UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Thursday 29 April 2004 9 to 10.30

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) A linear system with input vector $\underline{u}$, state vector $\underline{x}$, and output vector $\underline{y}$, has the state-space model

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}, \quad \underline{y} = C\underline{x} + D\underline{u}$$

Derive an expression for the transfer function matrix relating the inputs and outputs. [20%]

(b) The steering autopilot of a ship has the state-space model

$$\dot{x}_a = -\frac{1}{T_a}x_a + \frac{1}{10}e, \quad \rho = \frac{9K_a}{T_a}x_a + \frac{K_a}{10}e$$

where e is the ship's heading error and ρ is the rudder angle demanded by the autopilot. Find the transfer function relating e and ρ. [20%]

(c) The ship's heading angle ψ responds to changes in the rudder angle ρ according to the transfer function

$$\frac{\bar{\psi}(s)}{\bar{\rho}(s)} = \frac{K}{s(1+sT)}$$

Using ψ and $\dot{\psi}$ as state variables, find a state-space model corresponding to this transfer function. [20%]

(d) If ψ_r is the required heading of the ship and $e = \psi_r - \psi$, obtain a state-space model of the complete closed loop, comprising both the ship and the autopilot. [20%]

(e) Indicate how the state-space model would be changed if integral action were added to the autopilot. Is integral action likely to be needed in this application? [20%]

2 Figure 1 shows the block diagram of a light-source tracking system. The transfer function of each (linearised) component is shown in the corresponding block. K_1 and K_2 are positive gains.

(a) Sketch a root-locus diagram for this feedback system, which shows how the closed-loop poles depend on the amplifier gain K_2, if $K_1 < 9$. [30%]

(b) Explain how the root-locus diagram changes as the gain K_1 is increased towards 9. Comment on the significance of this change. [20%]

(c) If $K_1 = 9$, find the gain K_2 which gives a pair of critically-damped closed-loop poles. [20%]

(d) If K_2 remains at the value found in (c), sketch a root-locus diagram which could be used to investigate the effects of varying K_1. [30%]

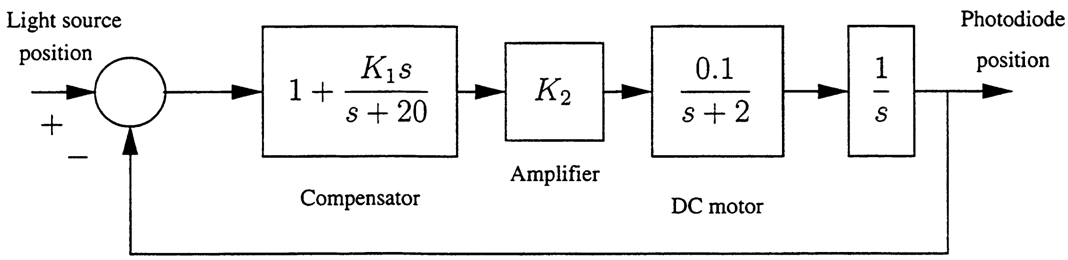

Fig. 1

3 Figure 2 shows the cross-section of a magnetically-levitated train. γ denotes the air gap between the vertical train position and the vertical track position. Magnets produce a vertical force which depends on the magnetising current I and on the air gap γ. The magnetising current I depends on a voltage u applied to the magnet's coils.

(a) Explain how one can determine the (open-loop) stability of a linear system given in the form

$$\frac{d\underline{x}}{dt} = A\underline{x} + Bu$$

[20%]

(b) For small perturbations about the equilibrium values, the linearised equations of motion of the magnetically-levitated train can be written as

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{S}{M} & 0 & -\frac{G}{M} \\ 0 & \frac{S}{G} & -\frac{R}{L} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u$$

if the state variables are chosen as $x_1 = \gamma$, $x_2 = d\gamma/dt$, and $x_3 = I$. Here M is the mass of the train, and G, L, R and S are positive constants.

Show that this system is not asymptotically stable. [20%]

(c) Show that the system in (b) is controllable if $G \neq 0$. [20%]

(d) If $M = 2000$ kg, $G = 500$ N A^{-1}, $S = 5 \times 10^5$ N m^{-1}, $R = 15\ \Omega$, and $L = 0.5$ H, find a set of state-feedback gains which will stabilise the system given in (b) and place the closed-loop poles at $-1 \pm 2j$ and -3. [40%]

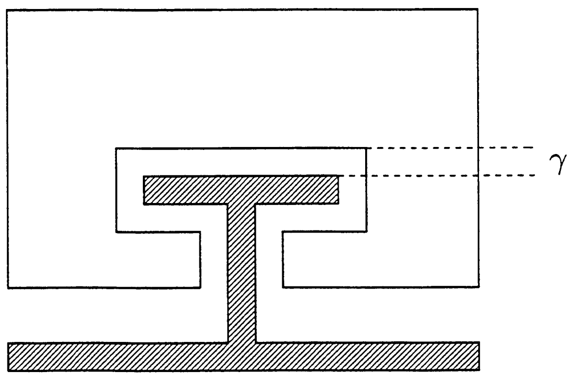

Fig. 2

4 (a) Explain, with the aid of a block diagram, what is meant by a *state observer* for a system of the form [30%]

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}, \qquad \underline{y} = C\underline{x}$$

(b) Consider the error

$$\underline{e} = \underline{x} - \hat{\underline{x}}$$

where $\hat{\underline{x}}$ is the state of the observer. Find an expression for the evolution with time of this error. [20%]

Hence obtain a condition for asymptotic stability of the observer. [20%]

(c) The linearised equations of motion of an underwater vehicle moving horizontally with forward speed u_0 are given by

$$\dot{\underline{x}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -u_0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix} \delta$$

where δ is the deflection of the stern plane, a_{ij} and b_i are non-zero constants, and the state vector is

$$\underline{x} = [w, q, \theta, z]^T$$

where w is the heave velocity (perpendicular to the pitch and roll axes), $q = d\theta/dt$ is the pitch rate, θ is the pitch angle, and z is the depth.

Determine whether the entire state vector can be estimated from measurements of θ and z only (assuming that δ is known). [30%]

END OF PAPER