UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIA

Saturday 8 May 2004 9 to 10.30

Module 3F3

SIGNAL AND PATTERN PROCESSING

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Use the matched z-transform to convert the analogue filter with transfer function

$$H(s) = \frac{s + 0.2}{(s + 0.2)^2 + 4}$$

into a digital IIR filter with transfer function $H(z)$. Use the sampling period $T = 0.2\,\mathrm{s}$ and compare the location of the poles and zeros in $H(z)$ with the locations of the poles and zeros obtained by applying the impulse invariance method in the conversion of $H(s)$. [40%]

(b) Explain the objectives and application in filter design of the bilinear transform

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} .$$

What is the main drawback of the bilinear transform? [20%]

(c) It is required to design a lowpass digital filter by approximating the following analogue transfer function

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} .$$

Using the bilinear transform, obtain the transfer function $H(z)$ of the digital filter assuming a 3 dB cutoff frequency of 150 Hz and a sampling frequency of 1.28 kHz. Sketch an implementation showing the required multiplier coefficients. [40%]

2 (a) Calculate the frequency response of the FIR filter given by

$$h(0) = 0.25, \;\; h(1) = 0.5, \;\; h(2) = 0.25,$$

exploiting symmetry to express its frequency response as the product of a pure delay term and a frequency-dependent gain. [15%]

(b) An FIR filter has an impulse response $h(n)$ which is defined over the interval $0 \leq n \leq N-1$. Show that if N is odd and $h(n)$ satisfies the positive symmetry condition, that is $h(n) = h(N-n-1)$, the filter has a linear phase response, and give an expression for the frequency response of the filter. [35%]

(c) A stationary random process $\{x_n\}$ has autocorrelation function:

$$r_{XX}[k] = 0.8^{|k|} \, .$$

The process is measured in additive, independent white noise v_n with unit variance, so that the observations y_n are:

$$y_n = x_n + v_n \, .$$

It is desired to estimate the underlying signal values $\{x_n\}$ based on the observations alone. One sample of time delay is allowable in the estimate. A 3-tap FIR filter is to be designed for this task such that the estimated value of x_{n-1} is given by

$$\hat{x}_{n-1} = \sum_{i=0}^{2} h(i) y_{n-i} \, .$$

Obtain a simplified expression for the mean-squared error of this estimator when the filter is constrained to have *linear phase response*. [15%]

Obtain the optimum mean-squared error filter coefficients, under the constraint of linear phase response. Determine the mean-squared error for this optimum filter. [35%]

3 A stationary random process is defined as:

$$x_n = u_n - \alpha u_{n-1}$$

where u_n is white noise with variance equal to 1 and α is a constant.

(a) Determine the autocorrelation function for the process. What type of process is this? [30%]

(b) Is the process mean ergodic? [10%]

(c) Determine and sketch the power spectrum of the process when $\alpha = 0.5$. [20%]

(d) The process is now to be approximated as an autoregressive (AR) process. Use the autocorrelation function determined in (a) above to obtain a first order ($P = 1$) autoregressive model for this process whose autocorrelation matches that in part (a) at lags 0 and 1. Sketch the power spectrum of this process for $\alpha = 0.5$ and comment on the quality of the approximation. Comment on whether the approximating AR model is stationary for all values of α . [40%]

4 A two class classification problem is to be solved by building a linear decision boundary between the two classes.

(a) Contrast the training and classification performance on the training data of the linear decision boundaries that are generated using the *perceptron* algorithm and using *least mean squares estimation*. When might it be useful to use one of these approaches rather than Bayes' decision rule? [25%]

(b) The following set of feature vectors are to be used to build a linear classifier for a 2-dimensional problem,

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_5 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathbf{x}_6 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Feature vectors $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ are labelled class ω_1 and $\mathbf{x}_4$, $\mathbf{x}_5$ and $\mathbf{x}_6$ are labelled class ω_2. The decision boundary is required to go through the origin.

(i) Sketch the position of the points on a diagram and mark the range of possible decision boundaries that correctly classify the training data. [15%]

(ii) What is the *cost function* for least mean squares estimation of a linear decision boundary? [10%]

(iii) Using least mean squares estimation and the *pseudo-inverse* method, compute the decision boundary. The target value for class ω_1 is 1, and for class ω_2 is -1. Show this decision boundary on the sketch of part (b)(i). What is the training data classification performance? [30%]

(iv) Rather than using the pseudo-inverse to estimate the decision boundary for least mean squares estimation, gradient descent optimisation is to be used. Derive an appropriate update rule to find the decision boundary. Contrast this form of parameter estimation with the pseudo-inverse approach for solving least mean squares classification problems. [20%]

END OF PAPER