UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Friday 23 April 2004 2.30 to 4.00

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Which statistical test would you choose to test that the numbers generated by a random number generator are indeed drawn from the correct distribution?

[8%]

(b) Suppose you are the manager of a soft drink stand in a football stadium. Before each football game starts, you need to plan how many boxes for soft drinks to order for one game. Assume D represents the demand for soft drinks in boxes. Suppose you order s boxes. When the demand is greater than the order, then the nominal inventory cost, associated with customer dissatisfaction and unrealised potential profits, is $40\ (D - s)$. When the demand is less than the order, then the real inventory cost for unsold drinks is $10\ (s - D)$. You want to find an optimal order level of boxes of soft drinks using a simulation model.

(i) What are the key uncertainties that determine your inventory cost? What data could be used to estimate the distribution of these uncertainty drivers? [12%]

(ii) Describe how you would use Monte Carlo simulation to determine a good value for the decision variable. [30%]

(c) Define the concepts of trend and seasonality for a time series in relation to forecasting. [8%]

(d) The Winters multiplicative method is being used to forecast quarterly US retail sales (in billions of dollars). At the end of the first quarter of 2003, $E_t = 300$, $T_t = 50$, and the seasonal indices are as follows: quarter 1, 0.90; quarter 2, 0.95; quarter 3, 0.95; quarter 4, 1.20. During the second quarter of 2003, retail sales are \$380 billion. Assume $\alpha = 0.2$, $\beta = 0.4$ and $\gamma = 0.5$.

(i) The forecasting function for the Winters multiplicative method is shown below. Explain the meaning of each equation. [12%]

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_t(k) = (E_t + kT_t)S_{t+k-c}$$

(Cont.

(ii) At the end of the second quarter of 2003, develop a forecast for retail sales during the fourth quarter of 2003. [20%]

(iii) At the end of the second quarter of 2003, develop a forecast for the second quarter of 2004. [10%]

2 A health agency collects data on the height and weight of individuals from the national population. A random sample of 11 males aged 18–24 years gave the following data, where x denotes height in inches, and y denotes weight in pounds.

x	65	67	71	71	66	75	67	70	71	69	69
y	175	133	185	163	126	198	153	163	159	151	155

(a) Assume that height is the independent variable and weight the dependent variable. Apply the linear regression model $y = a + b\,x + \varepsilon$ to the data given in the table. Derive the mathematical formulae for regression coefficients a and b from the least squares property. Find values for a and b. Use the regression equation obtained to predict the weight of an 18–24-year-old male who is 67 inches tall, and another 18–24-year-old male who is 73 inches tall. You may find the following data useful,

$\Sigma\, x_i = 761$, $\Sigma\, y_i = 1761$,

$\Sigma\, x_i\, y_i = 122{,}224$, $\Sigma\, x_i^2 = 52{,}729$, $\Sigma\, y_i^2 = 286{,}273$,

$(\Sigma\, x_i)^2 = 579{,}121$, $(\Sigma\, y_i)^2 = 3{,}101{,}121$, $(\Sigma\, x_i)(\Sigma\, y_i) = 1{,}340{,}121$. [70%]

(b) Explain why intercept a, and slope b in the linear regression model are random variables. [10%]

(c) Define the R-square statistic and describe its meaning. [20%]

(TURN OVER

3 Customers buy cars from three companies. Given the company from which a customer last bought a car, the probability that she will buy her next car from a given company is as follows:

	Will Buy Next From		
Last Bought From	A	B	C
A	0.80	0.10	0.10
B	0.05	0.85	0.10
C	0.10	0.20	0.70

(a) What is the probability distribution of a customer's next car purchase if she bought her current car from company A? What is the probability that at least one of the next two cars she buys will be a car from company A? [20%]

(b) Draw a transition network and determine the classes. Is this Markov chain irreducible and regular? Explain your answer. [20%]

(c) Does the probability distribution of the states tend to a limit as the number of transitions tends to infinity? If not, explain why not. If so, explain why and calculate the limiting distribution. [25%]

(d) At present, it costs company A an average of £10,000 to produce a car, and the average price a customer pays for one is £16,000. Company A is considering instituting a five-year warranty. It estimates that this will increase the cost per car by £1000, but a market research survey indicates that the probabilities will change as follows:

	Will Buy Next From		
Last Bought From	A	B	C
A	0.85	0.10	0.05
B	0.10	0.80	0.10
C	0.15	0.10	0.75

Should company A institute the five-year warranty? Support your claim with analysis. [35%]

4 (a) Explain traffic intensity and the relationship between traffic intensity and the utilization of servers in a queueing system. What is meant by a stationary state in a queueing system? When traffic intensity is one, explain whether the queueing system can eventually reach a stationary state. [20%]

(b) Suppose two single channel systems have the same mean exponential arrival rate and the same mean service rate, but the service time is constant in one and exponential in the other. Do they have the same average waiting time? Explain your answer. [10%]

(c) Drs Andy Smith and Bob Alan operate a medical centre. Each of them has a separate office to provide services to patients. The number of patients in the centre varies between 0 and 4. Let P_n be the probability of exactly n patients being in the centre. According to recent historical records, $P_0 = 1/16$, $P_1 = 4/16$, $P_2 = 6/16$, $P_3 = 4/16$, $P_4 = 1/16$.

(i) Calculate the expected number of "patients in the system", which can be denoted as L. How would you describe the meaning of L to Andy and Bob? [12%]

(ii) Determine the expected number of patients waiting for services. [12%]

(iii) Determine the expected number of patients being served. [10%]

(iv) If the patient arrival rate is 4 per hour, then determine the expected waiting time of patients and the expected total time patients spend in the centre. [16%]

(v) If Andy and Bob spend the same average time per patient, then what is the expected service time for each patient? [10%]

(vi) What is the average percentage of time that Andy and Bob have available to do things other than serve patients? [10%]

END OF PAPER