UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Wednesday 28 April 2004 2.30 to 4.00

Module 3E4

MODELLING CHOICE

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheet (S/27) (1 page).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) You are working on a project that will provide a new financial service to a major bank's customers.

Task	Description	Estimates of task durations (weeks)	Predecessor tasks
A	legal issues study	8	none
B	technical issues study	3	none
C	foundation implementation	20	A, B
D	preliminary marketing	6	A, B
E	advanced implementation	10	C, D
F	foundation marketing	12	C, D
G	retailing via branches	12	E, F
H	advanced marketing	9	E, F

(i) Draw an AON network for this project and find the critical activities using CPM. [10%]

(ii) A more realistic assessment of the task durations provides the following data: All tasks with estimated durations of 10 weeks or less will finish within (plus or minus) two weeks of their stated durations. Tasks F and G will be completed in 8 to 16 weeks. Task C could be completed in as few as 15 weeks, but may require as long as 30 weeks. Apply PERT, using the β-distribution for the duration of each task, to determine the latest times T_{95} and T_{99} that the project will finish with 95% and 99% probability, respectively. Give your answers in weeks rounded to one decimal place of accuracy. Recall that the β-distribution of a random variable with a most likely value m, a lowest value l and a highest value h, has a mean of $(l + 4m + h)/6$ and a variance of $(h-l)^2/36$. [20%]

(iii) Briefly explain to the project manager, using a simple example, the main difficulty of PERT regarding reliability of latest time estimates such as those in part (ii). [15%]

(Cont.

(b) A retailer will be opening its first Birmingham store in June. Its Sales and Marketing Department, SMD, has two aims in this calendar year: to achieve a volume of at least 80k units (i.e. 80,000 units) of sales, and to establish high brand recognition in the Birmingham area. Brand recognition is measured by consumer surveys that generate a brand index between 0 and 100, where a higher value of the index is preferred. SMD has a budget of £30k for promotion and advertising, for June-December, but would prefer to spend only up to £25k. SMD estimates that every £1k spent on advertising will generate 3k units of sales, and contribute 1.5 units to the brand index, while every £1k spent on promotions will generate 1k units of sales, and contribute 2.5 units to the brand index.

(i) Formulate a goal programming model for SMD using percentage deviations. [25%]

(ii) Sketch the sales and brand outcomes that can be generated without violating the "hard" constraints, giving coordinates of the vertices. Indicate the efficient frontier on your sketch. [30%]

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2 (a) Consider the following optimization problem where, for each fixed value of the parameter t, the variables are x_1, x_2 and x_3. For what values of t, if any, is this a convex program?

$$
\begin{aligned}
\text{Min} \quad & t x_1^2 - 4 t x_1 x_2 + x_2^2 - x_3 + \exp(x_3^2) \\
\text{subject to} \quad & t^2 \geq x_1^2 + x_2^2 + x_3^2 \\
& t x_2 \leq x_1 \\
& x_1, x_2 \text{ non-negative.}
\end{aligned}
$$

[35%]

(b) Briefly outline the projected gradient method for minimizing a smooth function $f: R^n \to R$ subject to linear constraints, including its convergence properties.

[25%]

(c) Consider the nonlinear program (NLP):

$$
\begin{aligned}
\text{Max} \quad & -x_1^2 - x_2^2 - x_3^2 \\
\text{subject to} \quad & -x_1 - 2x_2 + 3x_3 = 1 \\
& x_1, x_2, x_3 \text{ non-negative.}
\end{aligned}
$$

Write down the linear program (LP) that approximates this problem at $x^* = (0,0,1/3)$ and verify that x^* is optimal for this LP. Find the Lagrange multiplier for x^*. Estimate the optimal value of the NLP if the RHS value of the equality constraint is changed from 1 to 1.15. [40%]

3 (a) A company make three types of computers for the consumer market, basic, deluxe and laptop. The company is trying to optimise the product mix at its three production plants. All plants can produce all types of computers. Assume that all computers that are made can be sold. The production capacities for assembly and fabrication at the plants are given in the following table:

Plant	**Assembly (Hours)**	**Fabrication (Hours)**
1	20,000	100,000
2	30,000	100,000
3	10,000	70,000
Total	60,000	270,000

The company's profit is \$400 for a basic computer, \$750 for a deluxe computer, and \$980 for a laptop computer. The hours required to assemble and fabricate each type of computer are given in the following table:

Computer	**Assembly (Hours per computer)**	**Fabrication (Hours per computer)**
Basic	3	8
Duluxe	4	12
Laptop	8	16

Define decision variables and formulate this problem as a profit-maximising linear program. You are not required to solve this problem. [40%]

(b) Consider the following linear program:

Maximise $2x + 3y$

Subject to $x + 2y \leq 30$

$x + y \leq 20$

$x, y \geq 0$

(i) Solve the above linear program using the Simplex method. [40%]

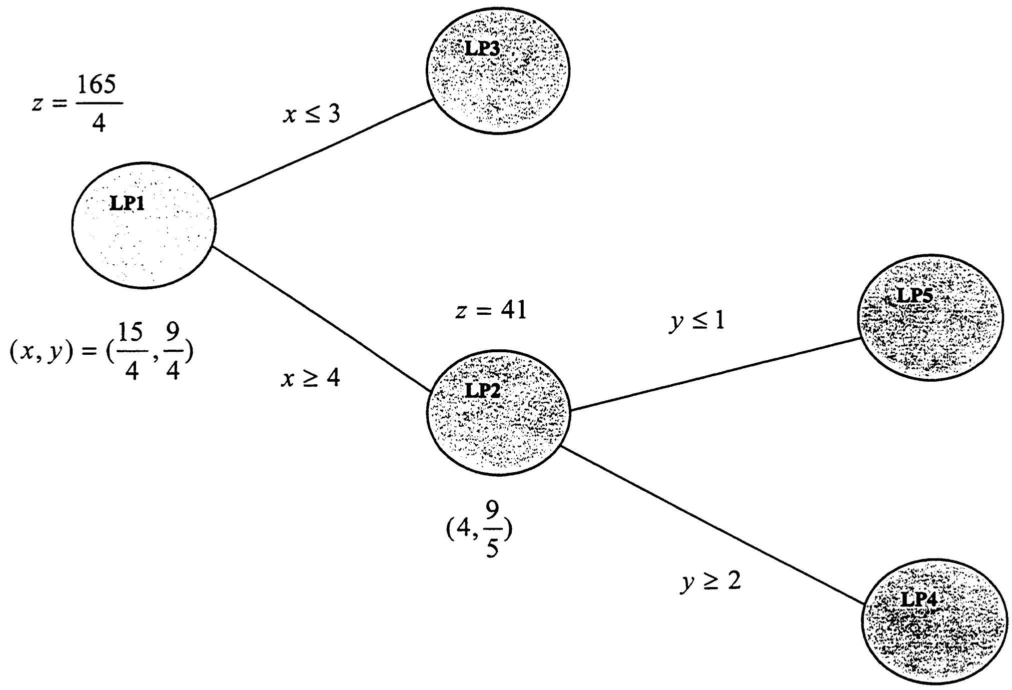
(ii) Calculate the shadow prices for the first two constraints. Explain the meaning of a shadow price. [12%]

(iii) What are the reduced costs of x and y in your final optimal solution? [8%]

4 (a) Consider the following integer linear program (ILP).

$$\begin{array}{ll} \text{Maximise} & z = 8x + 5y \\ \text{Subject to} & x + y \leq 6 \\ & 9x + 5y \leq 45 \\ & x, y \geq 0, \text{ integer} \end{array}$$

Someone has used the branch-and-bound method for solving the above problem as shown in the following branch-and-bound tree:



(i) Find the optimal solution of the ILP by completing the branch-and-bound tree. You may use the graphical approach for solving linear programs. Specify the order of nodes that you visited in the whole branch-and-bound process. [44%]

(ii) Write down the linear program formulation used at node LP5. [8%]

(iii) Sometimes integer linear programs can be solved using the following heuristic approach: (I) Solve the relaxed linear program (RLP) of the ILP and obtain an optimal solution of the RLP, (II) Apply the rounding technique to the optimal solution of the RLP (rounding up, down or to the nearest integer point). Give an example where this approach fails. [8%]

(Cont.

(b) A product can be produced on any one of three different machines. Each machine has a fixed set-up cost, variable production cost per-unit-processed, and a production capacity given in the table below. A total of 1800 units of the product must be produced. Define decision variables and formulate this problem as an integer linear program of minimizing the total cost. You are not required to solve this problem.

Machine	Fixed Cost	Variable Cost	Capacity
1	£1000	£20	1000
2	£950	£24	1200
3	£850	£22	1400

[40%]

END OF PAPER

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Copied from data table S/27. Statistical Tables 1

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THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^{2}0097$	$\cdot 9^{2}0358$	$\cdot 9^{2}0613$	$\cdot 9^{2}0863$	$\cdot 9^{2}1106$	$\cdot 9^{2}1344$	$\cdot 9^{2}1576$
2·4	$\cdot 9^{2}1802$	$\cdot 9^{2}2024$	$\cdot 9^{2}2240$	$\cdot 9^{2}2451$	$\cdot 9^{2}2656$	$\cdot 9^{2}2857$	$\cdot 9^{2}3053$	$\cdot 9^{2}3244$	$\cdot 9^{2}3431$	$\cdot 9^{2}3613$
2·5	$\cdot 9^{2}3790$	$\cdot 9^{2}3963$	$\cdot 9^{2}4132$	$\cdot 9^{2}4297$	$\cdot 9^{2}4457$	$\cdot 9^{2}4614$	$\cdot 9^{2}4766$	$\cdot 9^{2}4915$	$\cdot 9^{2}5060$	$\cdot 9^{2}5201$
2·6	$\cdot 9^{2}5339$	$\cdot 9^{2}5473$	$\cdot 9^{2}5604$	$\cdot 9^{2}5731$	$\cdot 9^{2}5855$	$\cdot 9^{2}5975$	$\cdot 9^{2}6093$	$\cdot 9^{2}6207$	$\cdot 9^{2}6319$	$\cdot 9^{2}6427$
2·7	$\cdot 9^{2}6533$	$\cdot 9^{2}6636$	$\cdot 9^{2}6736$	$\cdot 9^{2}6833$	$\cdot 9^{2}6928$	$\cdot 9^{2}7020$	$\cdot 9^{2}7110$	$\cdot 9^{2}7197$	$\cdot 9^{2}7282$	$\cdot 9^{2}7365$
2·8	$\cdot 9^{2}7445$	$\cdot 9^{2}7523$	$\cdot 9^{2}7599$	$\cdot 9^{2}7673$	$\cdot 9^{2}7744$	$\cdot 9^{2}7814$	$\cdot 9^{2}7882$	$\cdot 9^{2}7948$	$\cdot 9^{2}8012$	$\cdot 9^{2}8074$
2·9	$\cdot 9^{2}8134$	$\cdot 9^{2}8193$	$\cdot 9^{2}8250$	$\cdot 9^{2}8305$	$\cdot 9^{2}8359$	$\cdot 9^{2}8411$	$\cdot 9^{2}8462$	$\cdot 9^{2}8511$	$\cdot 9^{2}8559$	$\cdot 9^{2}8605$
3·0	$\cdot 9^{2}8650$	$\cdot 9^{2}8694$	$\cdot 9^{2}8736$	$\cdot 9^{2}8777$	$\cdot 9^{2}8817$	$\cdot 9^{2}8856$	$\cdot 9^{2}8893$	$\cdot 9^{2}8930$	$\cdot 9^{2}8965$	$\cdot 9^{2}8999$
3·1	$\cdot 9^{3}0324$	$\cdot 9^{3}0646$	$\cdot 9^{3}0957$	$\cdot 9^{3}1260$	$\cdot 9^{3}1553$	$\cdot 9^{3}1836$	$\cdot 9^{3}2112$	$\cdot 9^{3}2378$	$\cdot 9^{3}2636$	$\cdot 9^{3}2886$
3·2	$\cdot 9^{3}3129$	$\cdot 9^{3}3363$	$\cdot 9^{3}3590$	$\cdot 9^{3}3810$	$\cdot 9^{3}4024$	$\cdot 9^{3}4230$	$\cdot 9^{3}4429$	$\cdot 9^{3}4623$	$\cdot 9^{3}4810$	$\cdot 9^{3}4991$
3·3	$\cdot 9^{3}5166$	$\cdot 9^{3}5335$	$\cdot 9^{3}5499$	$\cdot 9^{3}5658$	$\cdot 9^{3}5811$	$\cdot 9^{3}5959$	$\cdot 9^{3}6103$	$\cdot 9^{3}6242$	$\cdot 9^{3}6376$	$\cdot 9^{3}6505$
3·4	$\cdot 9^{3}6631$	$\cdot 9^{3}6752$	$\cdot 9^{3}6869$	$\cdot 9^{3}6982$	$\cdot 9^{3}7091$	$\cdot 9^{3}7197$	$\cdot 9^{3}7299$	$\cdot 9^{3}7398$	$\cdot 9^{3}7493$	$\cdot 9^{3}7585$
3·5	$\cdot 9^{3}7674$	$\cdot 9^{3}7759$	$\cdot 9^{3}7842$	$\cdot 9^{3}7922$	$\cdot 9^{3}7999$	$\cdot 9^{3}8074$	$\cdot 9^{3}8146$	$\cdot 9^{3}8215$	$\cdot 9^{3}8282$	$\cdot 9^{3}8347$
3·6	$\cdot 9^{3}8409$	$\cdot 9^{3}8469$	$\cdot 9^{3}8527$	$\cdot 9^{3}8583$	$\cdot 9^{3}8637$	$\cdot 9^{3}8689$	$\cdot 9^{3}8739$	$\cdot 9^{3}8787$	$\cdot 9^{3}8834$	$\cdot 9^{3}8879$
3·7	$\cdot 9^{3}8922$	$\cdot 9^{3}8964$	$\cdot 9^{4}0039$	$\cdot 9^{4}0426$	$\cdot 9^{4}0799$	$\cdot 9^{4}1158$	$\cdot 9^{4}1504$	$\cdot 9^{4}1838$	$\cdot 9^{4}2159$	$\cdot 9^{4}2468$
3·8	$\cdot 9^{4}2765$	$\cdot 9^{4}3052$	$\cdot 9^{4}3327$	$\cdot 9^{4}3593$	$\cdot 9^{4}3848$	$\cdot 9^{4}4094$	$\cdot 9^{4}4331$	$\cdot 9^{4}4558$	$\cdot 9^{4}4777$	$\cdot 9^{4}4988$
3·9	$\cdot 9^{4}5190$	$\cdot 9^{4}5385$	$\cdot 9^{4}5573$	$\cdot 9^{4}5753$	$\cdot 9^{4}5926$	$\cdot 9^{4}6092$	$\cdot 9^{4}6253$	$\cdot 9^{4}6406$	$\cdot 9^{4}6554$	$\cdot 9^{4}6696$
4·0	$\cdot 9^{4}6833$	$\cdot 9^{4}6964$	$\cdot 9^{4}7090$	$\cdot 9^{4}7211$	$\cdot 9^{4}7327$	$\cdot 9^{4}7439$	$\cdot 9^{4}7546$	$\cdot 9^{4}7649$	$\cdot 9^{4}7748$	$\cdot 9^{4}7843$
4·1	$\cdot 9^{4}7934$	$\cdot 9^{4}8022$	$\cdot 9^{4}8106$	$\cdot 9^{4}8186$	$\cdot 9^{4}8263$	$\cdot 9^{4}8338$	$\cdot 9^{4}8409$	$\cdot 9^{4}8477$	$\cdot 9^{4}8542$	$\cdot 9^{4}8605$
4·2	$\cdot 9^{4}8665$	$\cdot 9^{4}8723$	$\cdot 9^{4}8778$	$\cdot 9^{4}8832$	$\cdot 9^{4}8882$	$\cdot 9^{4}8931$	$\cdot 9^{4}8978$	$\cdot 9^{5}0226$	$\cdot 9^{5}0655$	$\cdot 9^{5}1066$
4·3	$\cdot 9^{5}1460$	$\cdot 9^{5}1837$	$\cdot 9^{5}2199$	$\cdot 9^{5}2545$	$\cdot 9^{5}2876$	$\cdot 9^{5}3193$	$\cdot 9^{5}3497$	$\cdot 9^{5}3788$	$\cdot 9^{5}4066$	$\cdot 9^{5}4332$
4·4	$\cdot 9^{5}4587$	$\cdot 9^{5}4831$	$\cdot 9^{5}5065$	$\cdot 9^{5}5288$	$\cdot 9^{5}5502$	$\cdot 9^{5}5706$	$\cdot 9^{5}5902$	$\cdot 9^{5}6089$	$\cdot 9^{5}6268$	$\cdot 9^{5}6439$
4·5	$\cdot 9^{5}6602$	$\cdot 9^{5}6759$	$\cdot 9^{5}6908$	$\cdot 9^{5}7051$	$\cdot 9^{5}7187$	$\cdot 9^{5}7318$	$\cdot 9^{5}7442$	$\cdot 9^{5}7561$	$\cdot 9^{5}7675$	$\cdot 9^{5}7784$
4·6	$\cdot 9^{5}7888$	$\cdot 9^{5}7987$	$\cdot 9^{5}8081$	$\cdot 9^{5}8172$	$\cdot 9^{5}8258$	$\cdot 9^{5}8340$	$\cdot 9^{5}8419$	$\cdot 9^{5}8494$	$\cdot 9^{5}8566$	$\cdot 9^{5}8634$
4·7	$\cdot 9^{5}8699$	$\cdot 9^{5}8761$	$\cdot 9^{5}8821$	$\cdot 9^{5}8877$	$\cdot 9^{5}8931$	$\cdot 9^{5}8983$	$\cdot 9^{6}0320$	$\cdot 9^{6}0789$	$\cdot 9^{6}1235$	$\cdot 9^{6}1661$
4·8	$\cdot 9^{6}2067$	$\cdot 9^{6}2453$	$\cdot 9^{6}2822$	$\cdot 9^{6}3173$	$\cdot 9^{6}3508$	$\cdot 9^{6}3827$	$\cdot 9^{6}4131$	$\cdot 9^{6}4420$	$\cdot 9^{6}4696$	$\cdot 9^{6}4958$
4·9	$\cdot 9^{6}5208$	$\cdot 9^{6}5446$	$\cdot 9^{6}5673$	$\cdot 9^{6}5889$	$\cdot 9^{6}6094$	$\cdot 9^{6}6289$	$\cdot 9^{6}6475$	$\cdot 9^{6}6652$	$\cdot 9^{6}6821$	$\cdot 9^{6}6981$

Example: $\Phi(3{\cdot}57) = \cdot 9^{3}8215 = 0{\cdot}9998215$.