UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Thursday 6 May 2004 2.30 to 4.00

Module 3D2

GEOTECHNICAL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheets (19 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A 5 m diameter tunnel is to be constructed in clay below a pile foundation, as shown in Fig. 1. It can be assumed that the tunnel construction can be considered as an axisymmetric contracting cylindrical cavity under undrained conditions, analogous to cylindrical cavity expansion. The measured radial ground movement at the tunnel boundary is 25 mm.

(a) Calculate the ground movement at the toe of the pile foundation, ignoring any influence of the pile. [20%]

(b) In the elastic zone of the soil, at any radius r, the following expressions apply:

$$\sigma_r = \sigma_0 - \frac{G\delta A}{\pi r^2}$$

$$\sigma_\theta = \sigma_0 + \frac{G\delta A}{\pi r^2}$$

where σ_r and σ_θ are the radial and circumferential stresses respectively, σ_0 is the original insitu total stress in the ground, G is the elastic shear modulus of the clay, and δA is the contraction of the cavity (expressed as a change in the cross-sectional area A).

Show that the radius of the elastic/plastic boundary, r_p, is given by

$$(r_p/r_t)^2 = (G/c_u)\,(\delta A/A)$$

where r_t is the radius of the tunnel and c_u is the undrained shear strength of the clay. [30%]

(c) Using the attached Soil Mechanics Data Book, derive a relationship for r_p as a function of r_t, c_u and the reduction in support pressure in the tunnel $\delta\sigma_c$ from the original insitu total stress σ_0. [30%]

(d) Calculate the minimum tunnel support pressure that could be permitted if the toe of the pile foundation is to remain outside the plastic zone. Assume that the clay has a unit weight of 20 kN m^{-3} and an undrained shear strength c_u of 100 kPa. [20%]

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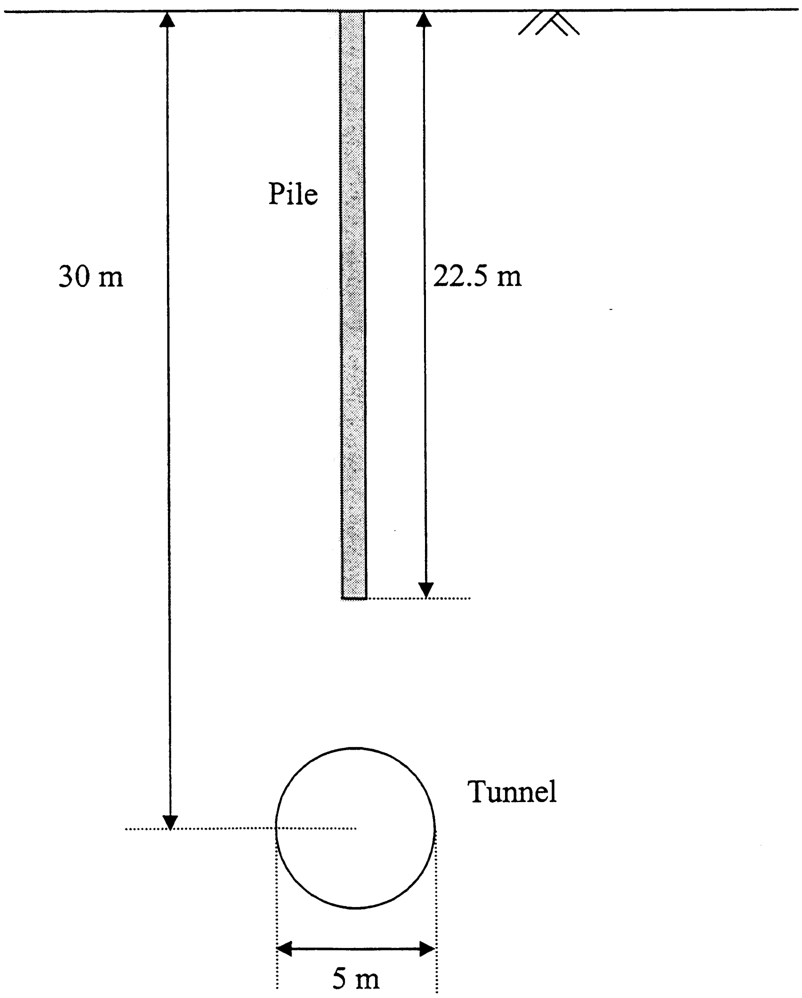

Fig. 1

(TURN OVER

2 A smooth retaining wall is installed in clay, as shown in Fig. 2, and the soil is rapidly excavated on one side. The excavated soil is simultaneously replaced by a support system with an equivalent triangular pressure distribution. Before excavation a self-boring pressuremeter test in the clay at a depth of 5 m showed the insitu total horizontal pressure to be 125 kPa. The unit weight of the clay is 20 kN m^{-3}, the critical state angle of friction is 25 degrees, and the water table is at the ground surface. A piezometer is installed at A prior to excavation to monitor the pore pressure change during excavation.

(a) Calculate the value of K_0, the coefficient of horizontal earth pressure at rest. What does this indicate about the previous stress history of the clay? [20%]

(b) A cylindrical sample of the clay is recovered from a depth of 5 m before construction of the wall. It is placed in a triaxial apparatus and a cell pressure of 100 kPa is applied without allowing any drainage. Assuming that the sample is recovered without any disturbance, such that the insitu mean normal effective stress p' does not change, what pore pressure would be measured? [20%]

(c) By means of jacks incorporated in the support system for the retaining wall, the support pressure p_s at 5 m depth is set at 75 kPa. Assuming plane strain conditions and isotropic elastic undrained behaviour of the clay, predict the piezometer measurement at A, and show the total and effective stress paths on a (σ_v, σ'_v) versus (σ_h, σ'_h) plot, where σ_v and σ'_v are total and effective vertical stresses and σ_h and σ'_h are total and effective horizontal stresses. [40%]

(d) If, in the long-term, the pore pressure at A returns to its original value before construction, what is the minimum value of p_s needed to prevent failure of the wall? Show the active failure K_a line on your (σ_v, σ'_v) versus (σ_h, σ'_h) plot and sketch the likely total and effective stress paths corresponding to this minimum value of p_s. [20%]

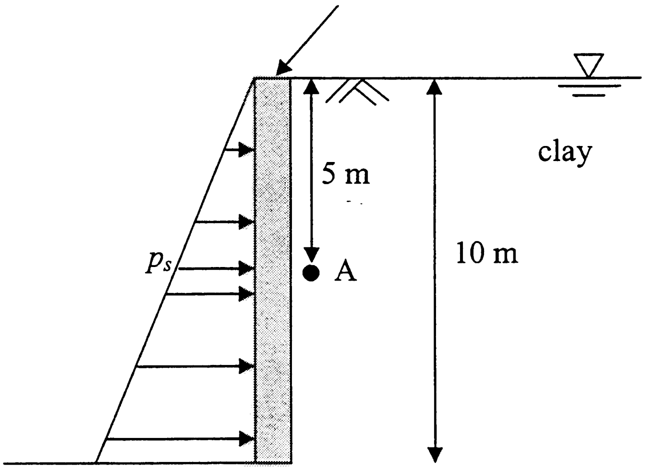

Fig. 2

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3 An excavation is to be made in overconsolidated clay as shown in Fig. 3. The unit weight of the clay is 18 kN m^{-3} and the clay is fully saturated up to the surface. The water table is located at 2 m below the ground surface.

(a) A soil sample was taken from the ground at a depth of 5 m. An unconfined compression test was performed on this sample and the sample failed when the axial stress was 100 kPa. The pore pressure at failure was -65 kPa. Draw the total and effective stress Mohr circles at failure and determine the friction angle at failure and the undrained shear strength. [25%]

(b) The excavation was 10 m deep and 5 m wide as shown in Fig. 3. The undrained shear strength of the sample will be used to evaluate the stability of the excavation.

(i) A triangular failure block with an angle θ is assumed to give a kinematic failure mechanism as shown in Fig. 4. Determine the external work rate when the block is sliding at a velocity of v. [20%]

(ii) Determine the internal dissipative work rate and evaluate the most critical angle of the failure surface θ and the corresponding mobilised shear stress along the failure surface. [20%]

(iii) Discuss whether the excavation is safe or not. Is there a more critical failure mechanism than the one shown in Fig. 4? If so, sketch the failure mechanism. No calculation is necessary. [15%]

(iv) With the aid of Fig. 5, evaluate the stability of the excavation bottom using the bearing capacity formula given in the attached Soil Mechanics Data Book. [20%]

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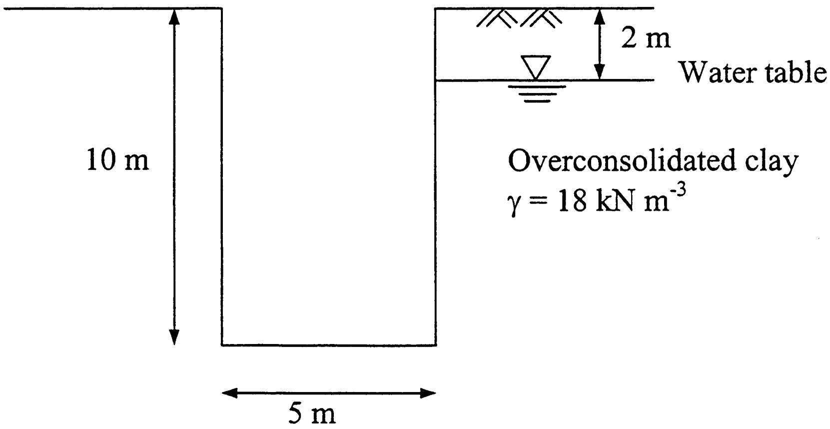

Fig. 3

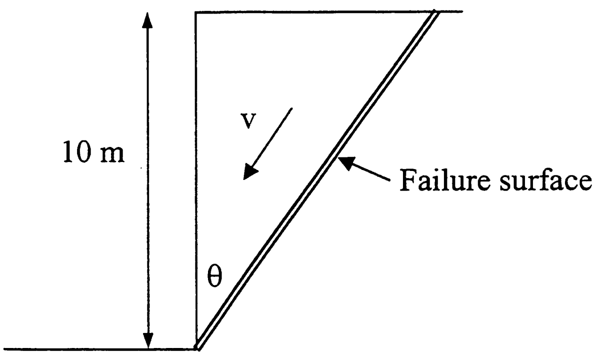

Fig. 4

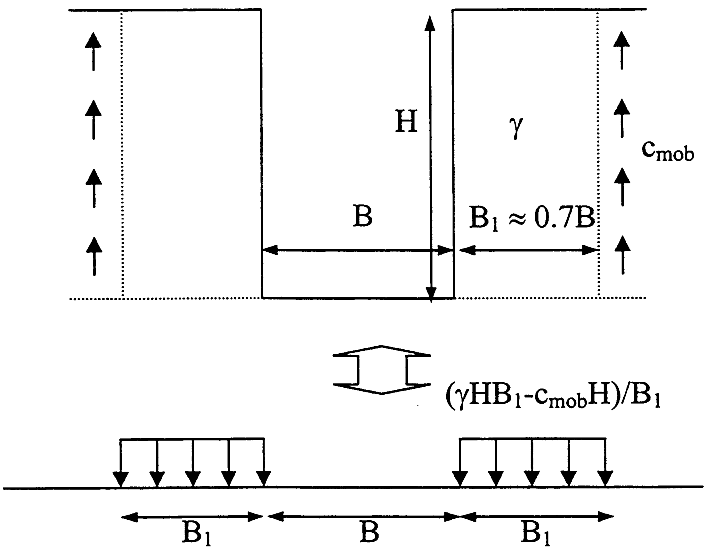

Fig. 5

(TURN OVER

4 A shallow footing is placed in clay for a new building as shown in Fig. 6. The width of the footing is 2 m and plane strain conditions are assumed. The clay has an undrained shear strength c_u of 75 kPa and an effective friction angle ϕ' of 25°. The water table is located 1.5 m below the ground surface. The unit weights of the clay below and above the water table are 18 kN m^{-3} and 16 kN m^{-3}, respectively. The weights of the footing itself and any soil directly above it should be ignored.

(a) Assuming the kinematic upper bound failure mechanism, which is circular with its centre at O, as shown in Fig. 7, determine the maximum load that can be applied to the footing. Assume that the foundation fails in undrained conditions. [20%]

(b) Assuming a static lower bound failure mechanism as shown in Fig. 8, determine the maximum load that can be applied to the footing using the stress discontinuity and stress fan concepts. Assume that the foundation fails in undrained conditions. [20%]

(c) Fig. 9 shows a failure mechanism under long term drained conditions and defines effective forces W'_p, W'_a, P'_p, P'_a, R'_p and R'_a which discount hydrostatic water pressures. Use the limit equilibrium method to answer the following questions:

(i) Evaluate force P'_a as a function of P_f, considering the forces acting on block a; [15%]

(ii) Evaluate force P'_p, considering the forces acting on block p; [15%]

(iii) By equating P'_a and P'_p, determine the maximum footing load P_f; [15%]

(iv) If the water table rises to the ground surface, how much decrease in the bearing capacity is expected? Assume the unit weight of the clay below the ground water becomes 18 kN m^{-3}. [15%]

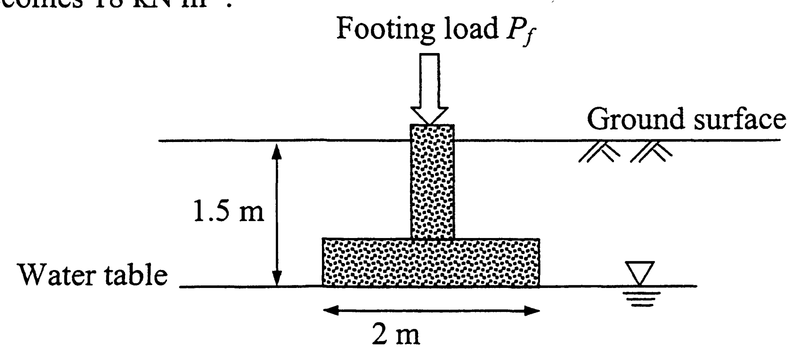

Fig. 6

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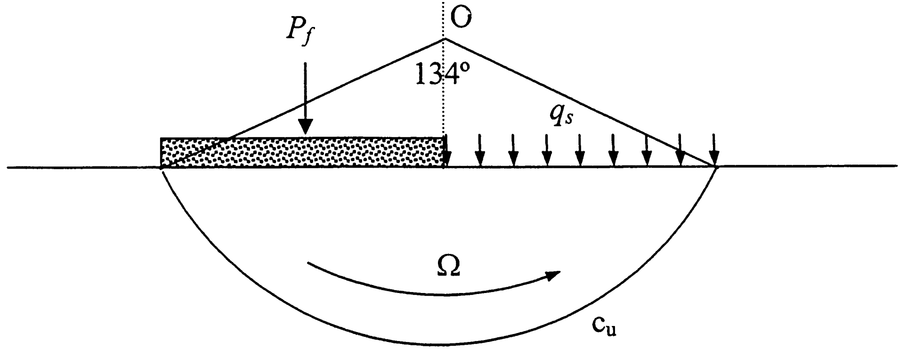

Fig. 7

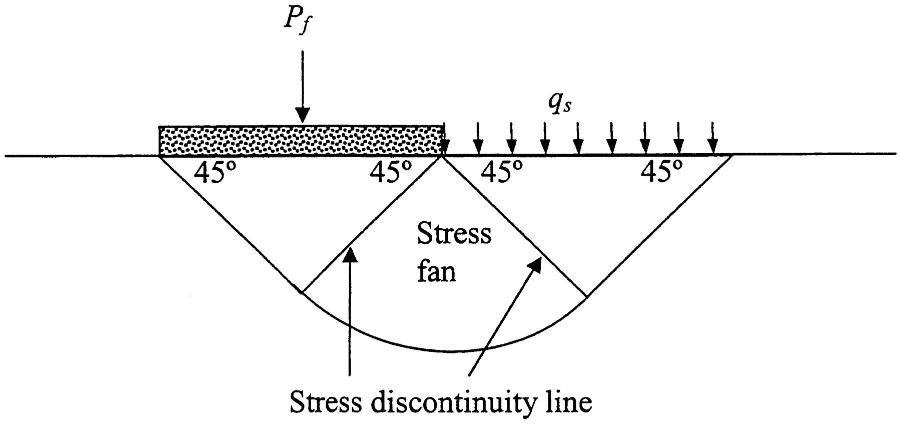

Fig. 8

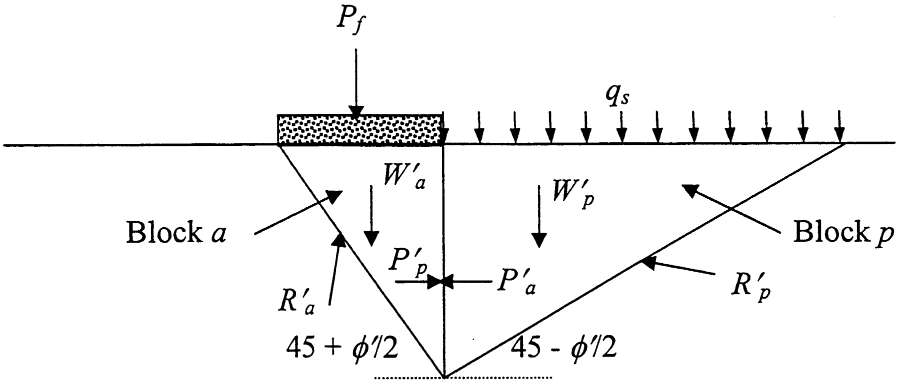

Fig. 9

END OF PAPER

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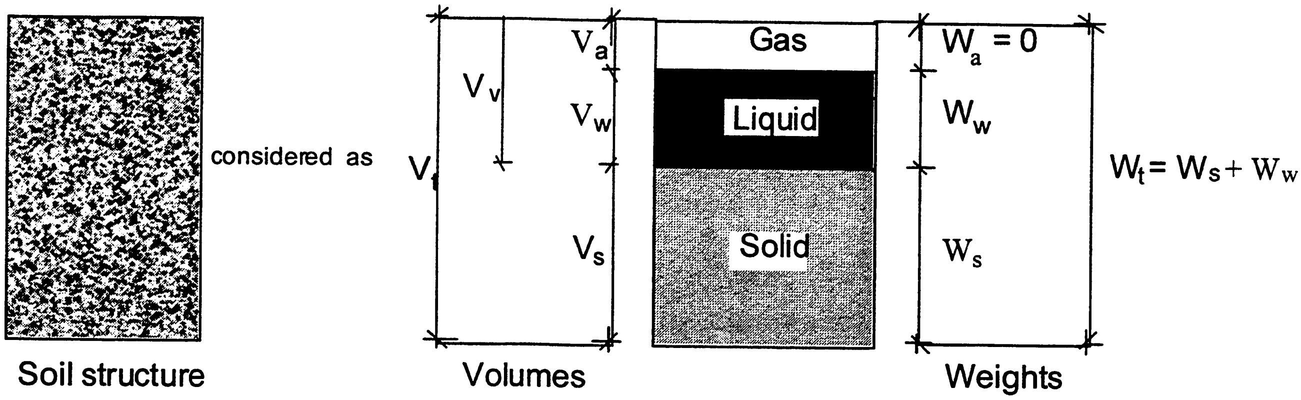
Engineering Tripos Part IIA

3D1 & 3D2
Soil Mechanics Data Book

Data Book 2003/2004

Contents	**Page**

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\, G_s/e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

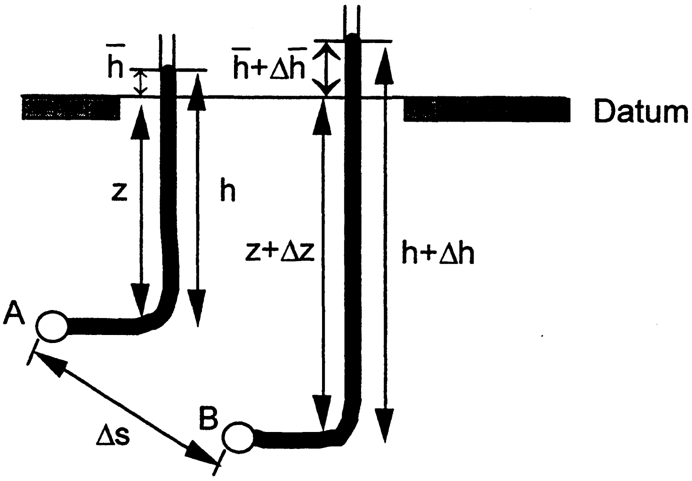
D — equivalent diameter of soil particle

D_{10}, D_{60} etc. — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U — uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
$10 \text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01 (D_{10} \text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

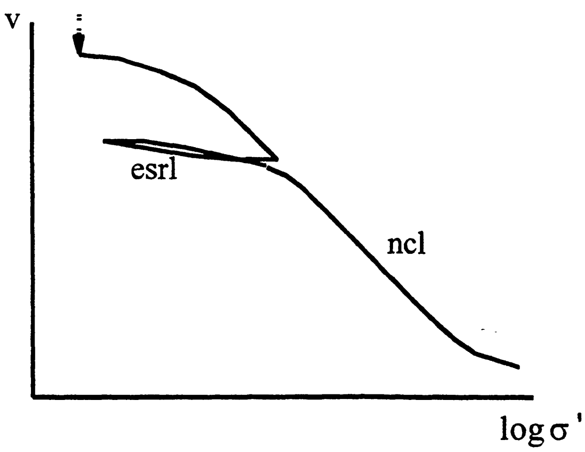
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

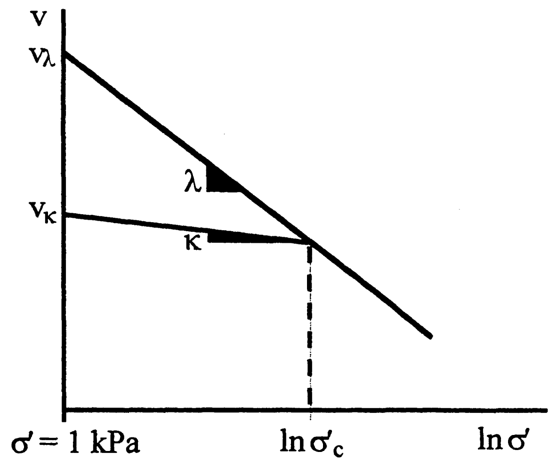
One-Dimensional Compression

✤ Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

✤ Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

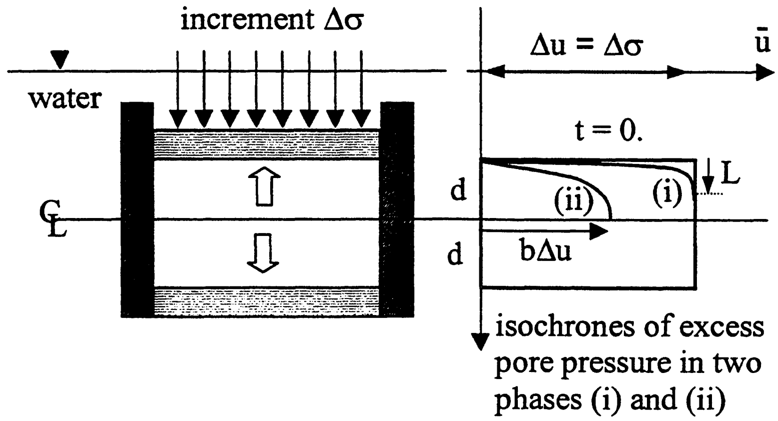
Tangent 1D plastic compression modulus $E_o = v\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

✣ Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

✥ Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

✥ Principal components of stress and strain

sign convention	compression positive
total stress	$\sigma_1,\ \sigma_2, \sigma_3$
effective stress	$\sigma_1',\ \sigma_2',\ \sigma_3'$
strain	$\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$

✥ Simple Shear Apparatus (SSA) (ε_2 = 0; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

✥ Biaxial Apparatus - Plane Strain (BA-PS) (ε_2 = 0; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

✣ Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	$= \sigma'_a + u$
total radial stress	σ_r	$= \sigma'_r + u$
total mean normal stress	p	$= (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	$= (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	$= \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	$= q/p'$
axial strain	ε_a	
radial strain	ε_r	
volumetric strain	ε_v	$= \varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	$= \frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	$= \sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	$= p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

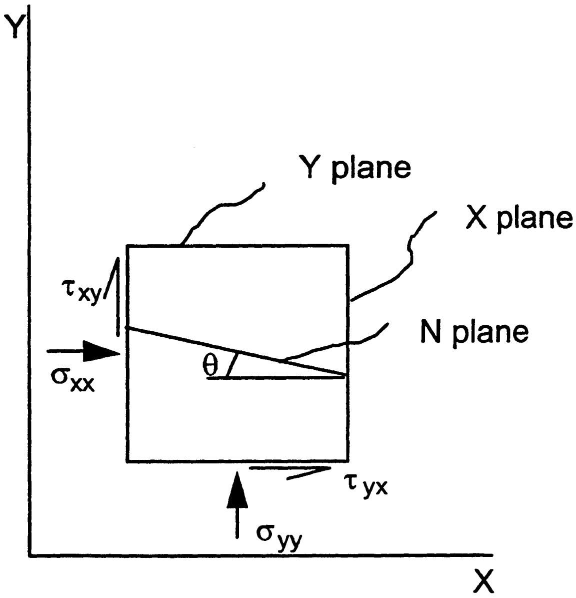
isotropic compression in which p' increases at zero q

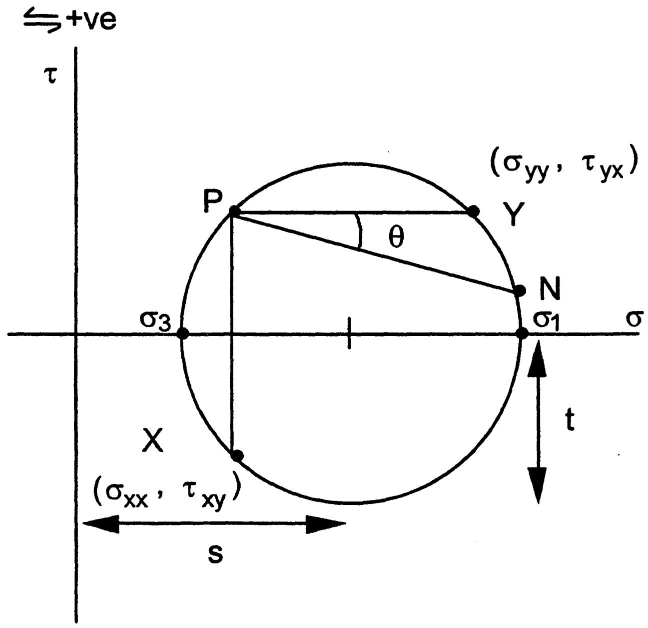
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

✣ Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P: the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

✣ Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

✣ General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

✣ General yield surface

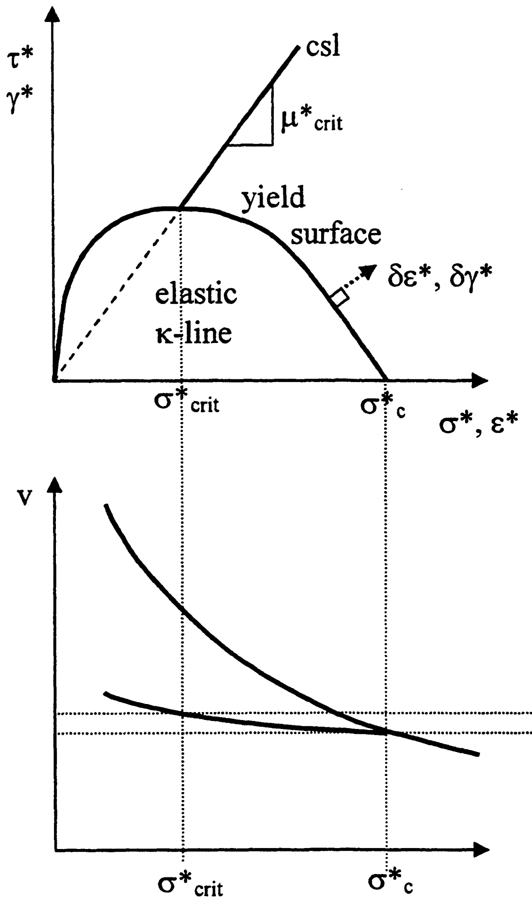
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

✣ Parameter values which fit soil data

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

✚ The yield surface in (σ^*, τ^*, v) space



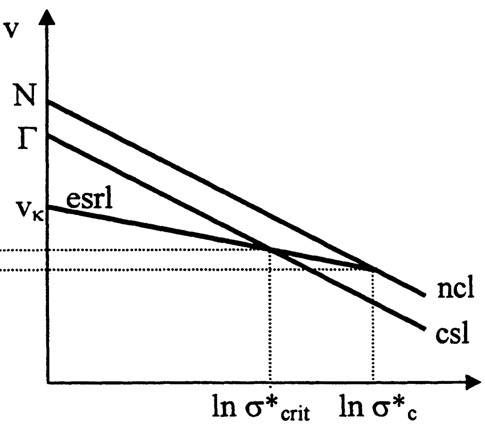
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

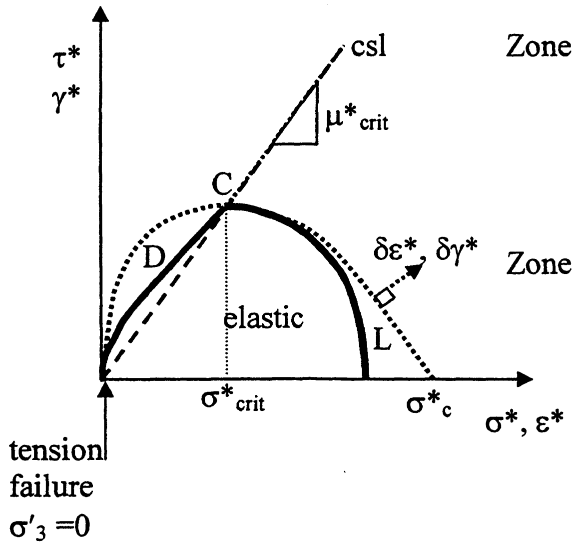
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



✚ Regions of limiting soil behaviour



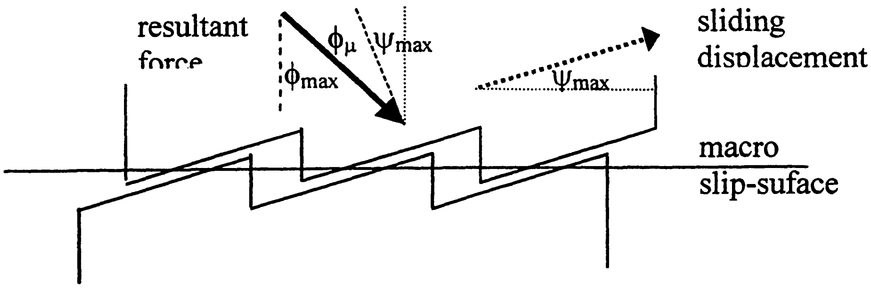
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

✣ Friction and dilatancy: the saw-blade model of direct shear

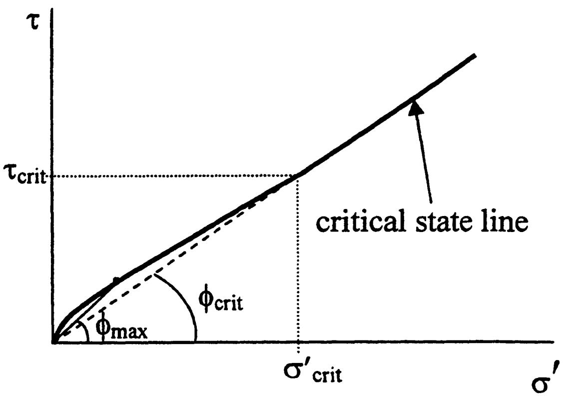


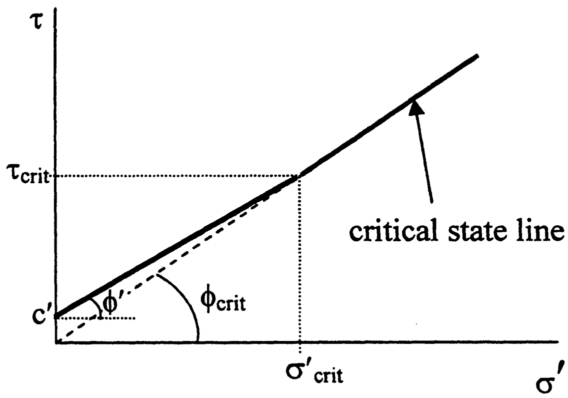
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

✣ Friction and dilatancy: secant and tangent strength parameters





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

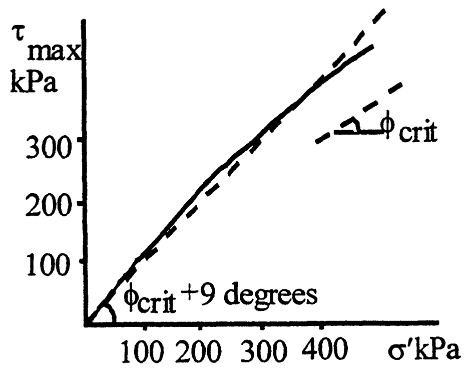
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

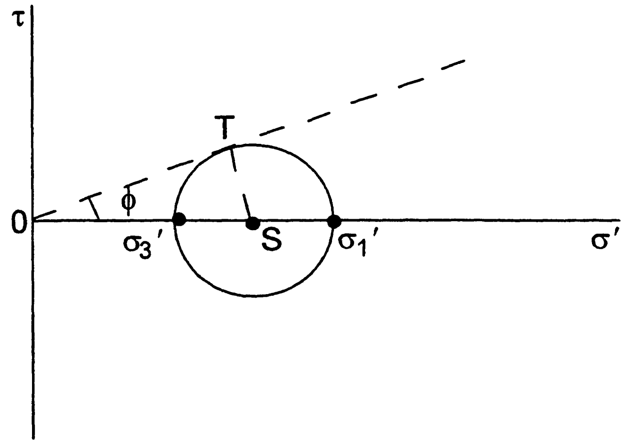
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' < 400$ kPa

✣ Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

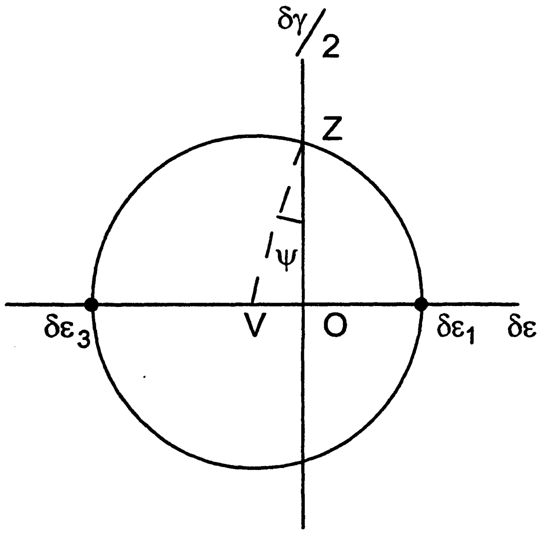
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ'_{crit} after large shear strains

✣ Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$

✚ Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

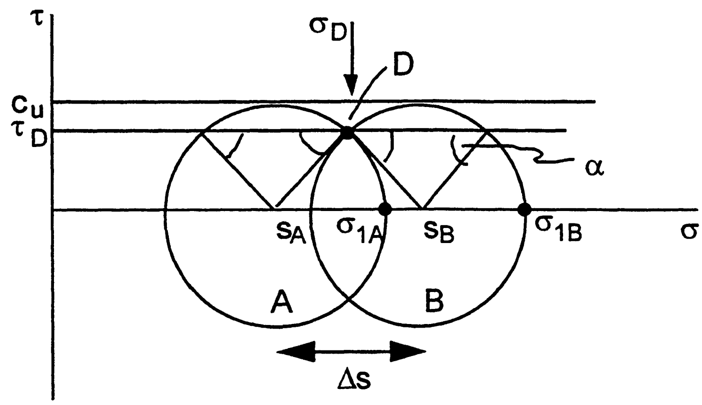
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

✚ Stress conditions across a discontinuity

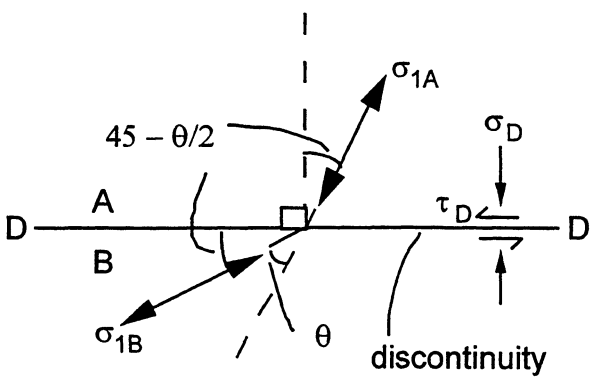


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1(B)} - \sigma_{1(A)} = 2c_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$\theta = 30°$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi'$

✚ Limiting stresses

$$\sin\phi' = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principle effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Earth pressure coefficient K: $\sigma'_h = K\sigma'_v$

Earth pressure at rest for normally consolidated soils $K_0 = 1 - \sin\phi'$

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure:

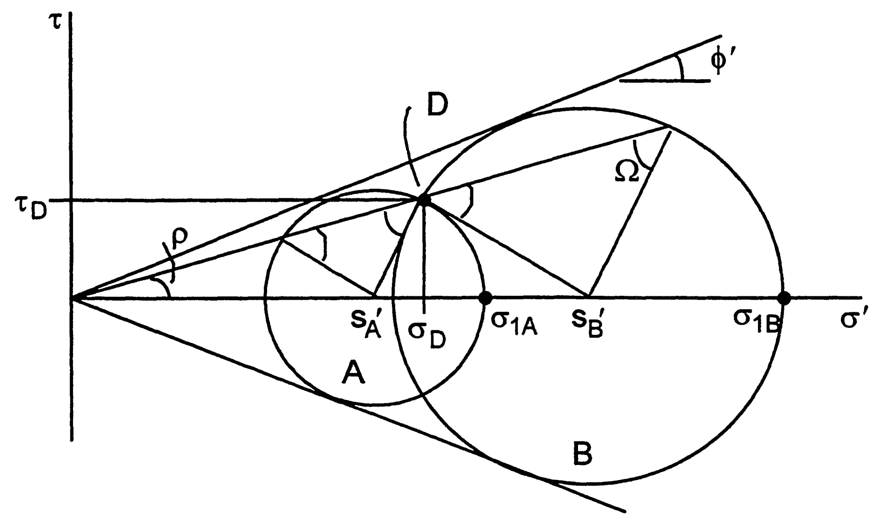
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

✚ Stress conditions across a discontinuity

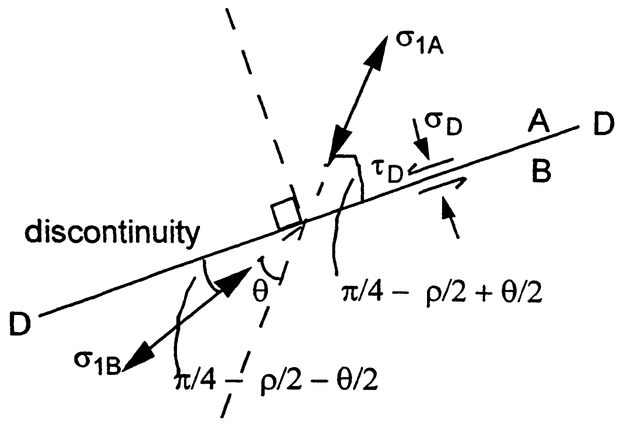


Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B



$\sin\rho = \cos\theta \sin\phi'$

$s'_B/s'_A = \cos(\theta-\rho)/\cos(\theta+\rho)$

In limit with $\delta\theta \rightarrow 0$

$\rho \rightarrow \phi'$,

$ds' = 2s' \, . \, \delta\theta \tan\phi'$.

Cylindrical cavity expansion

Expansion $\delta A = A - A_0$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Formula for shallow foundation design

For clay in undrained conditions (ϕ=0 and q_s calculated based on total stress)

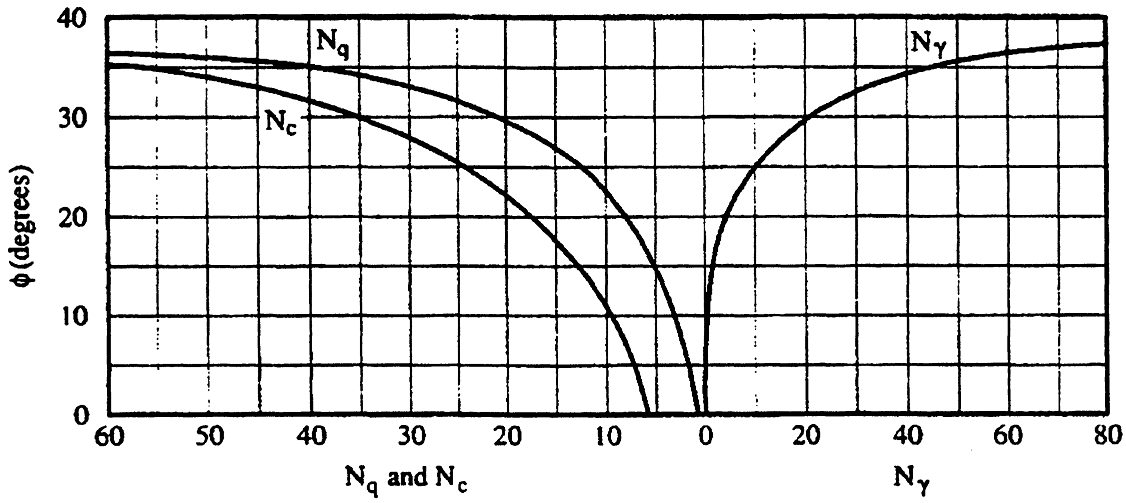
$$q_f = cN_c\zeta_c + q_s\zeta_s$$

For sand and clay in drained conditions (q_s calculated based on effective stress)

$$q_f = 0.5\gamma' BN_\gamma\zeta_\gamma + q_sN_q\zeta_s$$

$$\zeta_c = \zeta_{cd} \times \zeta_{cs} \times \zeta_{ci} \times \zeta_{c\beta} \times \zeta_{c\delta}$$
$$\zeta_r = \zeta_{\gamma d} \times \zeta_{\gamma s} \times \zeta_{\gamma i} \times \zeta_{\gamma\beta} \times \zeta_{\gamma\delta}$$
$$\zeta_s = \zeta_{sd} \times \zeta_{ss} \times \zeta_{si} \times \zeta_{s\beta} \times \zeta_{s\delta}$$

Correction factors		
	- Foundation depth	ζ_{cd} $\zeta_{\gamma d}$ ζ_{sd}
	- Foundation shape	ζ_{cs} $\zeta_{\gamma s}$ ζ_{ss}
	- Inclined loading	ζ_{ci} $\zeta_{\gamma i}$ ζ_{si}
	- Surface slope	$\zeta_{c\beta}$ $\zeta_{\gamma\beta}$ $\zeta_{s\beta}$
	- Base tilt	$\zeta_{c\delta}$ $\zeta_{\gamma\delta}$ $\zeta_{s\delta}$



[REMARKS]

(a) For clay in undrained conditions,

$N_c = 5.14$	for strip footing
$N_c = 5.69$	for circular footing (smooth)
$N_c = 6.05$	for circular footing (rough)
$N_c = 5(1+0.2\ B/L)$	for a rectangular footing of dimensions B x L (L>B)

(b) For sand and clay in drained conditions (use the above chart or the following equations)

$N_q = \tan^2(\pi/2 + \phi/2)e^{(\pi\tan\phi)}$

$N_\gamma = 2(N_q - 1)\tan\phi$

(c) For more complicated geometries, apply the correction factors using Table 1.

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Table 1 Correction factors

	Cohesion	Self-weight	Surcharge
1. Foundation shapes	$\zeta_{CS} = 1 + \frac{B'}{L'}\frac{N_q}{N_c}$	$\zeta_{\gamma S} = 1 - 0.4\frac{B'}{L'}$	$\zeta_{qS} = 1 + \frac{B'}{L'} tan\phi$
2. Inclined loading	$\zeta_{Ci} = (1 - 2i/\pi)^2$	$\zeta_{\gamma i} = (1 - i/\phi)^2$	$\zeta_{qi} = (1 - \frac{2i}{\pi})^2$
3 Foundation depth	$\zeta_{Cd} = 1 + 0.4\xi \ (\phi = 0)$ $= \zeta_{qd} - \frac{1-\zeta_{qd}}{N_c\, tan\phi} \ (\phi > 0)$	$\zeta_{\gamma d} = 1.0$	$\zeta_{qd} = 1 + 2tan\phi(1 - sin\phi)^2\xi$
4. Surface slope	$\zeta_{C\beta} = 1 - [2\beta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\beta} - \frac{1-\zeta_{q\beta}}{N_c\, tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\beta} = (1 - tan\beta)^2$ $Note: N_\gamma = -2 sin\beta (\phi = 0)$	$\zeta_{q\beta} = (1 - tan\beta)^2$
5. Base tilt	$\zeta_{C\delta} = 1 - [2\delta/(\pi + 2)] \ (\phi = 0)$ $= \zeta_{q\delta} - \frac{1-\zeta_{q\delta}}{N_c\, tan\phi} \ (\phi > 0)$	$\zeta_{\gamma\delta} = (1 - \delta\, tan\phi)^2$	$\zeta_{q\delta} = (1 - \delta\, tan\phi)^2$

V : vertical load, H : horizontal load, B : foundation width, L (>B) : foundation length, e_B : eccentricity parallel to B, e_L : eccentricity parallel to L; B' = B – $2e_B$, L' = L – $2e_L$; i = tan^{-1}(H/V) where i is in radians; ξ = D/B if D/B < 1; ξ = tan^{-1}(D/B) if D/B > 1, β < π/4 where β is in radians, δ < π/4 where δ is in radians.