

ENGINEERING TRIPOS      PART IIA

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Tuesday 27 April 2004    2.30 to 4.00

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Module 3D5

ENVIRONMENTAL ENGINEERING I

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments:*

*Special datasheet (6 pages).*

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

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1 The River Cam at Magdalene Bridge has a catchment area of approximately  $600 \text{ km}^2$ . The base flow under the bridge is typically  $3 \text{ m}^3\text{s}^{-1}$ . A storm crosses the catchment, dropping rain uniform in space and time of intensity  $12 \text{ mm}\cdot\text{hr}^{-1}$  for a duration of two hours.

(a) For a soil near Cambridge, the constants in the f-capacity equation for infiltration have been estimated to be around  $f_0 = 8 \text{ mm}\cdot\text{hr}^{-1}$ ,  $f_c = 2 \text{ mm}\cdot\text{hr}^{-1}$  and  $K_f = 0.7 \text{ hr}^{-1}$ . Assuming the soil to be dry before the storm, estimate the total proportion of rain that will run off. [25%]

(b) Assuming that there is no further rain, briefly describe various methods by which one could estimate the soil water content at a time a few days later. [25%]

(c) In a different storm, 30 mm of rain fell uniformly over the catchment within one day, and of this, 10 mm ran off. The associated runoff above baseflow through Magdalene Bridge had distribution percentages of 8, 31, 35, 18, 8 over each of the subsequent five days. By constructing an S-curve on graph paper (or otherwise) estimate the distribution percentages in half-day intervals for a storm of twelve hours duration in which 20 mm of rain fell, of which 8 mm ran off. Estimate also

(i) the peak flow (in  $\text{m}^3\text{s}^{-1}$ ),

(ii) the peak velocity of water flowing under the bridge (in  $\text{m}\cdot\text{s}^{-1}$ ) if the cross-sectional area at that point is  $20 \text{ m}^2$ ,

(iii) the volume of water that flows under the bridge over the five day period. [50%]

2 A river of rectangular cross section is flowing steadily with mean velocity  $0.5 \text{ m.s}^{-1}$  and depth 2 m. Then, at time  $t=0$ , a hydro-electric plant begins to release water into the river. Just downstream of the exit from the hydro-electric plant the level in the river rises at a rate of  $0.02 \text{ m.s}^{-1}$ . It may be assumed that bed friction and slope are negligible.

(a) What is the minimum distance downstream for the intersection of two positive characteristics? [50%]

(b) How long after time  $t=0$  does this occur? [20%]

(c) Without making additional calculations, indicate how the speed of propagation of this downstream bore might be calculated. [30%]

3 (a) Show from first principles that the velocity in the overlap layer in steady flow at constant depth over a smooth bed in a wide channel is given by

$$u = \frac{u_*}{K} \log_e \frac{y}{y_0}$$

where  $y_0$  is a constant and the other symbols are defined in the Data Sheet. [40%]

(b) Making use of the Prandtl-Karman formulae given in the Data Sheet, obtain an expression for the Manning number for both smooth and rough beds. Give reasons for caution when applying these expressions to real rivers. [60%]

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4 A wide river of depth 2 m has a bed slope of 0.001. The concentration of sediment at 0.006 m above the bed is  $10 \text{ kg.m}^{-3}$  and that at 0.06 m above the bed is  $0.3 \text{ kg.m}^{-3}$ .

(a) Assuming that the Karman constant is 0.4 and that the specific gravity of the sediment is 2.65, estimate the grain diameter of the suspended sediment. [35%]

(b) Estimate the sediment transport rate in suspension, per metre width of channel, between 0.006 m above the bed and the water surface. It may be assumed that the bed roughness size equals 0.006 m and that the kinematic viscosity of water is  $10^{-6} \text{ m}^2\text{s}^{-1}$ . [35%]

(c) Give reasons why these calculations should be treated with caution. [30%]

**END OF PAPER**

**Module 3D5: Environmental Engineering I**
**Data Sheet**

(SI units throughout)

**f-capacity equation**

$$f = f_c + (f_0 - f_c) e^{-K_f t}$$

**Spatially-varied flow (backwater curves)**

$$\delta d = \ell \left( \frac{S - \bar{U}^2 C_f / 2gR}{1 - \bar{U}^2 / gd} \right)$$

**Open-channel characteristics**

$$\bar{U} + 2c - g(S - S_f)t = \text{const} \quad \frac{dx}{dt} = \bar{U} + c$$

$$\bar{U} - 2c - g(S - S_f)t = \text{const} \quad \frac{dx}{dt} = \bar{U} - c$$

**Fall velocity of particles of sand in water (20°C)**

 For  $D < 0.0005$  m

$$W \doteq 56 \times 10^4 D^2 (\rho_s - \rho) / \rho$$

 For  $D > 0.002$  m (and shape factor=0.7)

$$W \doteq 3.3 D^{1/2} ((\rho_s - \rho) / \rho)^{1/2}$$

**Initial motion of sediment on a flat bed**

$$\frac{u_* k_s}{\nu} > 70$$

$$\frac{\tau_c}{(\rho_s - \rho)gD} = 0.05$$

**Velocity in uniform flow in a channel**

Chézy

$$\bar{U} = CR^{\frac{1}{2}} S^{\frac{1}{2}}$$

Manning

$$\bar{U} = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

**Prandtl-Karman**

$$\text{Hydraulically smooth } \left( \frac{u_* k_s}{\nu} < 5 \right) \quad \frac{u}{u_*} = 2.5 \log_e \left( \frac{9.05 y u_*}{\nu} \right)$$

$$\frac{\bar{U}}{u_*} = 2.5 \log_e \left( \frac{3.66 R u_*}{\nu} \right)$$

$$\text{Hydraulically rough } \left( \frac{u_* k_s}{\nu} > 70 \right) \quad \frac{u}{u_*} = 2.5 \log_e \left( \frac{30.2 y}{k_s} \right)$$

$$\frac{\bar{U}}{u_*} = 2.5 \log_e \left( \frac{12.1 R}{k_s} \right)$$

**Variation of the concentration of sediment in suspension with distance from the bed**

$$\frac{C}{C_a} = \left[ \left( \frac{d-y}{y} \right) \left( \frac{a}{d-a} \right) \right]^{K u_*} \frac{W}{K u_*}$$

**Sediment load in suspension**

$$\int_b^d C u dy = 11.6 u_* C_b b [I_1 \log_e (A d) + I_2]$$

where  $A = \frac{9.05 u_*}{\nu}$  for a hydraulically smooth bed  
 $= \frac{30.2}{k_s}$  for a hydraulically rough bed

b/d	W/Ku <sub>*</sub> = 0.2		W/Ku <sub>*</sub> = 0.6		W/Ku <sub>*</sub> = 1.0		W/Ku <sub>*</sub> = 1.5	
	I <sub>1</sub>	I <sub>2</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>1</sub>	I <sub>2</sub>
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113

Limiting shear stress for a particle on a slope

$$\frac{(\tau_c)_\theta}{\tau_c} = \cos \theta \left( 1 - \frac{\tan^2 \theta}{\tan^2 \phi} \right)^{\frac{1}{2}}$$

"Regime" formulae

$$\bar{U} = 0.635 f^{\frac{1}{2}} R^{\frac{1}{2}}$$

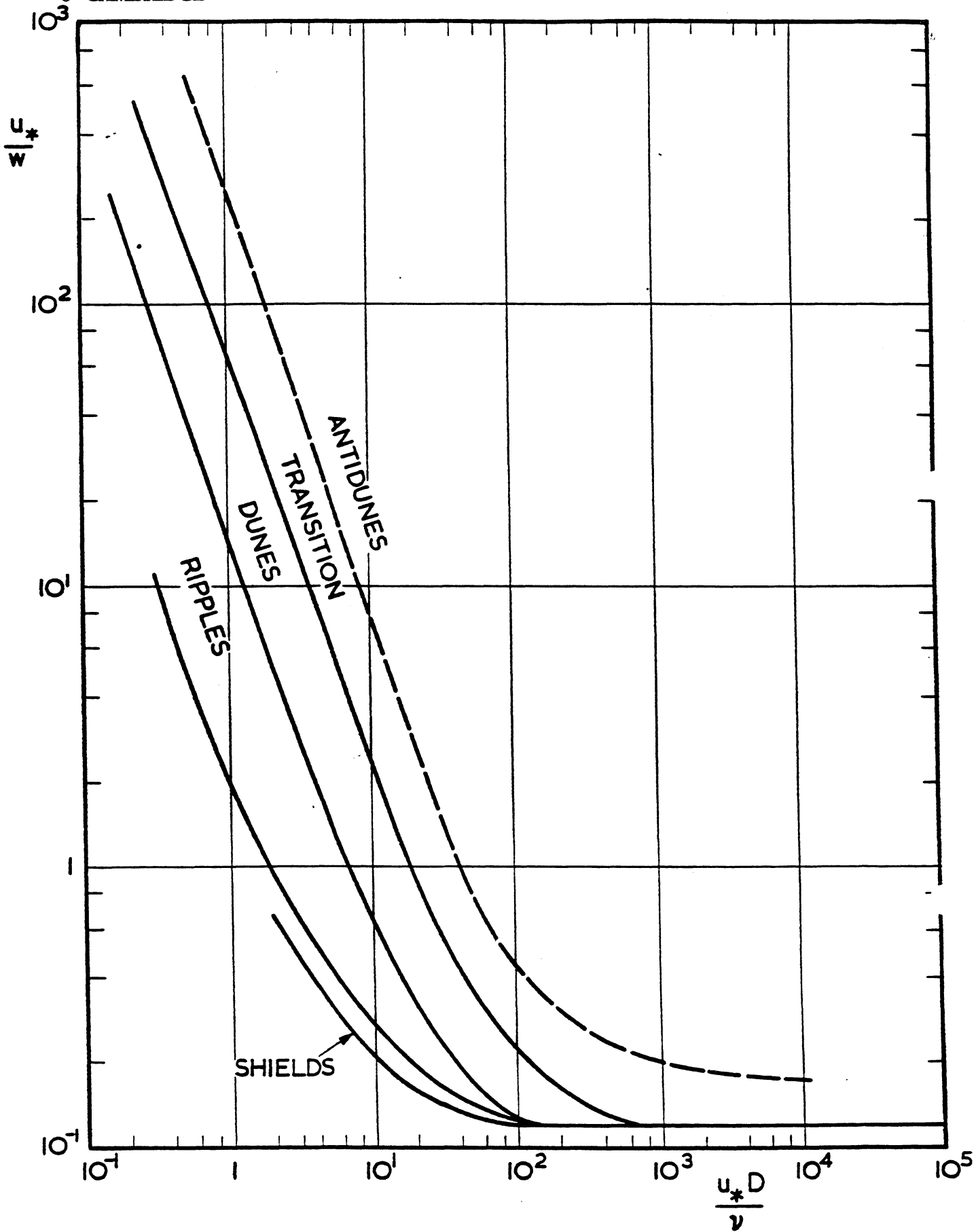
$$P = 4.83 Q^{\frac{1}{2}}$$

$$R = 0.4725 Q^{\frac{1}{3}} f^{-\frac{1}{3}}$$

$$S = 0.000303 f^{\frac{5}{3}} Q^{-\frac{1}{6}}$$

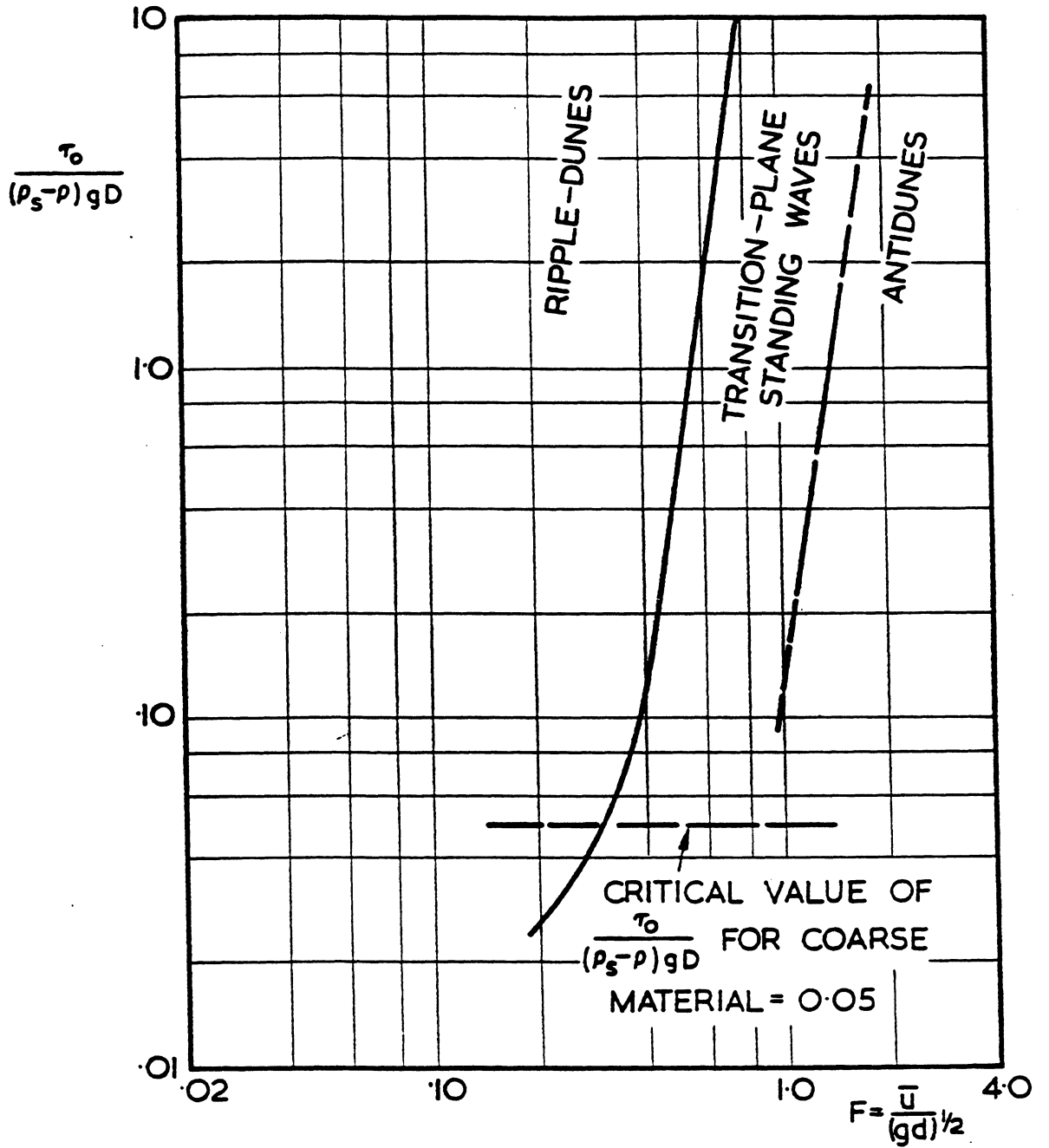
SYMBOLS

- C Concentration of sediment. Chezy roughness coefficient  $(= (2g/C_f)^{1/2})$
- $C_f$  friction coefficient
- D grain size or body diameter
- $D_{65}$  grain size for which 65% by weight of grains have a smaller diameter
- f Lacey silt factor
- $f_0, f_c$  coefficients in f-capacity equation
- $h_r$  ripple height
- K Karman constant
- $K_f$  coefficient in f-capacity equation
- $k_s$  roughness height
- P wetted perimeter of a channel
- Q total flow rate of water
- R hydraulic radius  $(= A/P)$
- S channel slope
- t time
- u horizontal component of fluid velocity
- $\bar{U}$  mean velocity
- $u_*$   $(\tau_o / \rho)^{1/2}$
- v vertical component of fluid velocity
- W fall velocity
- x,y co-ordinates
- $\theta$  angle of a slope to the horizontal
- $\phi$  angle of repose of sediment
- $\nu$  kinematic viscosity
- $\rho$  density of fluid
- $\rho_s$  density of sediment
- $\tau_o$  shear stress on the bed
- $\tau_c$  critical value of  $\tau_o$  for sand movement

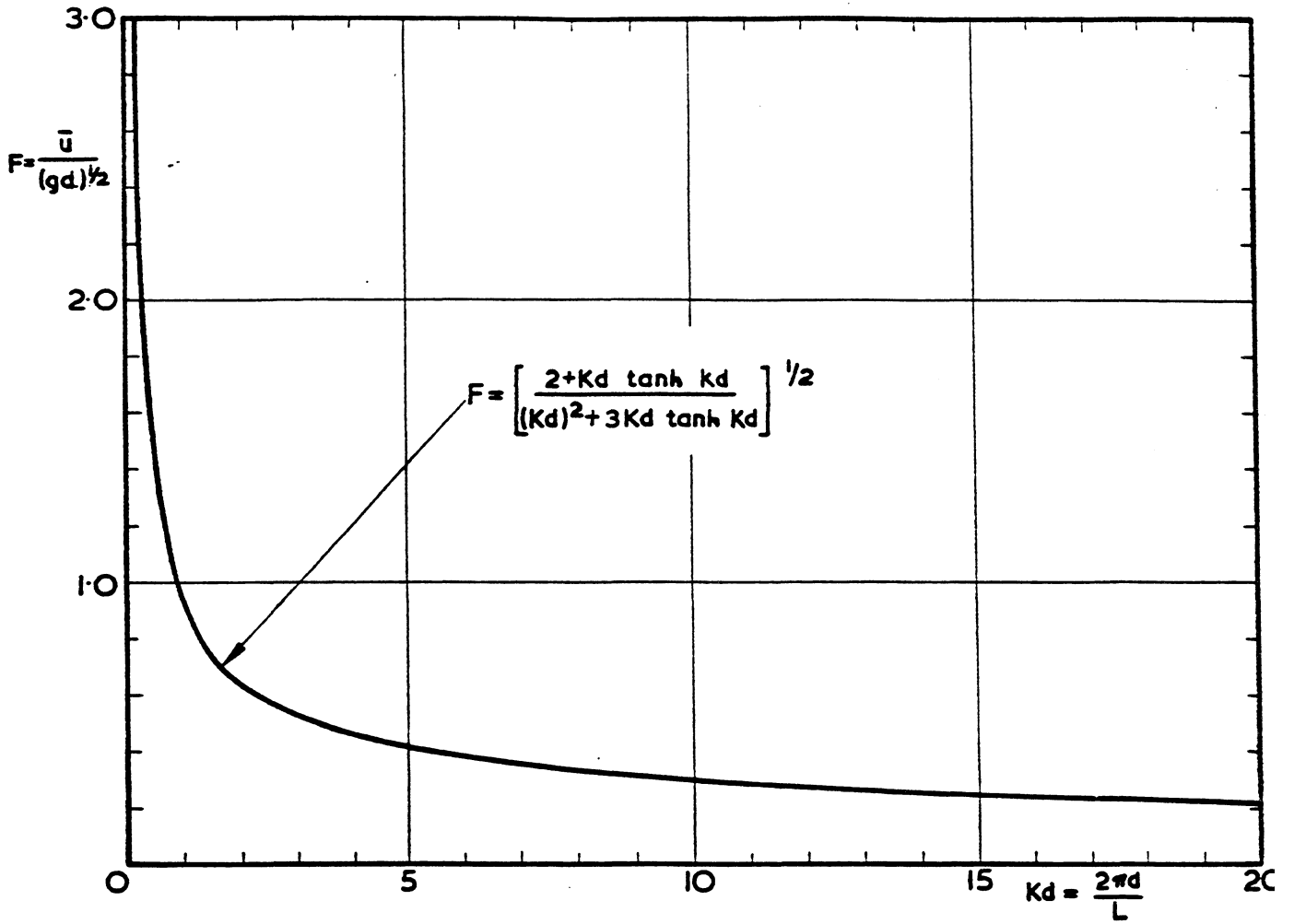


BED REGIMES IN OPEN CHANNEL FLOW (Albertson Simons & Richardson)

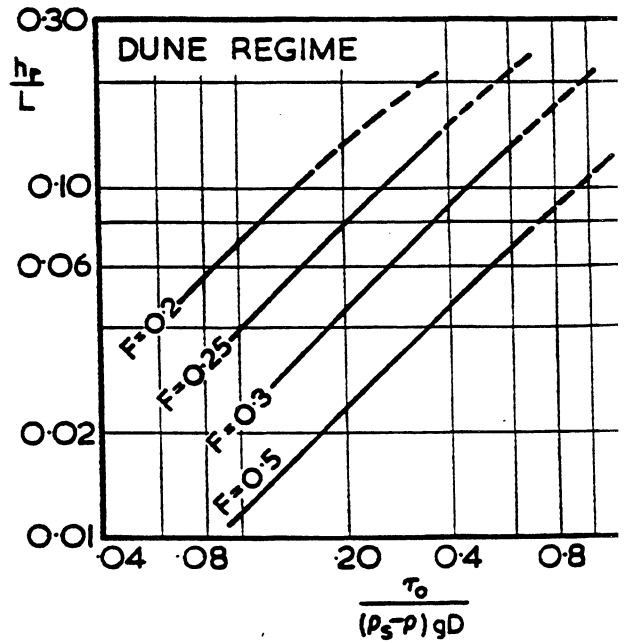
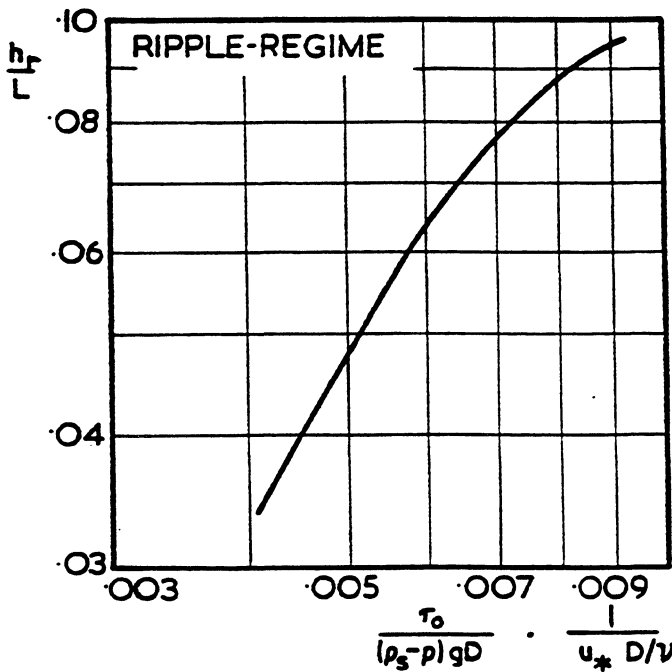




BED REGIMES IN OPEN-CHANNEL FLOW  
(Garde & Albertson)



WAVELENGTH OF DUNES / ANTIDUNES (Kennedy)



RIPPLE / DUNE STEEPNESS (Garde & Albertson)