UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Wednesday 5 May 2004 2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 A Z-section purlin, of overall depth $d = 3b$ and width $2b$, has a constant cross-section shown in Fig. 1. It is made of material with Young's modulus E and elastic shear modulus G. The uniform thickness t is small in comparison with b. The x, y coordinate system has its origin at the centroid O as shown.

(a) Determine the second moments of area I_{xx}, I_{yy} and I_{xy} and the location of the shear centre. [40%]

(b) The purlin is mounted as a horizontal cantilever of length L with the y axis vertically upwards. The cantilever is loaded by a tip force P acting vertically downwards along the line of the web. Stating any assumptions you make, determine the elastic deflection of the tip. [30%]

(c) The line of action of the tip load P is moved to pass through a point A in the cross section with coordinates $(b, 0)$. The dimensions of the section are set at $t = b/10$ and $L = 20b$.

(i) Determine the twist at the tip of the cantilever, assuming warping is completely unrestrained. [15%]

(ii) Estimate the twist at the tip of the cantilever when full warping restraint is provided at the cantilever base. [15%]

Note that the restrained warping torsion constant $\Gamma = \dfrac{d^2}{4} I_{yy}$ for a Z-section purlin and the characteristic length $\lambda = \sqrt{\dfrac{E\Gamma}{GJ}}$.

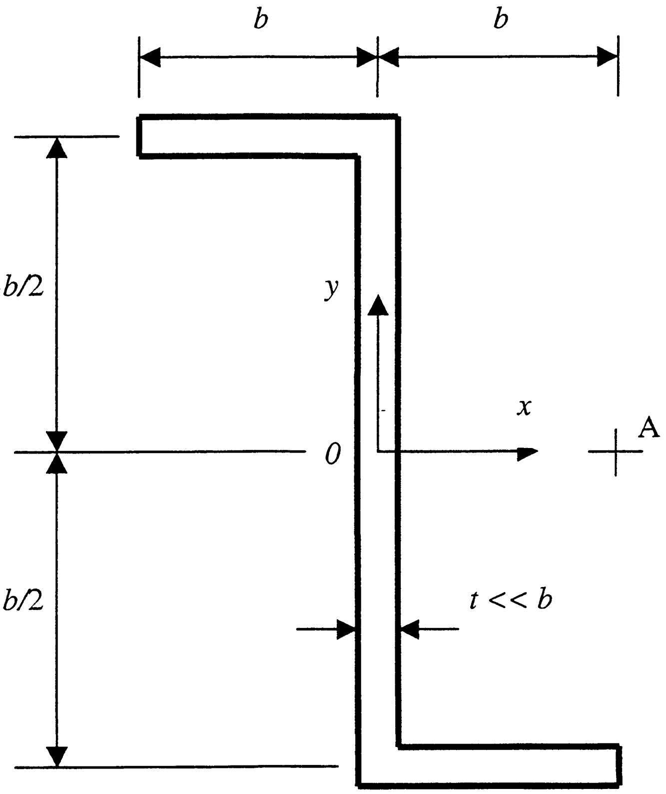

Fig. 1

2 A beam has uniform flexural stiffness EI and a total length of 20 m, divided into two equal spans. The beam is continuous over the central simple support and has simple supports at its ends.

(a) Sketch the elastic influence line for the sagging bending moment at a point A which is a distance of 1m to the right of the central support. [30%]

(b) Find a mathematical expression for this influence line and determine its value at the supports, at the centre of each span and at A. [50%]

(c) Without further calculation, describe how a bridge designer would use this influence line to calculate maximum and minimum bending moments at point A under a distributed load of any length. [20%]

3 (a) Briefly describe the difference in meaning of the eigenvalues in the classical and non-classical descriptions of elastic stability. [20%]

(b) The mechanism shown in Fig. 2 consists of two light rigid pieces ABCD and BEF. There are simple supports at the ends A and F. The pieces have a hinge connection at B, and are connected by a linear elastic spring of stiffness k between D and E. The spring is unstressed when ABEF is straight and DE is $0.7L$. All deformations remain within the plane of the page.

(i) Show that buckling occurs when $P = 3kL/2$. [20%]

(ii) When $P = kL$ a small external transverse load Q is applied at E in the direction of D. Assuming deflections remain small, determine the deflections of E and D in the x-direction. [20%]

(iii) What would be the effect on the small-deflection buckling behaviour if the spring were not stress-free when ABEF is straight? [10%]

(iv) Explain briefly how the buckling behaviour may be affected by asymmetries arising from higher order geometric effects or nonlinear elastic behaviour of the spring. [10%]

(c) Explain briefly Shanley's insight that resolved "The Column Paradox" of inelastic buckling. [20%]

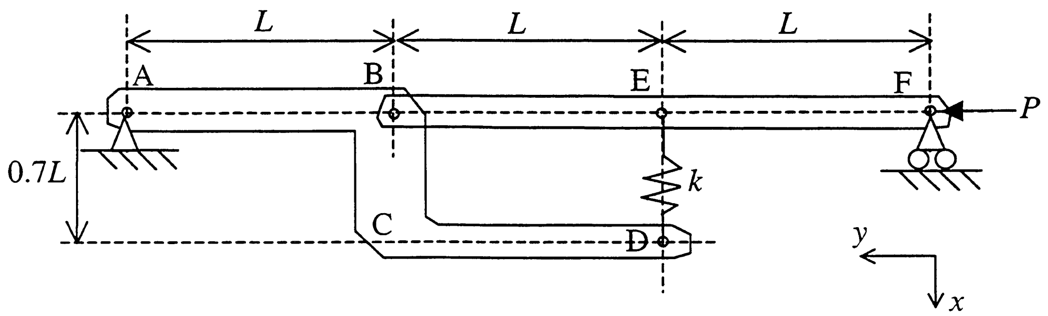

Fig. 2

4 (a) Fig. 3a defines the usual notation for the elastic slope-deflection equations of a beam-column. The rigid-jointed frame shown in Fig. 3b has two beams and a column. The relevant factor $k = EI/L$ is the same for all three members. A point load P is applied at B as shown. Stating your assumptions, write down an appropriate set of slope-deflection equations in matrix form and show that, by this theory, instability occurs when

$$s^2(1-c^2)+7s+12=0$$

where s and c are the stability functions for the column BC. Sketch the shape of the buckled structure. [50%]

(b) Explain briefly how residual stresses can arise in the production of hot-rolled steel Universal Beams and explain briefly how modern design codes of practice take account of their effect on buckling behaviour even though according to the Lower Bound Theorem a pre-existing state of self-stress does not affect the collapse load of a structure. [25%]

(c) Explain briefly how the Perry-Robertson approach to column buckling deals with inelastic behaviour. [25%]

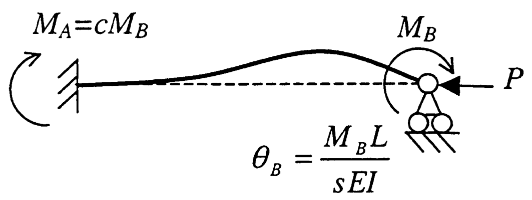

Fig. 3a

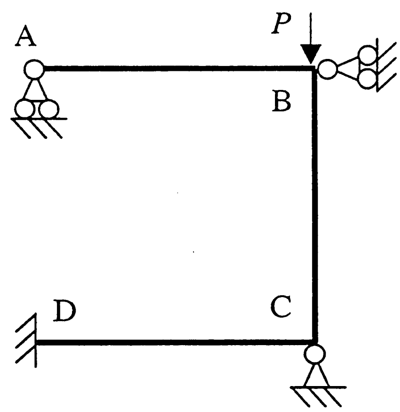

Fig. 3b

END OF PAPER