore pressures to become atmospheric. Clay at about 1 m depth in the stockpile has a total vertical stress of about 20 kPa as it comes into drained equilibrium in state D. [20%]

(ii) The clay is dug out once again and compacted to state E in its final location by being remoulded at constant water content. [20%]

(c) Give two reasons why the undrained strength of the compacted clay in state E will be significant to the engineers on site. Deduce some simple rules to avoid, as far as possible, the deterioration of clay fill between extraction and final compaction. [20%]

4 A calcium carbonate (shelly) sand which seems superficially similar to Dog's Bay Sand (as described in the attached Data Book) is found to considerable depth beneath shallow water in a region of oil exploration. Spud-cans (circular pad foundations) for jack-up rigs will apply vertical stresses of 300 kPa to the sand. Unfortunately the sand seems quite variable in strength giving rise to concern that one leg may punch through a soft spot and cause a rig to topple in a storm.

(a) In order that an engineer can begin to make assessments of bearing capacity, predict the peak secant angles of friction that might be observed in fully drained triaxial tests on both the densest and loosest states of Dog's Bay Sand, each tested at cell pressures simulating the state of stress beneath a spud-can. Use these values to place estimated bounds on the angle of friction that might be used in bearing capacity analyses. [50%]

(b) Block samples of the sand from the sea bed are obtained and trimmed to make triaxial test specimens 100 mm diameter and 200 mm high. It is noticed that the sand contains many fine particles, reducing its D_{10} value to 0.002 mm. Suggest a rate of axial compression for the triaxial tests such that the expected peak rate of dilatancy could be satisfied without creating negative excess pore pressures greater than 1 kPa, permitting the samples to drain through both top and bottom platens. Use suitable empirical expressions for permeability and rate of dilatancy given in the Data Book. [50%]

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2003/2004

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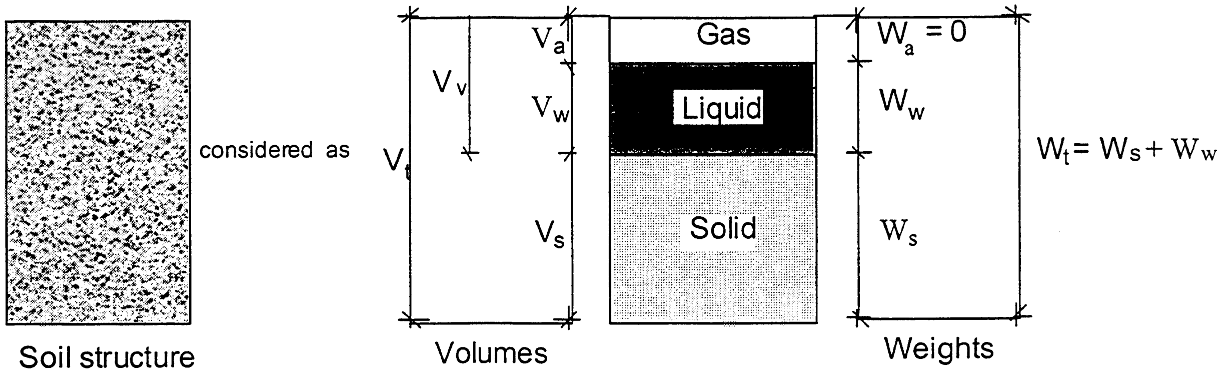
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81 \text{ kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit	w_L
Plastic Limit	w_P
Plasticity Index	$I_P = w_L - w_P$
Liquidity Index	$I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

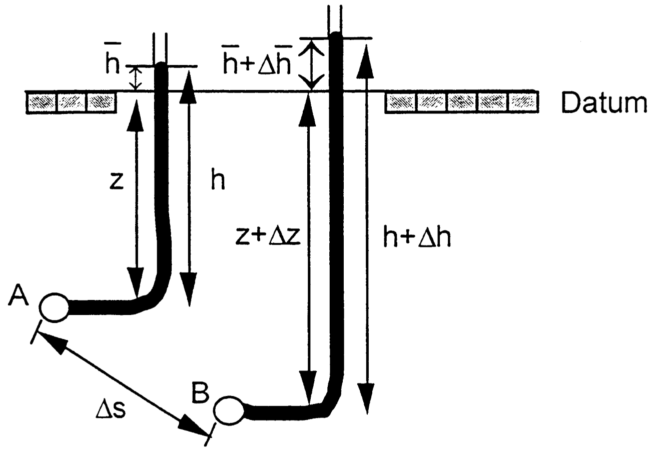
Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc.	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.
C_U	uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta \bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla \bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
$10 \text{ mm} > D_{10} > 1 \mu\text{m}$	:	$k \cong 0.01$ (D_{10} in mm)^2 m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

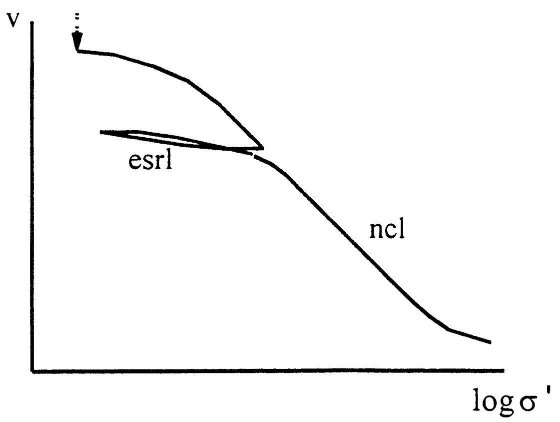
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3 \times 10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

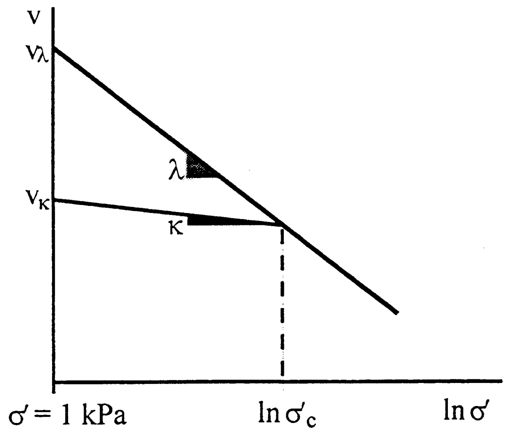
One-Dimensional Compression

Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

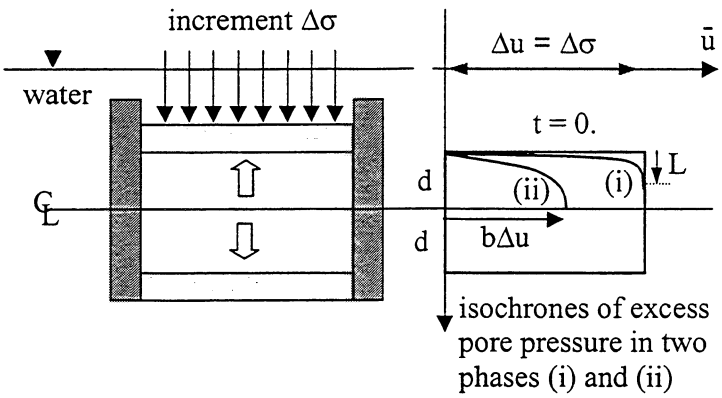
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $\quad L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $\quad b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

Principal components of stress and strain

sign convention	compression positive
total stress	$\sigma_1,\ \sigma_2, \sigma_3$
effective stress	$\sigma_1',\ \sigma_2',\ \sigma_3'$
strain	$\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$

Simple Shear Apparatus (SSA) ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\, \delta\gamma + \sigma' \delta\varepsilon$

Biaxial Apparatus - Plane Strain (BA-PS) ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1' \delta\varepsilon_1 + \sigma_3' \delta\varepsilon_3$
	$\delta W = s' \delta\varepsilon_v + t \delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

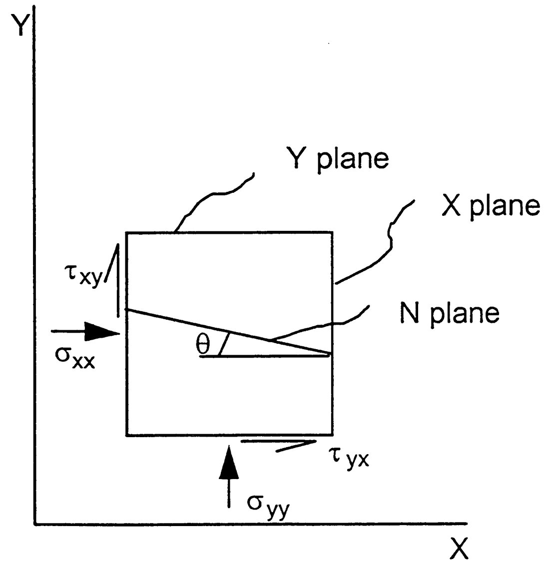
isotropic compression in which p' increases at zero q

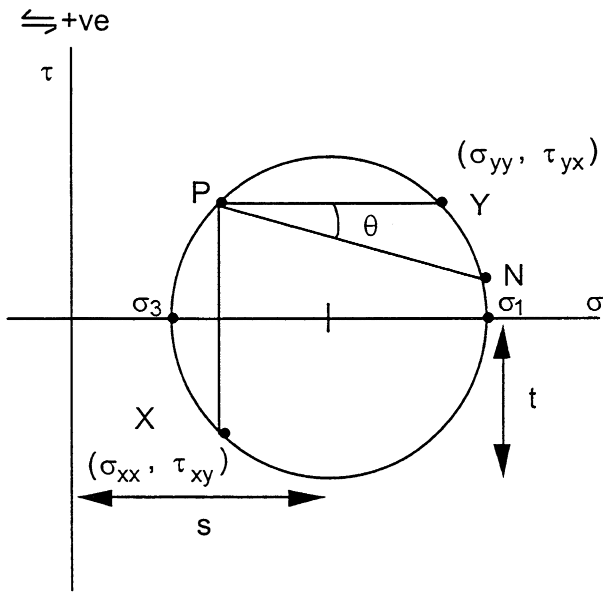
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P: the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility	$m_v = d\varepsilon / d\sigma'$
constrained modulus	$E_o = 1 / m_v$

Physically fundamental parameters

shear modulus	$G' = dt / d\varepsilon_\gamma$
bulk modulus	$K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus	$G_u = G'$
undrained bulk modulus	$K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli	E' (effective), E_u (undrained)
Poisson's ratios	ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

General yield surface

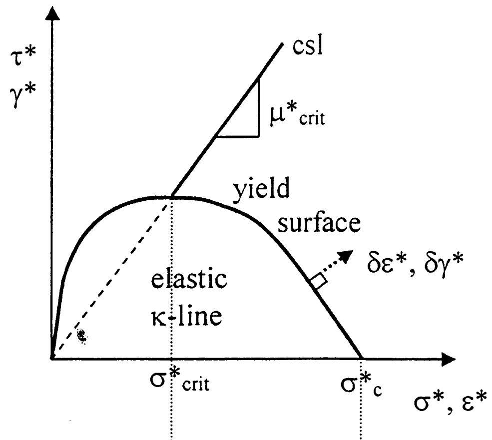
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500 Dense 1500	Loose 2500 Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

The yield surface in (σ^*, τ^*, v) space



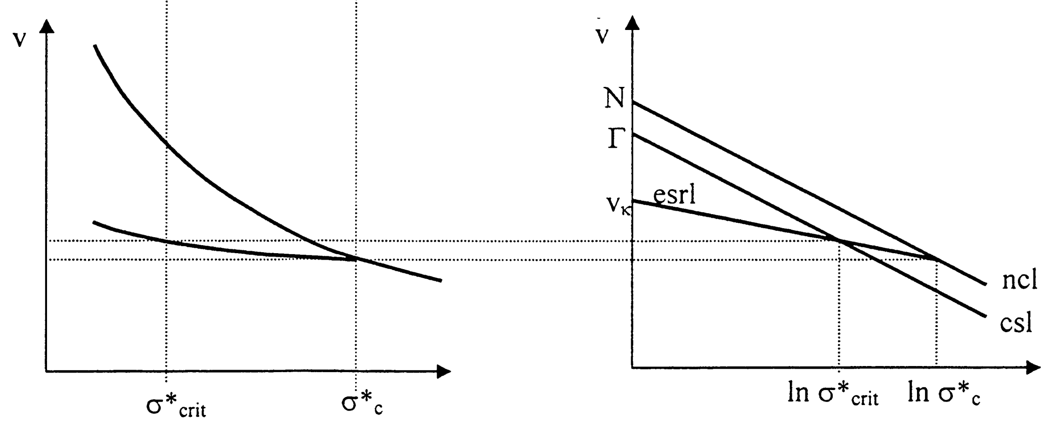
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

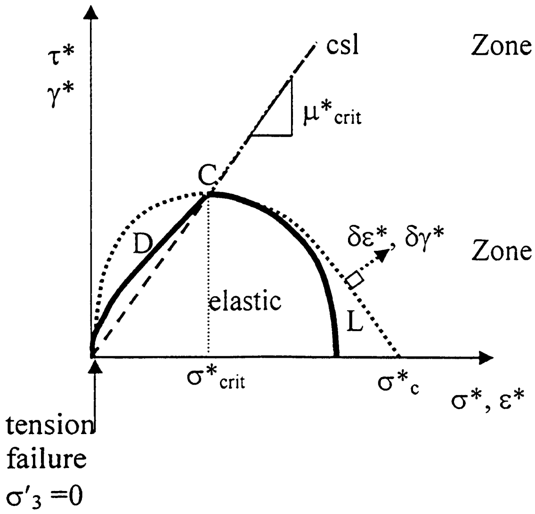
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



Regions of limiting soil behaviour



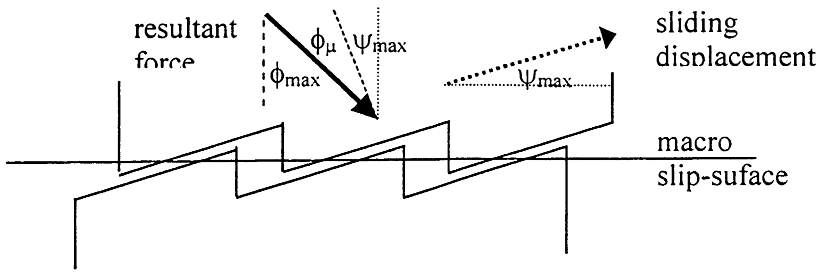
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

Friction and dilatancy: the saw-blade model of direct shear

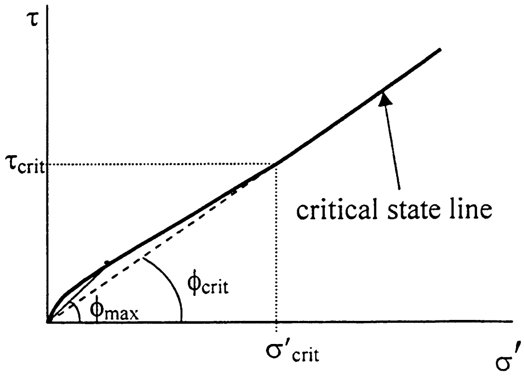


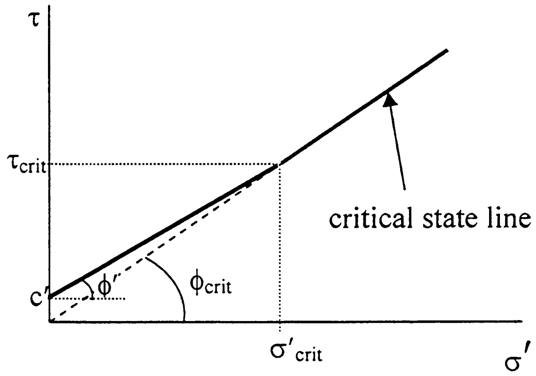
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

Friction and dilatancy: secant and tangent strength parameters





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})} \quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p') \quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

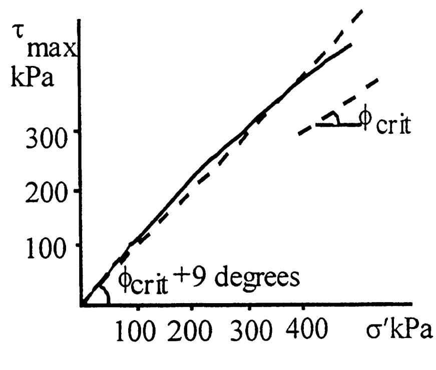
Relative dilatancy index $I_R = I_D I_C - 1 \quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

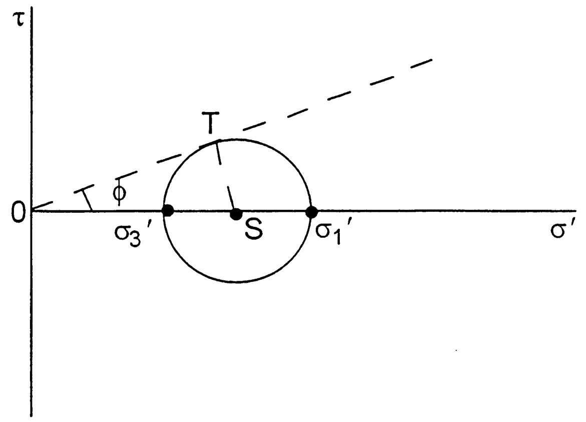
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' < 400$ kPa

Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

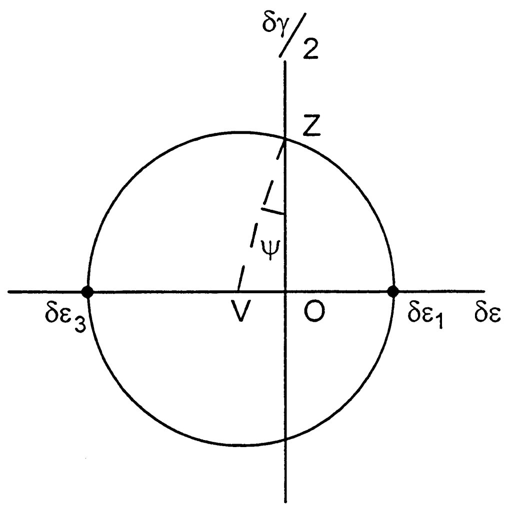
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ'_{crit} after large shear strains

Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$

Limiting stresses

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

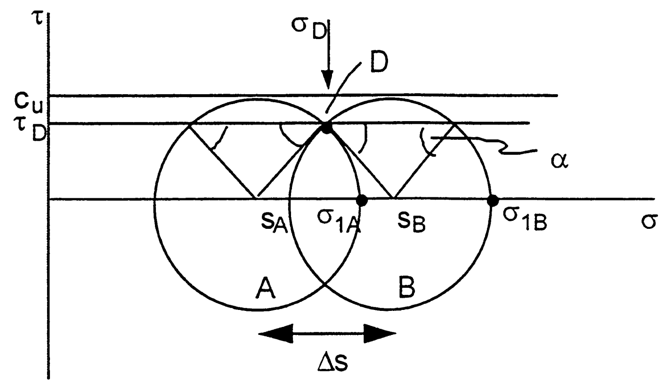
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = Ac_u x$$

Stress conditions across a discontinuity

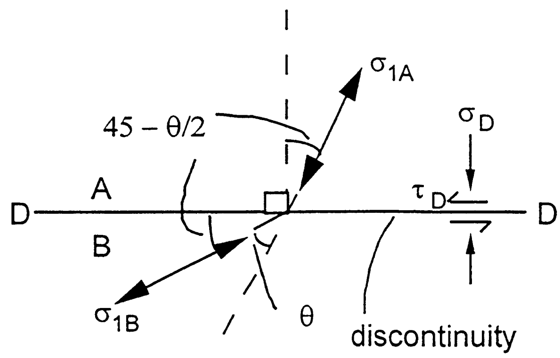


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1(B)} - \sigma_{1(A)} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u d\theta$$



Useful example:

$\theta = 30°$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi'$

Limiting stresses

$$\sin\phi' = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principle effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Earth pressure coefficient K: $\sigma'_h = K\sigma'_v$

Earth pressure at rest for normally consolidated soils $K_0 = 1 - \sin\phi'$

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure:

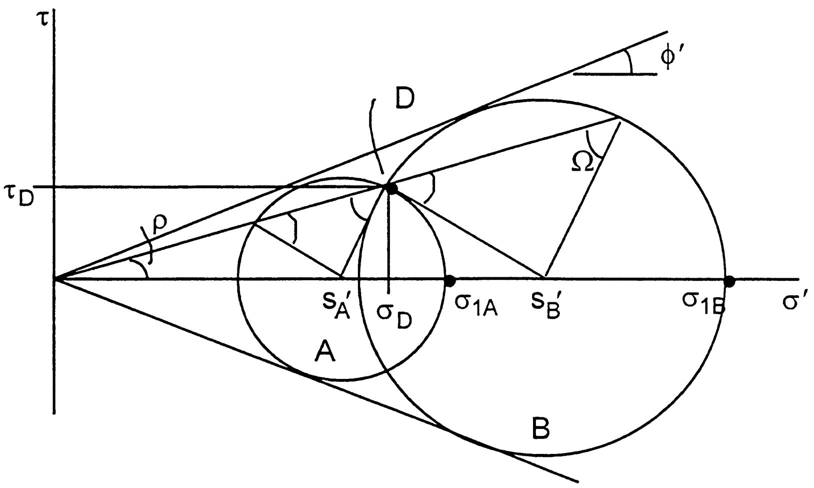
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \frac{1}{K_a}$$

Stress conditions across a discontinuity

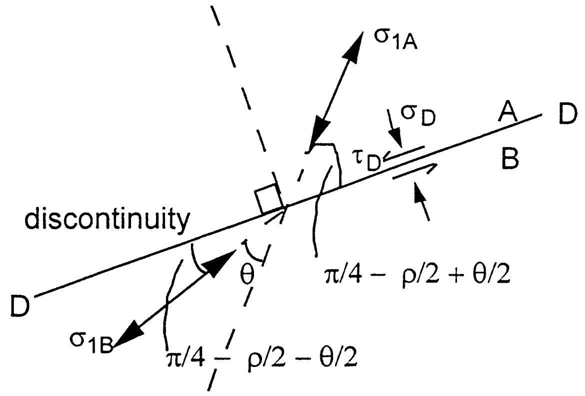


Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B



$\sin\rho = \cos\theta \sin\phi'$

$s'_B/s'_A = \cos(\theta-\rho)/\cos(\theta+\rho)$

In limit with $\delta\theta \rightarrow 0$

$\rho \rightarrow \phi'$,

$ds' = 2s' \,.\, \delta\theta \tan\phi'$.

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Formula for shallow foundation design

For clay in undrained conditions (ϕ=0 and q_s calculated based on total stress)

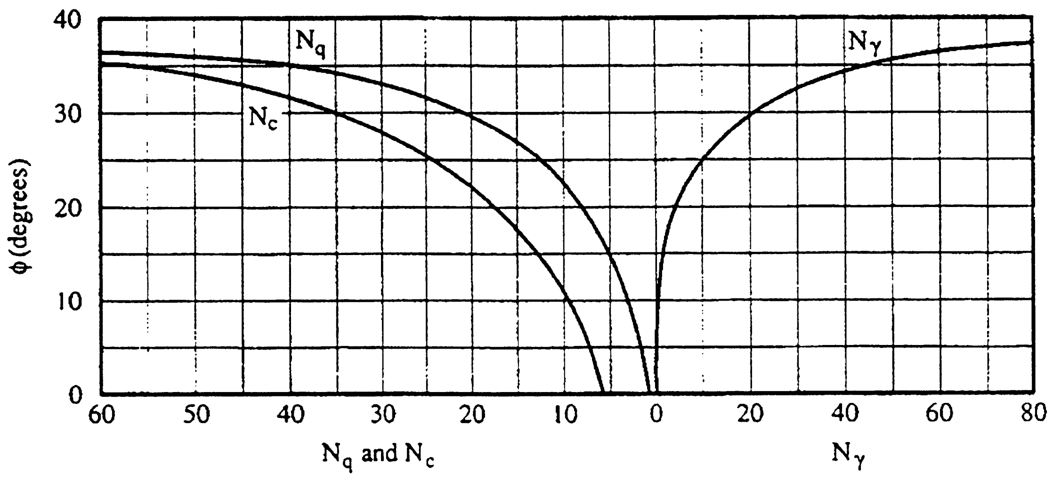
$$q_f = cN_c\zeta_c + q_s\zeta_s$$

For sand and clay in drained conditions (q_s calculated based on effective stress)

$$q_f = 0.5\gamma' B N_\gamma \zeta_\gamma + q_s N_q \zeta_s$$

$$\zeta_c = \zeta_{cd} \times \zeta_{cs} \times \zeta_{ci} \times \zeta_{c\beta} \times \zeta_{c\delta}$$
$$\zeta_r = \zeta_{\gamma d} \times \zeta_{\gamma s} \times \zeta_{\gamma i} \times \zeta_{\gamma\beta} \times \zeta_{\gamma\delta}$$
$$\zeta_s = \zeta_{sd} \times \zeta_{ss} \times \zeta_{si} \times \zeta_{s\beta} \times \zeta_{s\delta}$$

Correction factors	- Foundation depth	ζ_{cd} $\zeta_{\gamma d}$ ζ_{sd}
	- Foundation shape	ζ_{cs} $\zeta_{\gamma s}$ ζ_{ss}
	- Inclined loading	ζ_{ci} $\zeta_{\gamma i}$ ζ_{si}
	- Surface slope	$\zeta_{c\beta}$ $\zeta_{\gamma\beta}$ $\zeta_{s\beta}$
	- Base tilt	$\zeta_{c\delta}$ $\zeta_{\gamma\delta}$ $\zeta_{s\delta}$



[REMARKS]

(a) For clay in undrained conditions,

$N_c = 5.14$	for strip footing
$N_c = 5.69$	for circular footing (smooth)
$N_c = 6.05$	for circular footing (rough)
$N_c = 5(1+0.2\ B/L)$	for a rectangular footing of dimensions B x L (L>B)

(b) For sand and clay in drained conditions (use the above chart or the following equations)

$N_q = \tan^2(\pi/2 + \phi/2)e^{(\pi\tan\phi)}$

$N_\gamma = 2(N_q - 1)\tan\phi$

(c) For more complicated geometries, apply the correction factors using Table 1.

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Table 1 Correction factors

	Cohesion	Self-weight	Surcharge
1. Foundation shapes	$\zeta_{cs} = 1 + \frac{B'}{L'}\frac{N_q}{N_c}$	$\zeta_{\gamma s} = 1 - 0.4\frac{B'}{L'}$	$\zeta_{qs} = 1 + \frac{B'}{L'}\tan\phi$
2. Inclined loading	$\zeta_{ci} = (1 - 2i/\pi)^2$	$\zeta_{\gamma i} = (1 - i/\phi)^2$	$\zeta_{qi} = (1 - \frac{2i}{\pi})^2$
3 Foundation depth	$\zeta_{cd} = 1 + 0.4\xi \ (\phi = 0)$ $= \zeta_{qd} - \frac{1-\zeta_{qd}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma d} = 1.0$	$\zeta_{qd} = 1 + 2\tan\phi(1 - \sin\phi)^2\xi$
4. Surface slope	$\zeta_{c\beta} = 1 - [2\beta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\beta} - \frac{1-\zeta_{q\beta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\beta} = (1 - \tan\beta)^2$ $Note: N_\gamma = -2\sin\beta(\phi = 0)$	$\zeta_{q\beta} = (1 - \tan\beta)^2$
5. Base tilt	$\zeta_{c\delta} = 1 - [2\delta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\delta} - \frac{1-\zeta_{q\delta}}{N_c \tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\delta} = (1 - \delta\tan\phi)^2$	$\zeta_{q\delta} = (1 - \delta\tan\phi)^2$

V : vertical load, H : horizontal load, B : foundation width, L (>B) : foundation length, e_B : eccentricity parallel to B, e_L : eccentricity parallel to L; B' = B – $2e_B$, L' = L – $2e_L$; i = $\tan^{-1}$(H/V) where i is in radians; ξ = D/B if D/B < 1; $\xi = \tan^{-1}$(D/B) if D/B > 1, $\beta < \pi/4$ where β is in radians, $\delta < \pi/4$ where δ is in radians.