ENGINEERING TRIPOS PART IIA

---

Thursday 6 May 2004 9 to 10.30

---

Module 3F4

DATA TRANSMISSION

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.**

(TURN OVER

1 The spectrum of a single received pulse in a baseband digital transmission system can be written as

$$H(\omega) = H_T(\omega)H_C(\omega)H_R(\omega).$$

(a) Explain with the aid of a block diagram the terms $H_T(\omega)$, $H_C(\omega)$ and $H_R(\omega)$. Indicate how the effects of channel noise may be included and state the usual assumptions made about the noise. [25%]

(b) The received pulse is proportional to $p_R(t)$, where $p_R(t)$ satisfies the Nyquist pulse shaping criterion and $p_R(0) = 1$. Show that the energy of the transmitted pulse is

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{k^2 |P_R(\omega)|^2}{|H_C(\omega)|^2 |H_R(\omega)|^2} d\omega$$

where $k$ is a gain term and $P_R(\omega)$ is the Fourier transform of $p_R(t)$. [25%]

(c) If the channel noise power spectrum at the receiver input is given by

$$N(\omega) = \frac{N_0}{2}|H_C(\omega)|^2$$

and $P_R(\omega)$ is real and positive, derive an expression for the maximum possible value of

$$\frac{\text{Eye Opening}}{\sqrt{\text{Noise Power}}}$$

at the data slicer in terms of $N_0$ and $E_T$. [50%]

Note: The Schwartz inequality is

$$\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \geq \left| \int_{-\infty}^{\infty} F(\omega)G(\omega) d\omega \right|^2$$

2 (a) Define what is meant by a systematic linear binary block code. [20%]

(b) Explain how the process of linear block coding may be represented using a generator matrix G, and show that provided that the parity check matrix H is suitably defined, the syndrome is a function only of the error vector and not of the transmitted codeword. [20%]

(c) A systematic linear block code has a generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

Deduce its minimum Hamming distance and hence its error detecting and correcting abilities. [30%]

(d) Determine the syndromes for all the single-bit errors for the code in (c) and establish if this code is perfect. [30%]

(TURN OVER

3 (a) Explain how a modulated signal $s(t)$ is related to its carrier wave of frequency $\omega_c$ and to the phasor waveform $p(t)$, which represents the modulation. [25%]

(b) Sketch the block diagram of a Quadrature Demodulator, and derive expressions which show how the real and imaginary parts, $i(t)$ and $q(t)$, of the phasor waveform $p(t)$ can be obtained from the input signal $s(t)$. [25%]

(c) Define the phasor constellation for 16-level Quadrature Amplitude Modulation (16-QAM) and label the constellation points with a binary representation which minimises the decoded bit error rate. Explain your choice of labeling and show the optimum decision boundaries for a data detector, assuming equally probable data symbols. [25%]

(d) Describe, with the aid of a block diagram, how a 16-QAM demodulator and data detector could in principle be built with the following components: a quadrature demodulator, two *integrate and dump* matched filters, six threshold detectors and some simple decoding logic. (You need not include any phase-locked loops for recovering the carrier phase or symbol timing.) [25%]

4 (a) Briefly describe the principles on which Orthogonal Frequency Division Multiplexing (OFDM) works and explain the main advantages of this multi-carrier approach to data transmission. [30%]

(b) Derive an expression for the approximate spectral efficiency (ratio of the user data rate in bits per second to the bandwidth in Hz) achieved by a Coded OFDM (COFDM) signal with the following parameters:

Total number of carriers, $N$
Number of carriers used as pilot tones, $n$
FFT analysis block period, $T$
Guard period, $\Delta T$
$M^2$-QAM modulation on each carrier
Error correction code rate, $R$ (where $R < 1$). [40%]

(c) A digital video broadcast system has the following parameters: $T = 224$ μs, $\Delta T = 7$ μs, $M = 8$, $R = 0.6$ and a user data rate of 24 Mbit/s. Explain how COFDM allows multiple transmission paths to be accommodated, and calculate the maximum path length difference (in metres) that can be tolerated before significant performance degradation occurs. How does this compare with the path length difference that could be accommodated by a conventional single-carrier modulation system? [30%]

**END OF PAPER**