UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Wednesday 5 May 2004 9 to 10.30

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special Datasheets (10 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Derive an expression for the limiting span/depth ratio L/d for a uniform simply-supported beam, in linear-elastic/perfectly-plastic material, which is to carry a total uniformly-distributed load $\gamma_f W$ using the material strength σ_f to the full, with a maximum elastic deflection of L/F under load cW. [30%]

(b) An engineer wishes to design a beam with a span of 15 m and chooses not to rely on any ductility in basic calculations. The beam is to be simply supported with $\gamma_f = 1.5$, $c = 0.9$, $F = 300$ and a partial safety factor of 1.05 on material strength. However, the only beam available is 762 × 267 × 197 UB in S355 steel. Discuss whether the basic bending design will be governed by deflection or strength considerations, and hence estimate what value of total uniformly-distributed working load W could be carried. [30%]

(c) The engineer decides to make a 'castellated beam' by flamecutting the UB along the dotted line indicated in Fig. 1(a) and welding the upper part back to the lower in the displaced and raised position shown in Fig. 1(b). Assume that the new beam can be treated throughout by standard bending theory, with plane cross sections remaining plane right across the depth and the only effective material in bending being that above and below the hexagonal holes. By what factor and to what value has the 'shape efficiency' factor ϕ_e for elastic bending been increased? What value of W could now be carried, according to simple theory? Discuss the limitations of the simplifying assumptions and mention what other checks would be required in designing this beam. [40%]

(cont.

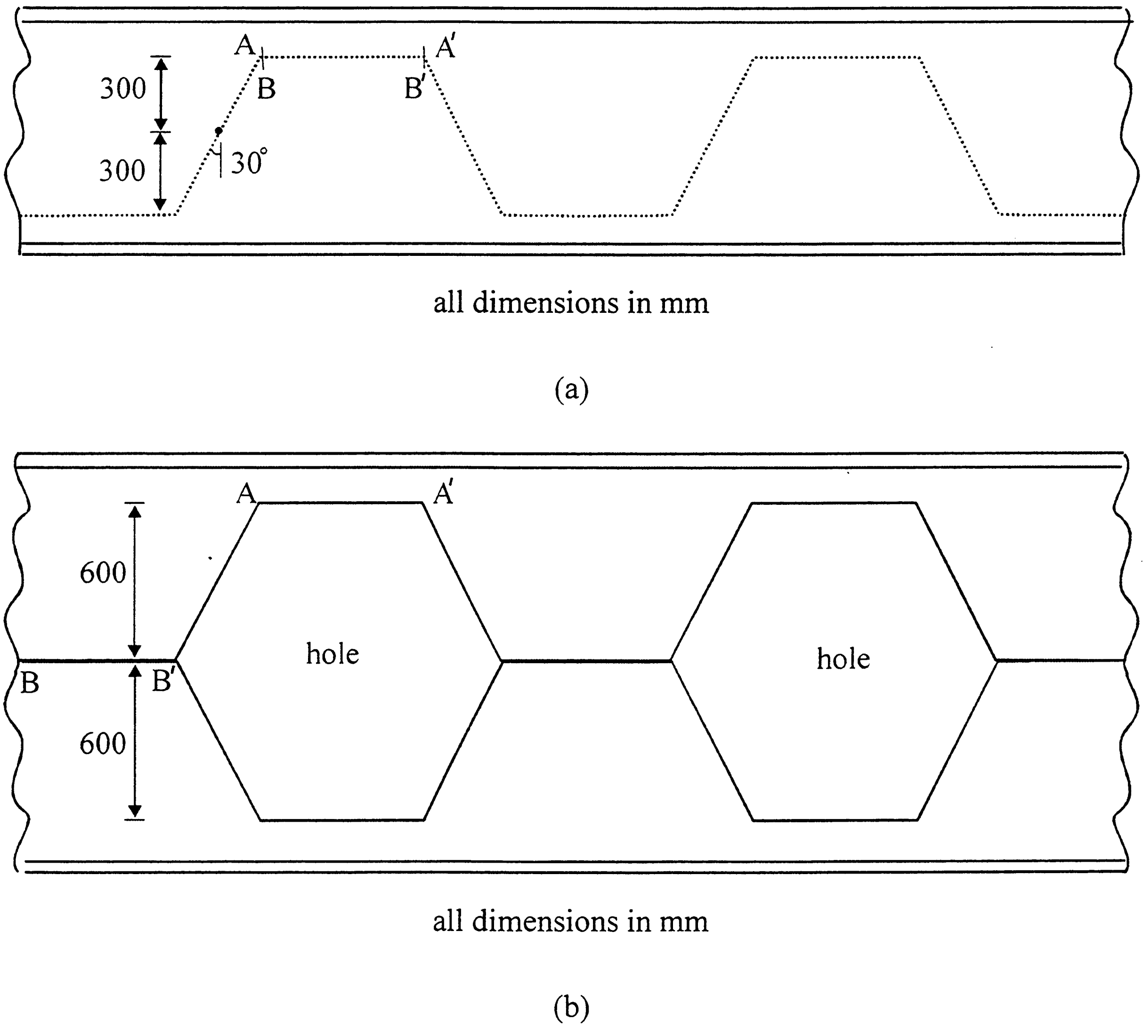


Fig. 1

(TURN OVER

2 (a) Outline the concept of a 'load path' in the design of structures to carry given applied loads, illustrating your answer by an example. In what circumstances, and with what limitations in the different structural materials, may an engineer designing or checking a structure at the ultimate limit state (ULS) choose a load path arbitrarily, without having to know or calculate the actual behaviour of the structure under load? [20%]

(b) Figure 2 shows a bolted connection of a secondary beam to a primary beam, intended to transfer a vertical reaction of 200 kN at ULS, using M20 bolts in 22 mm diameter holes.

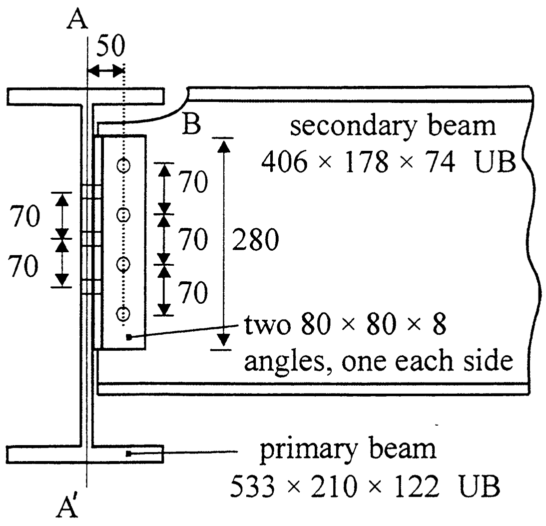
(i) Explain why the connection should be designed with the reaction on to the secondary beam acting in the central plane AA′ of the primary beam. [10%]

(ii) Describe the load path through the connection. Estimate the force transmitted by each bolt which passes through the primary web, and determine the mean shear stress in the bolt and the bearing stresses. Estimate also the maximum resultant force transmitted by a bolt passing through the secondary web, and the corresponding shear stress in the bolt and bearing stresses. If the maximum allowable nominal bolt shear stress and bearing stress are 160 MPa and 450 MPa, respectively, by what factor could the transmitted reaction be increased? [40%]

(iii) Sketch the free-body diagram for one of the 80 × 80 × 8 angles. What further checks could be performed to establish that the angle would be reasonably satisfactory? [15%]

(iv) To make this connection, part of the top of the secondary beam has to be cut away, as shown near B in Fig. 2. Could this have an adverse effect on the performance of the secondary beam? If so, what steps might be taken to mitigate that? [15%]

(cont.



all dimensions in mm

Fig. 2

(TURN OVER

3 You are asked to design a reinforced concrete beam. The beam is rectangular with a width of 370 mm, an overall height of 475 mm and a clear span of 5.5 m. The concrete cover is 40 mm. The concrete has a compressive cube strength of 25 MPa and the reinforcement steel has a yield stress of 460 MPa. The partial material safety factors for concrete and steel are $\gamma_c = 1.5$ and $\gamma_s = 1.15$ respectively. The beam is to carry a factored uniformly distributed vertical load of $w = 90$ kN/m.

(a) In the first instance, the beam is designed to be simply supported (see Fig. 3(a)). Determine the required longitudinal steel reinforcement lay-out at mid-span. [50%]

(b) You now wish to consider the case where the beam is continuous and the sagging moment at midspan is assumed to be equal to the hogging moments over the supports (see Fig. 3(b)). Based on experience gained from a previous project, you expect that four 20 mm diameter tensile reinforcement bars should be sufficient to sustain the maximum moment. Check that this design is satisfactory. [25%]

(c) Sketch the beam cross-section at mid-span and a schematic elevation of the longitudinal reinforcement along the length of the beam for the simply-supported and continuous cases (use your answers for parts (a) and (b), and do not carry out any further calculations). Briefly discuss the advantages and disadvantages of each system. What additional factors would need to be considered when detailing the reinforcement? [25%]

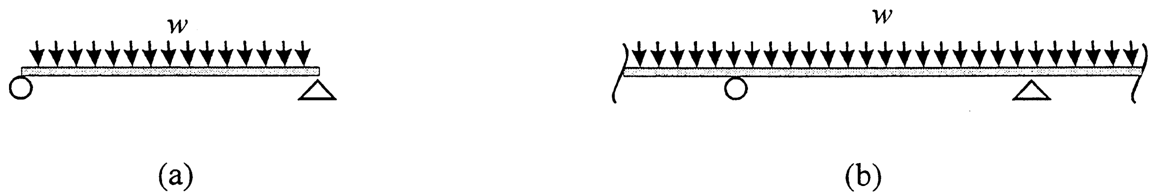


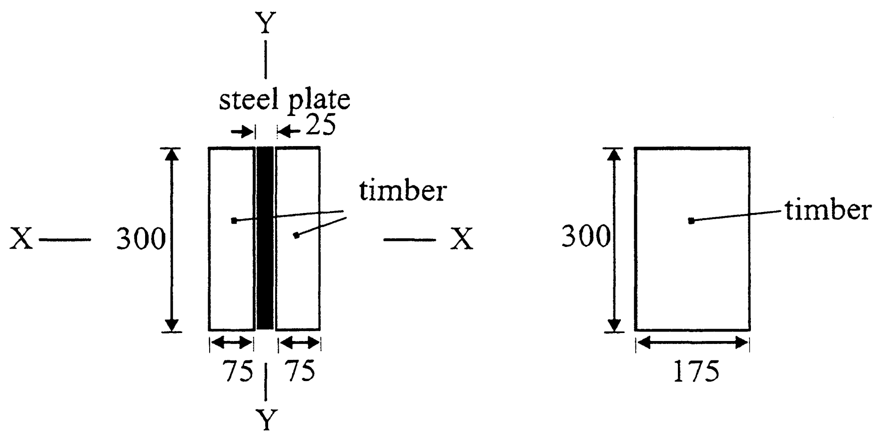
Fig. 3

4 A flitch beam consisting of two rectangular 75 × 300 C24 timber sections bolted to a central rectangular steel section is shown in Fig. 4(a). The steel and the timber can be assumed to be perfectly bonded. The steel section has a thickness of 25 mm, Young's modulus 210 GPa and a yield stress of 245 MPa. The partial material safety factors for steel and timber are 1.15 and 1.3 respectively. A solid C24 timber beam with equal overall dimensions is shown in Fig. 4(b). Both structures are located in an environment corresponding to service class 3 and subjected to short-term loading. Relevant timber properties can be found on the 3D3 Timber Datasheet. Use $E_{0,mean}$, $k_h = 1$, $k_{ls} = 1$ and, in the first instance assume $k_{crit} = 1$.

(a) By transforming the steel to timber, calculate EI_{XX} and EI_{YY} for the flitch beam. Calculate EI_{XX} and EI_{YY} for the solid timber structure. What do you conclude about the relative values? [30%]

(b) If the flitch beam is subjected to bending about the X-X axis, determine whether the steel will yield before the timber fails. Compare the ultimate bending moment capacity about X-X of the flitch beam with that of the solid timber section. [30%]

(c) Briefly describe the phenomenon of lateral torsional buckling. Compare qualitatively the anticipated lateral torsional buckling performance of the flitch beam with that of an equal solid timber section. In design, how can the likelihood of lateral torsional buckling be reduced? [40%]



all dimensions in mm

(a) (b)

Fig. 4

5 (a) A simply supported glass fibre reinforced polymer (GFRP) plate has a span of 70 mm and a width of 20 mm. The lay-up is specified to be balanced and symmetric consisting of layers of 0.125 mm GFRP plies. The constituent material properties of the E-glass/epoxy are consistent with those used on the 3D3 Advanced Composites Datasheet. The laminate is to be produced with plies laid at 0°, ±45° and 90° angles. The x-direction is at angle 0°.

(i) A Young's modulus E_x of 29 GPa is required and 33% of plies are to be oriented in the ±45° directions. Find the percentages of plies required in the different directions in order to meet this specification. [15%]

(ii) Find G_{xy}, ν_{xy} and E_y for the laminate found in part (i). [10%]

(iii) A uniform line load of total force 145 N is applied across the width of the plate at mid-span. Find the minimum laminate thickness required so that the failure strain limits of e_T = 0.3% and e_C = 0.7% are not exceeded. Suggest an appropriate lay-up. Assume that the possibility of a failure in shear can be disregarded. [35%]

(b) Derive an expression for the elastic shear deflection at mid-span of a rectangular simply supported beam with a point load V at mid-span. The beam is of width b, height h, span l and made of a material with shear modulus G. Would you expect the shear deflections in advanced composite structures to be significant? Give reasons. [40%]

END OF PAPER

Module 3D3 Selection of material and shape – design for bending

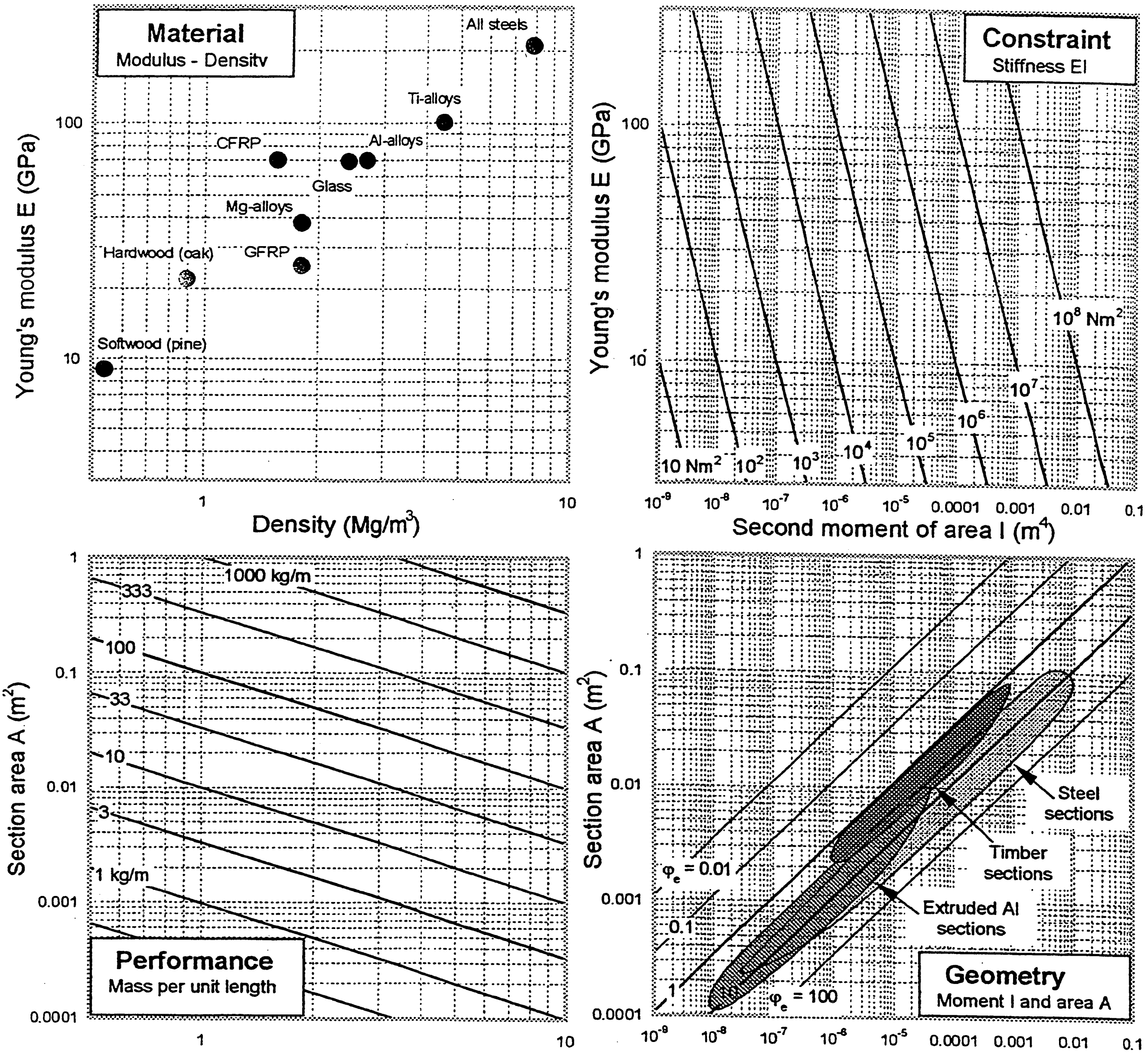


Figure 1. The chart-assembly for exploring structural sections for stiffness limited design. Each chart shares its axes with its neighbours.

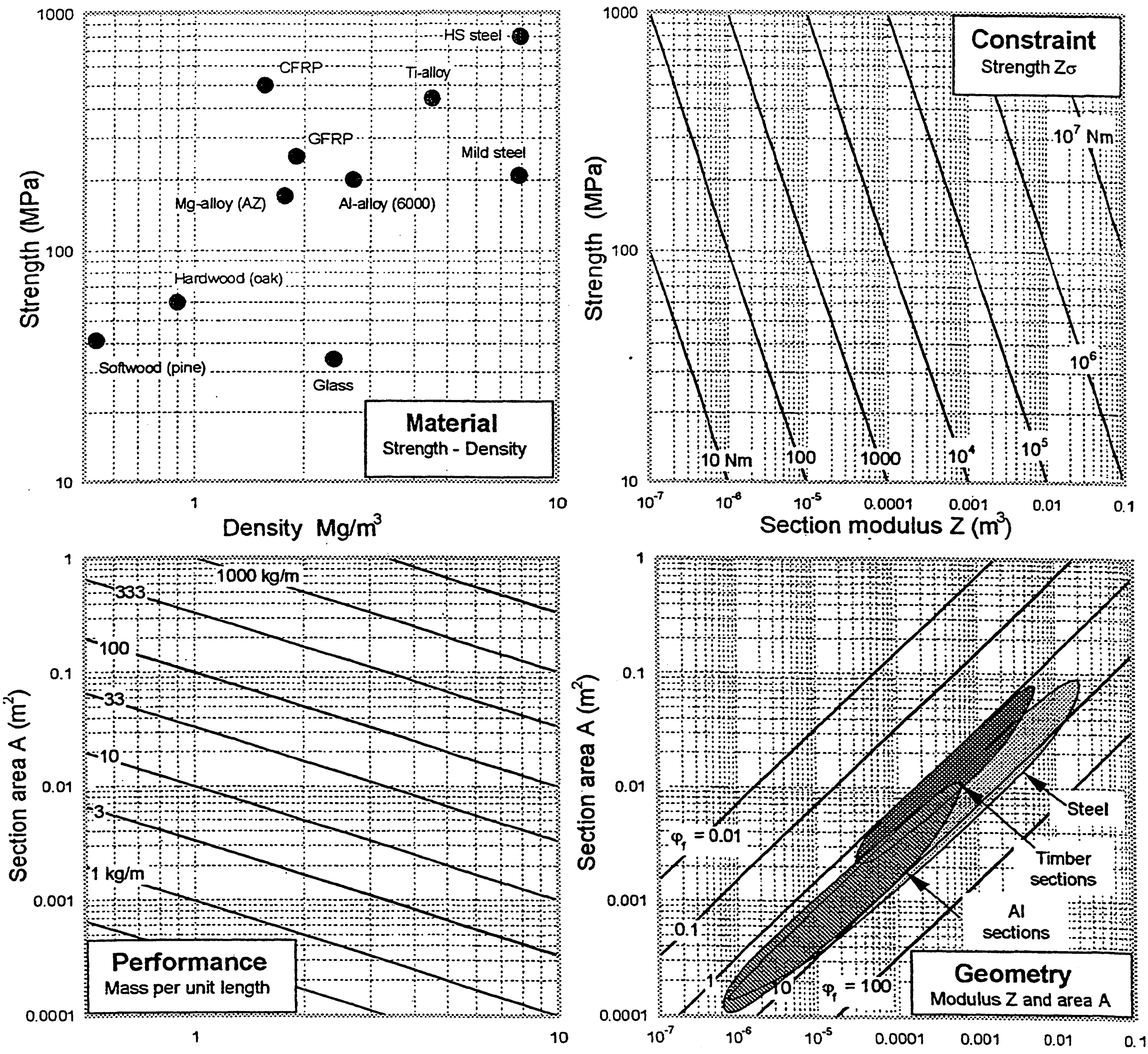


Figure 2 The chart-assembly for exploring structural sections for strength limited design. Like that for stiffness, each chart shares its axes with its neighbours.

Note: concept and charts are due to Prof. Michael Ashby, 2002
(see M.F. Ashby 'Materials selection in mechanical design' 2nd Edition, Butterworth 1999, JA163).

Module 3D3 – Structural Materials and Design — Steel Datasheet

Materials available (see Structures Data Book 1999 pp. 1 and 11)

The two common structural steels BS EN – S275 and S355 have characteristic yield strengths σ_y of 275 and 355 MPa respectively. Both satisfy the usual criteria for plastic design (adequate ratio of UTS to yield strength; adequate elongation to fracture). In design, the calculated strength (e.g. buckling resistance) for either material would be divided by a specified partial safety factor γ_m, often 1.1. Strength design is at ULS, with specified partial safety factors γ_f on loads, often 1.4 on dead load, 1.6 on live.

Tension members (axial force only)

Gross area A; net area A_n is A minus hole(s). Effective section A_e is KA_n but not greater than A, where factor K is 1.2 for S275, 1.1 for S355.

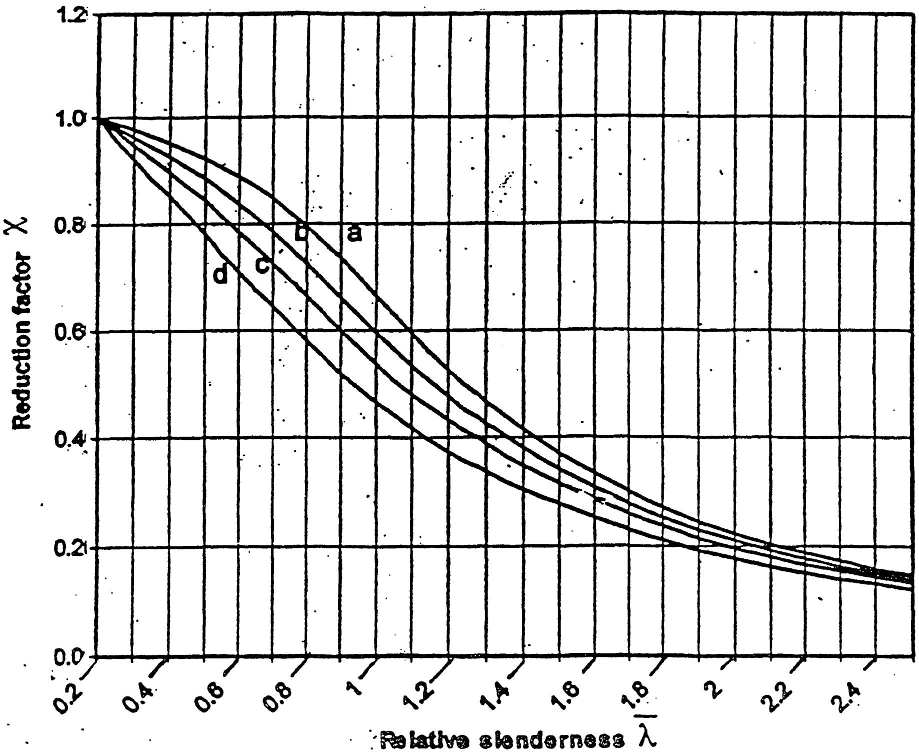
For eccentric connection, with area a_2 not connected at joint, effective area is often taken as $A_e - ca_2$, where factor c is 0.5 for bolted connections, 0.3 for welds.

Compression members (axial force only)

Joints do not normally control design, though criteria above should be checked. Base design against buckling on gross area A, yield strength σ_y, radius of gyration r, and column effective length L between points of contraflexure. Slenderness $\lambda = L/r$.

Define λ_0 as the slenderness at which the elastic critical stress for a perfect column equals the yield strength, so $\lambda_0 = \pi \{ E/\sigma_y \}^{1/2}$. Relative slenderness $\bar{\lambda} = \lambda/\lambda_0$

To allow for interaction between yield and buckling, use curves of reduction factor χ (on the full yield axial strength) plotted against the relative slenderness $\bar{\lambda}$. Typical curves are as shown below (these from the IStructE EC3 (Steel) Design Manual). Choice of empirical curve a to d depends on section type (extreme fibre distance y/r) and typical imperfection and residual stress magnitudes.



Beams (without axial force)

Moment - check maximum moment less than $\sigma_y Z_p$. Sections are Class 1 to 4, for ability to provide enough ductility for full plastic behaviour without local buckling.

Shear – yield strength q_w in shear often taken as $0.6\sigma_y$ for simplicity. But check for buckling in thin webs, for depth/thickness d/t and aspect ratio a/d, e.g. for a/d > 1, elastic critical average stress $q_{cr} = (0.75 + \{d/a\}^2)\,\{1000t/d\}^2$ in MPa.

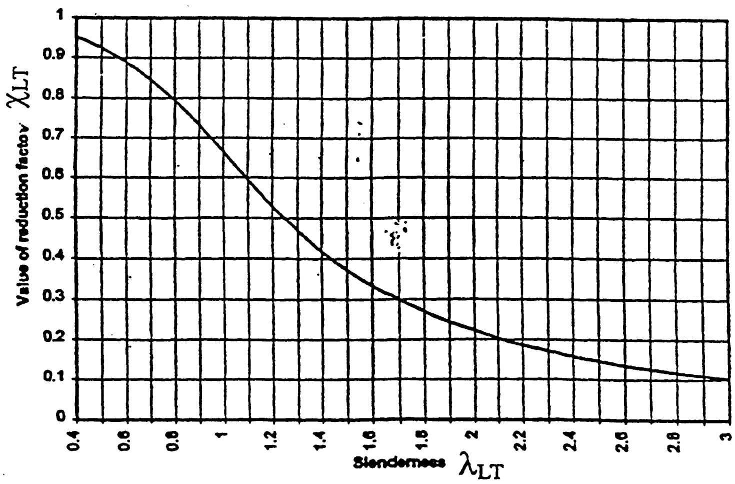
Bearing – check, using special formulae for the various failure modes, at supports and under high local loads, in case stiffener(s) are required.

Lateral-torsional buckling - for uniform bending moment M over a distance L between points where lateral displacement and rotation about beam axis are prevented, elastic critical value, with significant torsional stiffness due to restraint of warping,

$$M_c = \frac{\pi}{L}\sqrt{EI_{yy}\left\{GJ + \frac{\pi^2}{L^2}EC_w\right\}}.$$

For typical I-beam, basic torsion constant $J = \Sigma\, bt^3/3$ where b is the width of component plates of thickness t; and warping-restraint factor $C_w = D^2 I_{yy}/4$ where D is the distance between centres of flanges.

To allow for interaction between buckling and yield, again use empirical curves of the capacity reduction factor χ_{LT} against relative slenderness $\lambda_{LT} = \{M_p/M_{cr}\}^{1/2}$ (here M_{cr} is the hinge-position moment for elastic buckling).



Joints

Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design – often with a transmitted force uniformly distributed across the various fasteners involved (particularly if they have similar properties). For a bolted joint in shear, a couple C about its centre can be taken simply by forces F_i on each bolt, perpendicular to the line to the centre and proportional to the distance d_i from the centre, so that $F_i = Cd_i/\Sigma d_i^2$, but other equilibrium systems can be envisaged.

Applied shear forces F on bolts are commonly checked against the shear strength (say $0.6\sigma_y$) of the bolt, depending on the number of shear planes activated; and against the bearing strength $\sigma_b dt$ in each plate (d is bolt diameter, t is plate thickness), despite the fact that bolts may not actually transmit force in this way. For M20 bolts, σ_y is typically 600 MPa, and in Codes nominal σ_b is typically of order 400 MPa in S275 steel.

3D3 – Structural Materials and Design – Concrete Datasheet

	Span/effective depth ratio			
Structural system	IStructE*		EC2**	
	beam	slab	high	light
1. Simply supported beam One-way or two-way spanning simply supported slab	15	21	18	25
2. End span of continuous beam: one-way continuous slab; or two-way spanning slab continuous over one long side	20	27	23	32
3. Interior span of: beam; one-way or two way spanning slab	21	30	25	35
4. Slab supported on columns without beams (flat slab), based on longer span	n/a	26	21	30
5. Cantilever	6	8	7	10

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 4.2 [10.1], ** Table 4.14 [10.2]

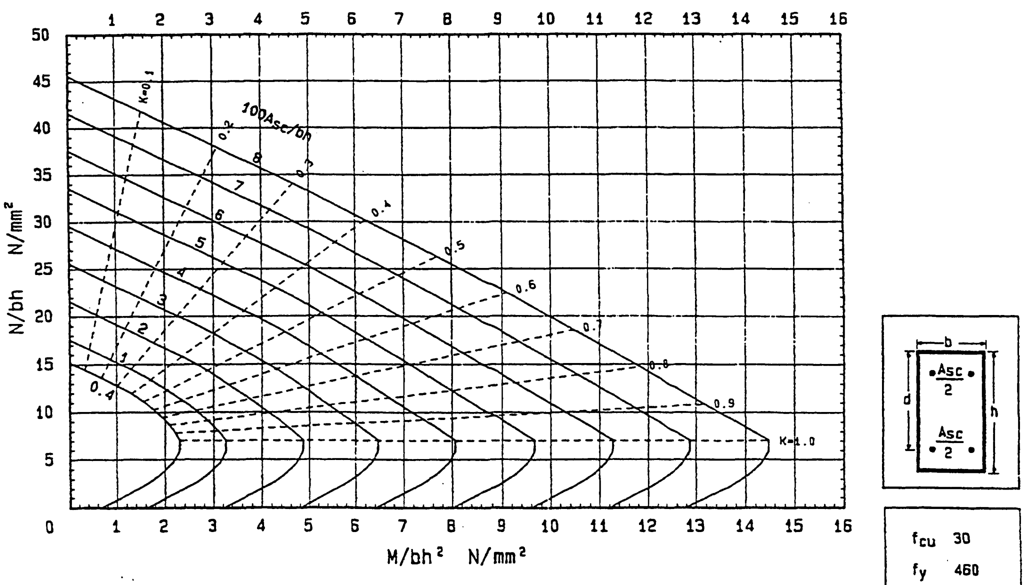
Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

Chart No. 29



Rectangular columns

Fig 10.14 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures DD ENV 1992-1-1:1992 – note** draft document
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1 - 0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if $x/d = 0.5$

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y \, r}{2\beta_a \sqrt{f_{cu}}}$$

Cracking

$$w_k = \beta s_{rm} \varepsilon_{sm}$$

$$s_{rm} = 50 \text{ mm} + 0.25 k_1 k_2 \phi / \rho_r$$

where: w_k is the design crack width

β is a statistical variation coefficient, usually 1.7

s_{rm} is the average final spacing

ε_{sm} is the mean steel strain (cracked-elastic theory)

ϕ is the average bar size

ρ_r is the effective steel ratio A_s/A_{ceff}

Shear

Concrete contribution

$V_{Rd1} = \{\tau_{Rd} k (1.2 + 40\rho_l)\} b_w d$

where: τ_{Rd} is the basic shear strength of the concrete.

k is a factor that varies with effective depth, d ($k = 1.6 - d$ but $k \geq 1$ and d in metres),

b_w is the minimum width

ρ_l is the reinforcement ratio $= A_s/b_w d$ but $\rho_l \leq 0.02$.

Shear stirrup contribution

$v_s = f_y A_v / \gamma_s b_w s$

Columns

Axial loading only

$$\sigma_u = 0.6 \frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm)

6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\|\|$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\|\|$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\|\|$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	2	3
Solid and glued laminated timber Plywood.			
Permanent	0,60	0,60	0,50
Long-term	0,70	0,70	0,55
Medium-term	0,80	0,80	0,65
Short-term	0,90	0,90	0,70
Instantaneous	1,10	1,10	0,90

Extract from [11.2] – Modification Factors for Service Class and Duration of Load

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k} / \sigma_{m,crit}}$

For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e).

$$k_{crit} = \begin{cases} 1 & \text{for } \lambda_{rel,m} \le 0{,}75 & (5.2.2c) \\ 1{,}56-0{,}75\lambda_{rel,m} & \text{for } 0{,}75 < \lambda_{rel,m} \le 1{,}4 & (5.2.2d) \\ 1/\lambda^2_{rel,m} & \text{for } 1{,}4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1-0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

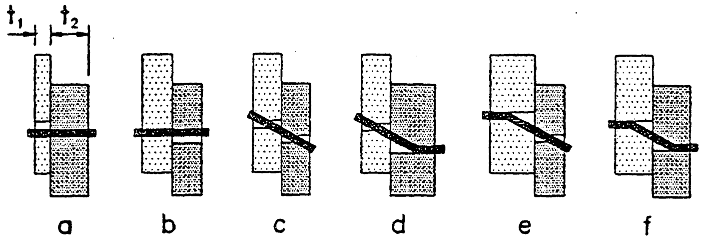
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear:

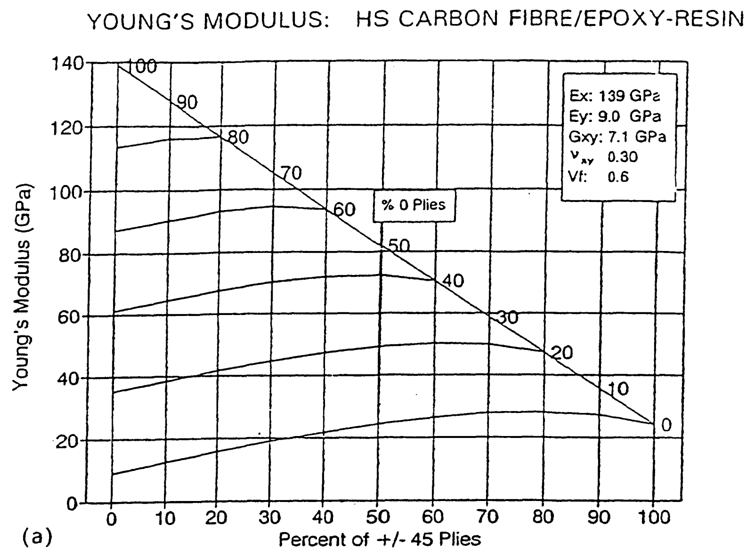
$$R_d = \min. \begin{cases} f_{h,1,d}t_1 d & (6.2.1a) \\ f_{h,1,d}t_2 d\,\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1+\dfrac{t_2}{t_1}+\left(\dfrac{t_2}{t_1}\right)^2\right]+\beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1+\dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1{,}1\dfrac{f_{h,1,d}t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}d t_1^2}} - \beta\right] & (6.2.1d) \\ 1{,}1\dfrac{f_{h,1,d}t_2 d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}d t_2^2}} - \beta\right] & (6.2.1e) \\ 1{,}1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d}f_{h,1,d}d} & (6.2.1f) \end{cases}$$

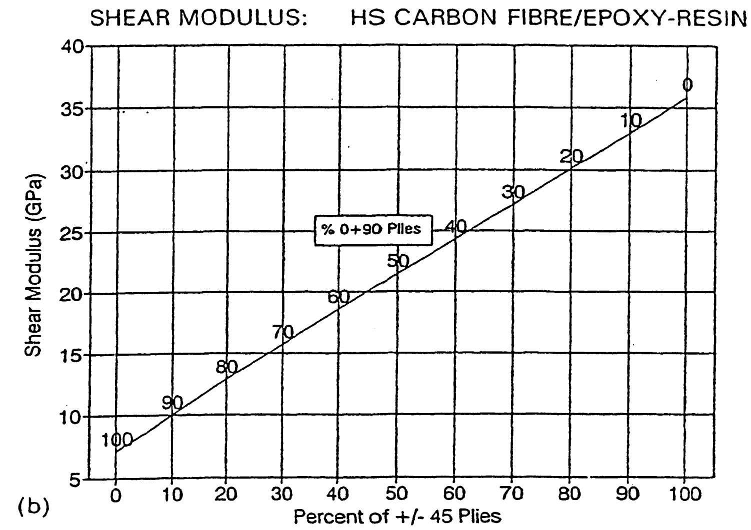


Single shear

Extract from [11.2] – Timber-to-timber and panel-to-timber joints

3D3 – Structural Materials and Design – Advanced Composites Datasheet





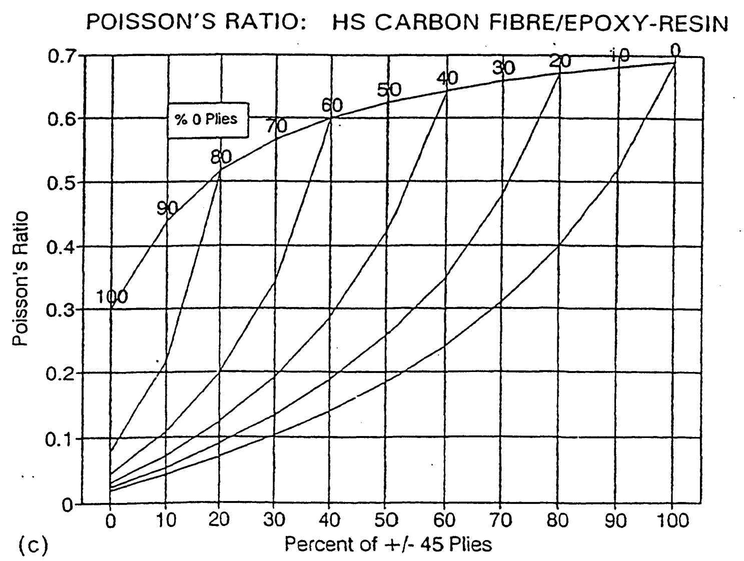


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

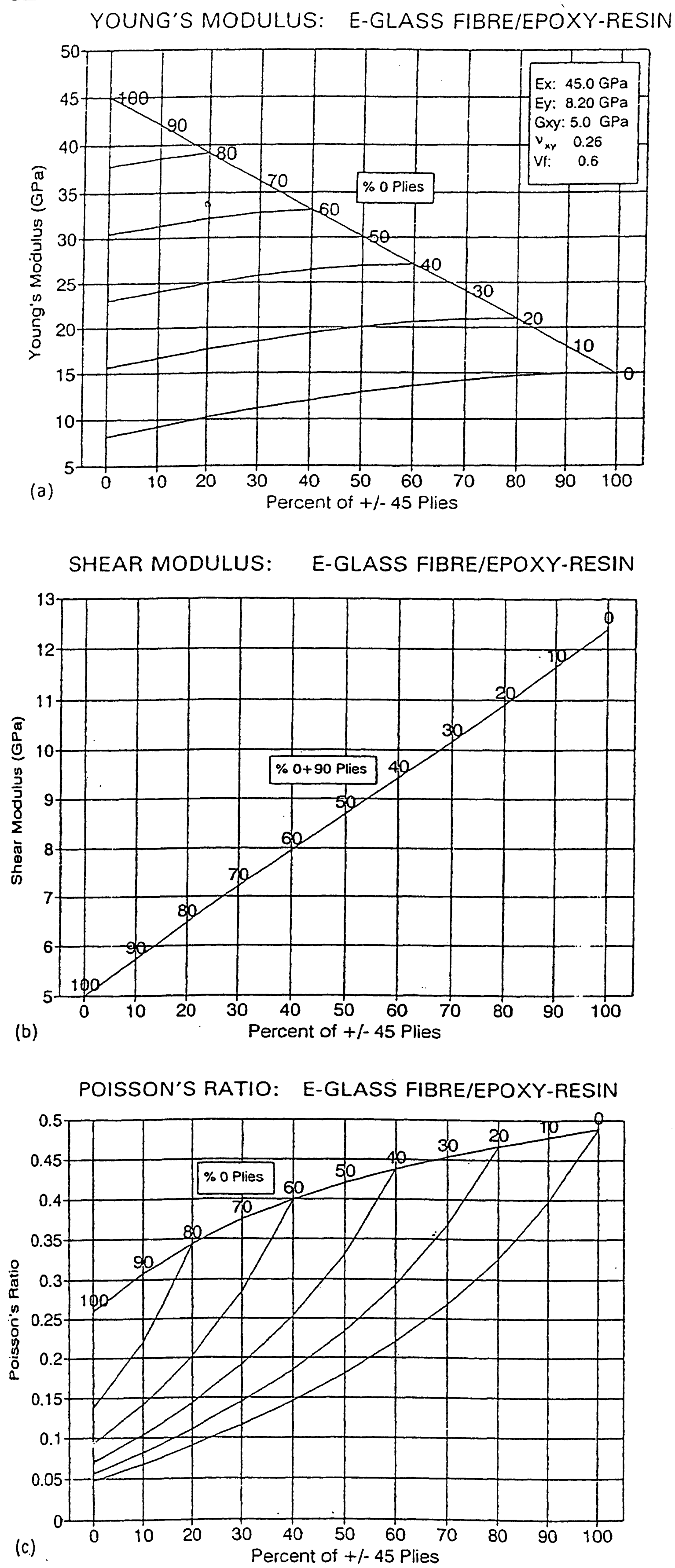


Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.