PART IIA 2005
3A1: Fluid mechanics I (double module)
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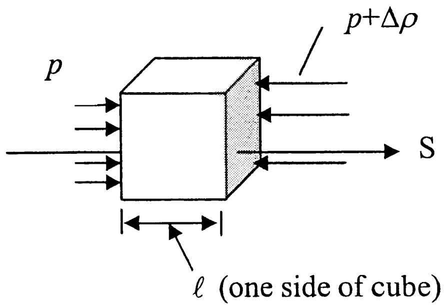
Datasheet: Applications of External Flows; Viscous Flow & Boundary Layers Data Card; Incompressible Flow Data Card

3A1 Examination Crib 2005

1 (a)



Pressure behind: p
Pressure ahead: $p+\Delta p$

Resultant force: $F = -\Delta p.\ell^2$

Estimate Δp from gradient: $\Delta p = \ell.\frac{dp}{dS} \Rightarrow F = -\ell^3.\frac{dp}{dS}$ (note '-' sign!)

(b) Newton: F = ma

Here: m=$m = \rho.\ell^3$, $a = \frac{F}{m}$ note: a is particle acceleration.

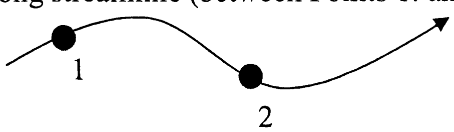
$$\Rightarrow a = -\frac{1}{\rho}\frac{dp}{dS}$$

(c) Assume steady flow

Hence particle acceleration = $\boxed{\cancelto{0}{\frac{dv}{dt}} + V\frac{dV}{dS}}$

Therefore: $V\frac{dV}{dS} = -\frac{1}{\rho}\frac{dp}{dS}$ *or* $-\rho V dV = dp$

Integrate along streamline (between Points 1. and 2.)

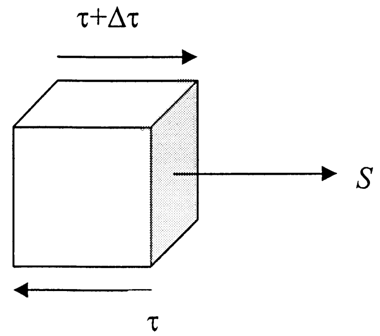


Assuming <u>constant density</u> (incompressible flow) and no other forces (friction, gravity etc.)

$$-\rho\int_1^2 V dV = \int_1^2 dp \Rightarrow p_2 - p_1 + \rho\frac{V_2^2}{2} - \rho\frac{V_1^2}{2} = 0$$

$$\underline{p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2}$$

(d) In a shear flow, there are shear forces acting in flow direction:



Introducing normal coordinate n and using Newton's law for viscous forces:

$$\tau = \mu\frac{\partial V}{\partial n}$$

$$\Delta\tau = \ell.\frac{\partial \tau}{\partial n} = \ell.\mu.\frac{\partial^2 V}{\partial n^2}$$

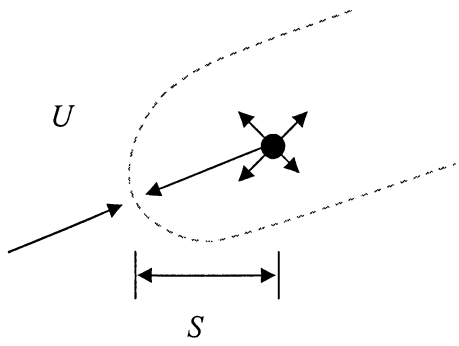
Add this term to result of (b):

Effec. Shear force: $F_s = \Delta\tau.\ell^2 = \mu\ell^3\frac{\partial^2 V}{\partial n^2} \Rightarrow a = \underbrace{-\frac{1}{\rho}\frac{dp}{dS}}_{pressure\ force} + \underbrace{\frac{\mu}{\rho}\frac{\partial^2 V}{\partial n^2}}_{viscous\ force}$

2 (a) Velocity due to point source at origin is $\dfrac{m}{4\pi r^2}$ radially outwards.

At stagnation point this must be equal to the velocity U to give zero velocity

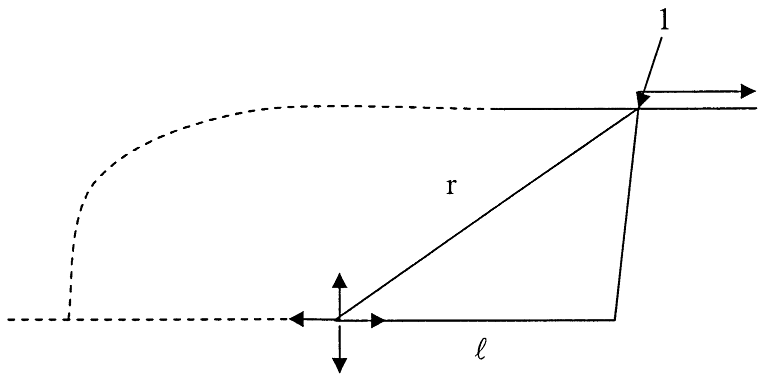
$$\frac{m}{4\pi S^2} = U \qquad S = \sqrt{\frac{m}{4\pi U}}$$



(b) Far downstream the influence of the point source is negligible hence the velocity is very close to uniform. The surface of the tube is a stream-surface (equivalent to a streamline in 2D), hence all the flow inside this surface must come from the source

$$m = U\frac{\pi D^2}{4} \qquad D = \sqrt{\frac{4m}{\pi U}} \qquad Note: \ \frac{S}{D} = \sqrt{\frac{\frac{m}{4\pi U}}{\frac{4m}{\pi U}}} = \frac{1}{4}$$

(c)



$$r \approx \ell$$

upstream value

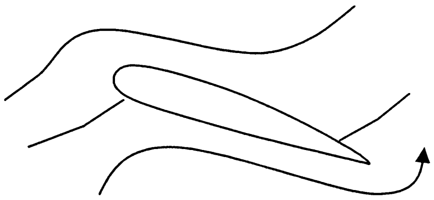
$$U_{SOUR} \approx \frac{m}{4\pi r^2} = \frac{\pi D^2 U}{16\pi \ell^2} \therefore \frac{U_{SOUR}}{U} = \frac{D^2}{16\ell^2}$$

$$\therefore \frac{1}{16}\frac{D^2}{\ell^2} = 0.01$$

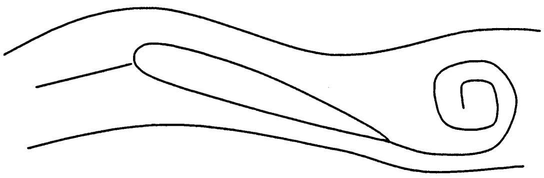
$$\frac{\ell}{D} = \sqrt{6.25} = 2.5$$

$+.25D$ <u>so nose 2.75 diam back</u>

3. (a) As the wing accelerates from stationary the boundary layer separates from the trailing edge and rolls up to form a standing vortex



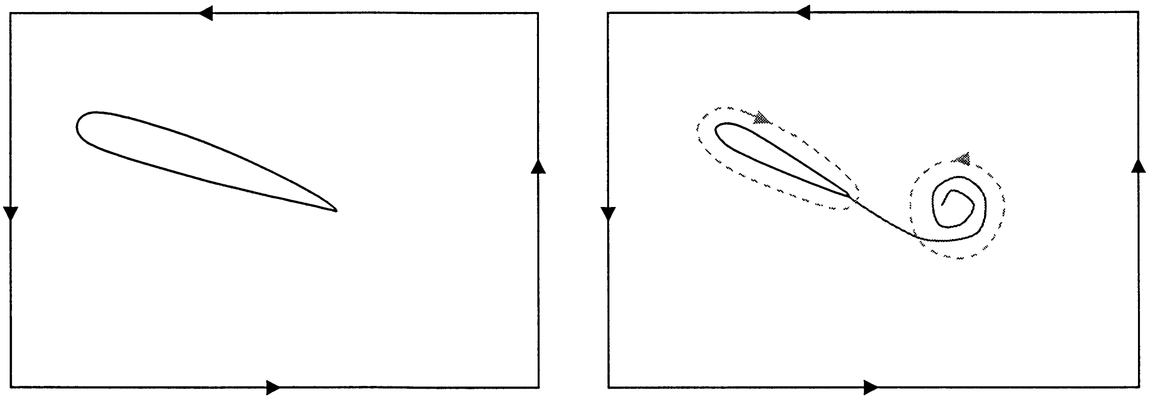
Strong adverse pressure gradient separation



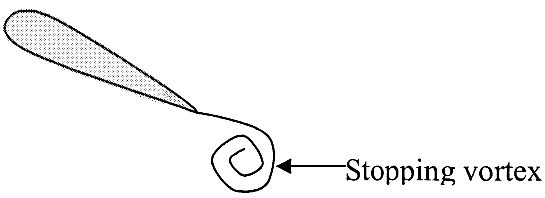
Induced velocity of vortex shifts stagnation points

Kelvin's circulation theorem says that the total circulation must remain zero

$t=0 \quad \Gamma=0$ $\qquad$ $t = t_1 \quad \Gamma = 0$ by Kelvin



Hence wing must have –ve circulation. When aircraft stops its circulation is shed as a stopping vortex

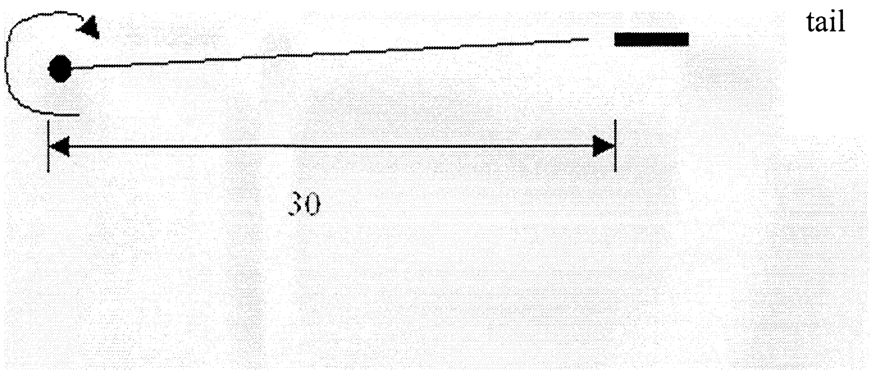


(b) Lift = weight →level flight

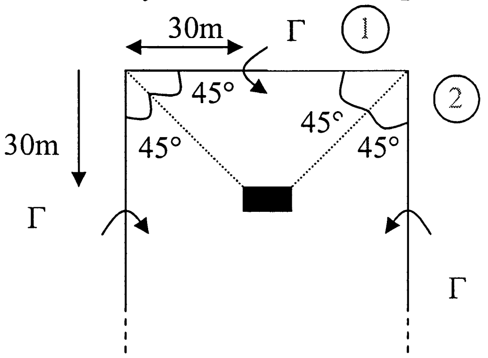
Lift = $-\rho U \Gamma 2\ell$ $\quad$ ℓ is span of one wing

Can model wing as a vortex of circulation

$$\Gamma = \frac{-400{,}000\ g}{\rho U.2\ell} = \frac{-400{,}000\ x\ 9.8}{1.2\ x\ 200\ x\ 60} = 272.2\frac{m^2}{s} \qquad \rho{=}1.2$$



A symmetrical aerofoil gives zero lift at zero angle of attack to the oncoming flow.

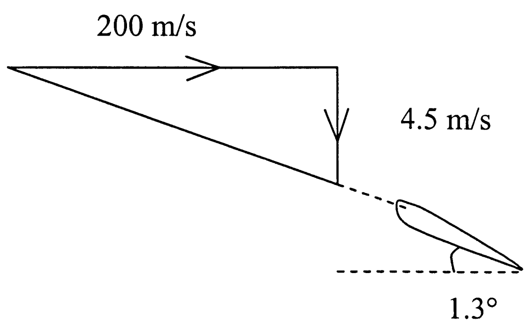


At tail, downwash, $w = \frac{\Gamma}{4\pi 30}\left(\cos 45^\circ + \cos 45^\circ\right)$ $\quad$ 1

$+\frac{2x\Gamma}{4\pi 30}\left(\cos 45^\circ + \cos 0^\circ\right)$ $\quad$ 2 x 2

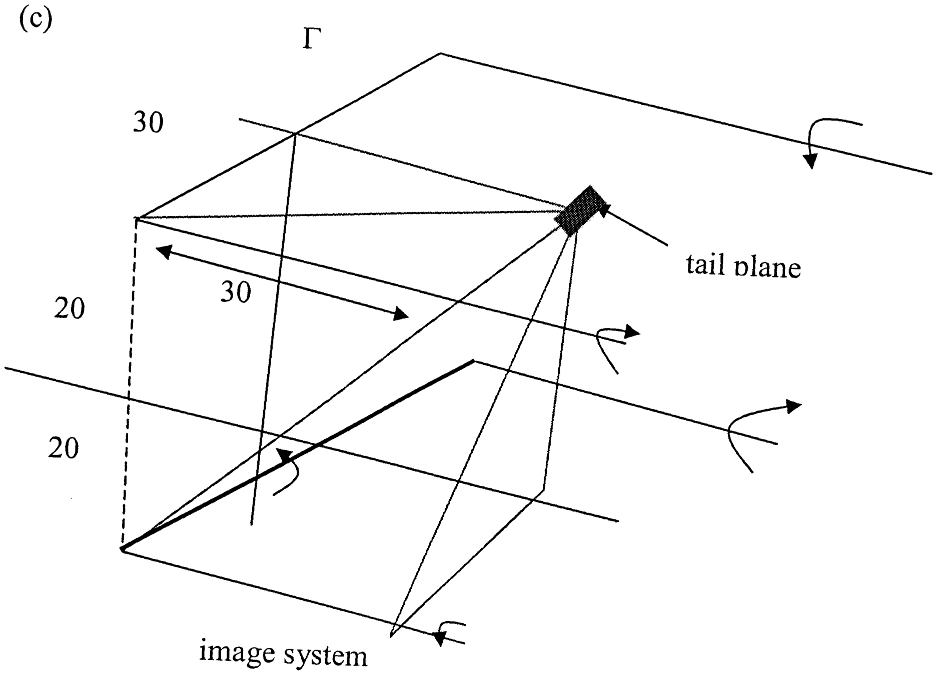
$= \frac{272.2}{4\pi 30}\left[1.41 + 2x(1.41 + 1.1)\right]$

$\therefore$ w = 4.5 m/s

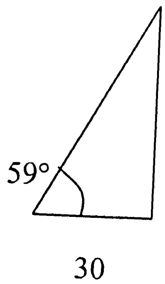


$$\tan(angle) = +\frac{4.5}{200} \Rightarrow angle = 1.3^\circ$$

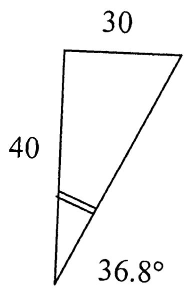
(c)

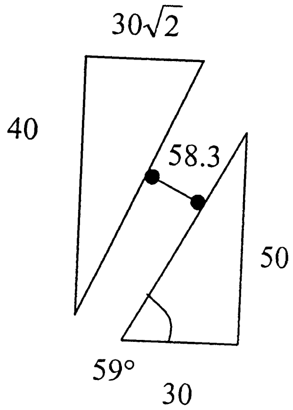


sides:



$\sqrt{30^2 + 40^2} = 50$





At tail, downwash (+ve down)

$$w = 9.0\left\{\overbrace{\frac{\Gamma}{4\pi\sqrt{30^2+40^2}}\left(\cos 59^\circ + \cos 59^\circ\right)}^{from \quad images}\right.$$

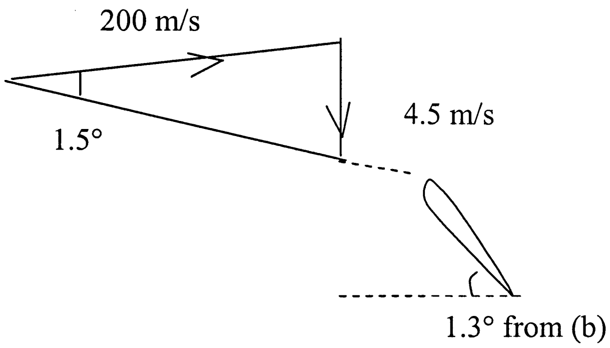
$$\left.\frac{+2\Gamma}{4\pi\sqrt{30^2+40^2}}\left(\cos 59^\circ + \cos 0^\circ\right)\right\}\text{Sin} \quad 36.8^\circ$$

$$= 9.0 - 0.43[1.030 + 2(0.515+1)]x \sin 36.8^\circ$$

$$\therefore w = 5.2\ m/s$$

$\therefore$ angle now 1.5°

Hence



$\Rightarrow$ lifting

4 (a) $F(z) = Uz + \frac{m}{2\pi}\ln(z+b) - \frac{m}{2\pi}\ln(z-b)$

(b) Stagnation points where

$$\frac{dF}{dz} = u - iv = 0$$

$$= \frac{U.4\pi^2(z^2-b^2) + m2\pi(z-b) - m2\pi(z+b)}{4\pi^2(z^2-b^2)} = 0$$

$$2\pi Uz^2 - 2\pi Ub^2 + mz - mb \cancel{-} mz - mb \cancel{=} 0$$

$$z^2 - b^2 - \frac{2mb}{2\pi U} = 0$$

$$z^2 = b^2 + \frac{mb}{\pi U}$$

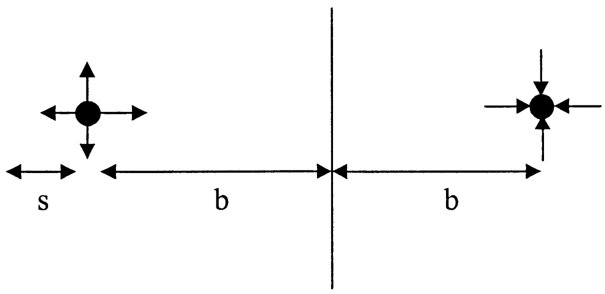
$$z = \pm\sqrt{b^2 + \frac{mb}{\pi U}}$$

$$\frac{z}{b} = \pm\sqrt{1 + \frac{m}{\pi Ub}}$$

Total length is twice this.

Alternatively you could get the same result by looking on the x-axis and equating velocities (see below):

ALTERNTIVE (b) METHOD



s is distance to stagnation point.

Velocity at s (on x-axis)

$$U - \frac{m}{2\pi s} + \frac{m}{2\pi(2b+s)} = 0$$

$$2\pi U s(2b+s) - m(2b+s) + ms = 0 \quad \text{NUMERATOR}$$

$$2\pi U s^2 + s(4\pi U b) - 2bm = 0$$

$$s^2 + 2bs - \frac{bm}{\pi U} = 0$$

$$s = \left(-2b \pm \sqrt{4b^2 + 4\frac{bm}{\pi U}}\right) \Big/ 2$$

$$\frac{s}{b} = \left(-2 \pm \sqrt{4 + 4\frac{m}{\pi U b}}\right) \Big/ 2$$

$$= \left(-1 \pm \sqrt{1 + \frac{m}{\pi U b}}\right)$$

Half width is s+b

$$\frac{s+b}{b} = \frac{s}{b} + 1$$

$$\frac{1}{b} = \frac{s+b}{b} = \pm\sqrt{1 + \frac{m}{\pi U b}}$$

Just as much work.

(c) (i) $$\frac{z}{b} = \pm\sqrt{1+\frac{m}{\pi U b}}$$ as $b \to 0$ mb = constant
$$z = \pm\sqrt{b^2+\frac{mb}{\pi U}}$$

$$z = \pm\sqrt{\frac{mb}{\pi U}}$$ So total length is twice this

(ii)

$$F(z) = Uz + \frac{m}{2\pi}\left(\ln(z+b) - \ln(z-b)\right)$$

$$= Uz + \frac{m}{2\pi}\left(\ln\left(1+\frac{b}{z}\right) + \cancel{\ln(z)}\right) - \left[\ln\left(1-\frac{b}{z}\right) + \cancel{\ln(z)}\right]$$

$$= Uz + \frac{m}{2\pi}\left(\frac{b}{z} - \left(-\frac{b}{z}\right)\right) \qquad \text{as } \frac{b}{z} \to 0$$

$$= Uz + \frac{m}{2\pi}\frac{2b}{z} \qquad 2bm = \text{constant} = \mu \text{ as } b \to 0$$

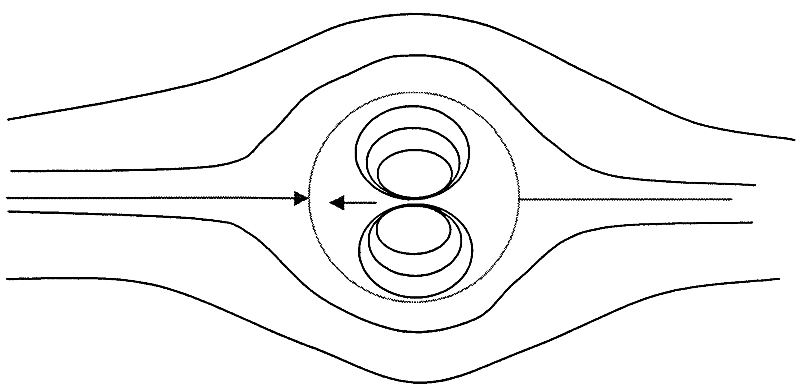
$$= Uz + \frac{\mu}{2\pi z}$$

Which is a doublet and free-stream $\rightarrow$ flow over a cylinder.

(iii) it's a circle so

$$|z| = \sqrt{\frac{mb}{\pi U}}$$ would be one way to write it.

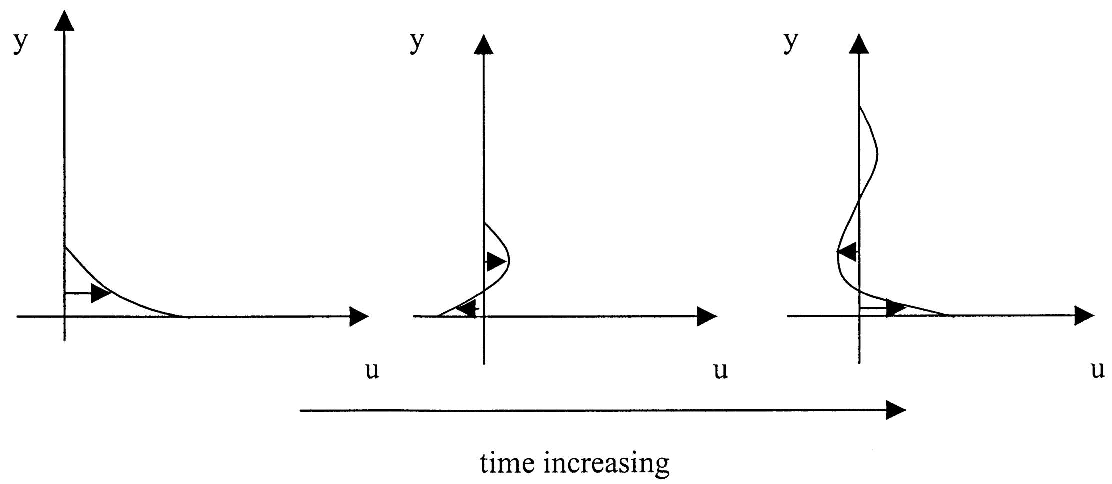
(iv) Flow over a "cylinder"



It could be used to model flow over a cylinder but would only be good around the front (upstream) at the rear the strong adverse press. Gradient will lead to separation and a large wake.

5.

a) Sketches



b) Due to horizontal homogeneity there is no $\partial/\partial x$ or $\partial/\partial z$ of velocity or pressure; no $\partial p/\partial z$ because streamlines are parallel. Hence the required result.

c) The governing equation is a linear equation driven by harmonic forcing. The response will be harmonic at the same frequency. Viscosity will ensure that the amplitude of the harmonic oscillation decreases with distance from the wall.

d)

$$\partial u/\partial t = \nu\, \partial^2 u/\partial y^2$$

Scale u with U_0, scale t with $1/n$ and scale y with δ to show that

$$nU_0 \sim \nu U_0/\delta^2$$

and so δ (the penetration distance) $\sim (\nu/n)^{1/2}$

Alternatively dimensional analysis might be used. If we can assume that $\delta = f(\nu,n)$ then we have 3 variables with two dimensions and these can be written in terms of 3 – 2 = 1 non-dimensional groups or terms.

That is

$$\delta/(\nu/n)^{1/2} = f(0) = constant \text{ or}$$

δ (the penetration distance) $\sim (\nu/n)^{1/2}$ as before however there is no reason why we can assume that $\delta = f(\nu,n)$ rather than $\delta = f(U_0, \nu, n)$

Physically we do not expect the amplitude at some specific distance from the plate to increase indefinitely with time to equal the plate amplitude. The larger the viscosity the further from the plate the effect of the movement of the plate. The more rapidly the velocity reverses the easier it will be for viscosity to smooth (cancel) out the spatial velocity variations.

d) By inspection the equations and boundary conditions for the unsteady heat conduction and the flow driven by the oscillating flat plate are identical. Thus the solutions will also be identical.

$$u(y,t) = U_0\, e^{-ky} \cos\,(\, nt - ky)$$

e) Substitute the solution in d) into the fundamental equation obtained in b) to show that $1/k = (2\nu / n)^{1/2}$

$1/k$ is the penetration distance , interpreted here as where the amplitude of the velocity oscillation is $1/e$ of the velocity amplitude of the plate.

6.

a) A "boundary layer" is the region of flow near a surface that is directly influenced by the surface. The use of the term is restricted to situations where the thickness of this region is small compared with other length scales of the problem.

b) The laminar boundary thickens due to the molecular diffusion of momentum/vorticity while the turbulent boundary layer thickens due to the turbulent mixing between the free stream flow and the slower moving fluid in the boundary layer. The latter process is stronger and the turbulent boundary layer grows more than the laminar one. Nearly linear growth compared with parabolic. The stronger mixing in the turbulent case leads to a near-uniform velocity profile with a sharp decrease to zero at the wall. Thus the turbulent boundary layer has the larger surface shear stress and skin friction coefficient. Arguments based on the momentum deficit in the boundary layers provide the same conclusion. Separation occurs with smaller adverse pressure gradients in the laminar case than in the turbulent case. The strong mixing in the turbulent case assists in keeping the slower moving fluid near the wall from reversing direction leading to separation.

c) In general they are very similar. The turbulence will transfer both the momentum and heat by the same fluid motions. If the Prandtl number is unity then both molecular and turbulent processes are similar and this leads to Reynolds analogy that connects the Stanton number to the Skin friction coefficient. The transport processes are slightly different in that momentum can also be transported by pressure variations.
This becomes particularly evident for flow over rough walls where there is no analogy between the force on the roughness elements due to flow separation and the heat transfer process from the wall which is always by conduction right at the wall

d) It is the same molecular and turbulent transfer processes in gases that transfer heat and mass; arising basically from the kinetic theory of gases. Note that this is not valid for liquids.

e) Given that $F = St\ U\ (\rho_s - (\rho_a = 0))\ kg/s/m^2$

Or $F = 0.037\ Re_x^{-0.2}\ U\ (\rho_s - (\rho_a = 0))$

This expression gives the sublimation rate per unit plate area and is a function of the distance x from the leading edge of the plate.

Thus

$$F_{avarage} = 0.037\ (U/\upsilon)^{-0.2}\ U\ (\rho_s - (\rho_a = 0))\ L^{-1} \int_0^L x^{-0.2}\, dx$$

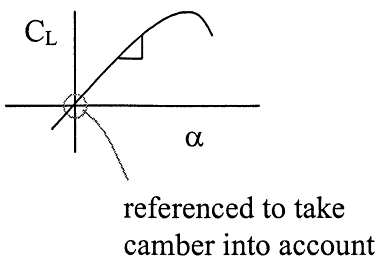
$$F_{avarage} = (0.037/0.8)\ (UL/\upsilon)^{-0.2}\ U\rho_s$$

And if $U = 8\ m/s,\ L = 1\ m,\ \upsilon = 15 \times 10^{-6}\ m^2/s$ then

$$F_{avarage} = 2.6 \times 10^{-6}\ kg/s/m^2$$

This can be re-arranged into a sublimation velocity (velocity at which the naphthalene surface is eroded) by dividing the above expression with the density of the solid naphthalene.

7 (a) For airfoils at small to moderate angles of attack (far away from stall) the local lift coefficient is proportional to the local angle of attack.



$$\therefore\ C_L = a_o \alpha_{eff}$$

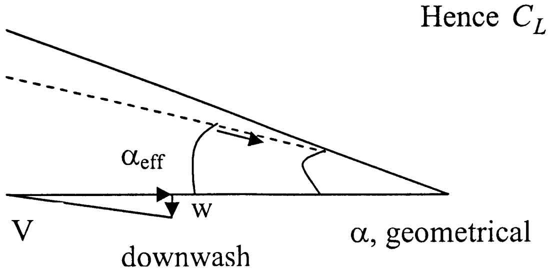
effective angle of attack

some constant = 2π for 2D
thin airfoil theory

For a finite wing the effective angle of attack depends on the downwash,

$\therefore \alpha_{eff} = \alpha - w/V$

Hence $C_L = a_o(\alpha - w/V)$

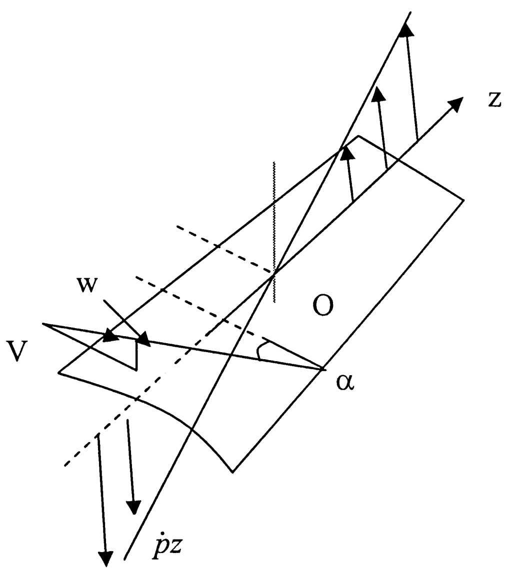


The local lift and local circulation are related by

$$L(z) = \rho V\Gamma(z) \therefore C_L = \rho V\Gamma \Big/ \frac{1}{2}\rho V^2 c = 2\Gamma\!\Big/_{Vc}$$

$$\therefore \Gamma(z) = \frac{1}{2}Vca_o\left(\alpha - \frac{w(z)}{V}\right)$$

(b) Rolling changes the effective angle of attack:



So $\alpha_{eff} = \alpha - \dfrac{w}{V} - \dfrac{\dot{p}z}{V}$

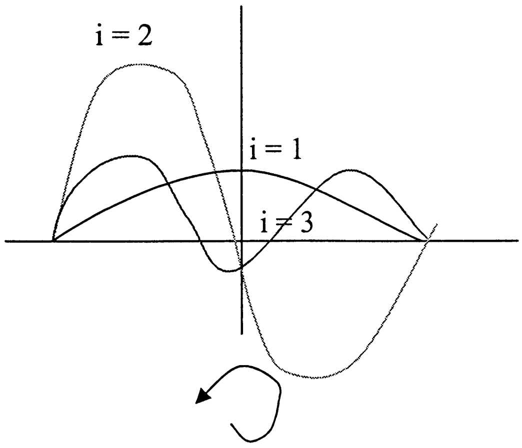
$$\therefore \Gamma(z) = \frac{1}{2}Vca_o\left(\alpha - \frac{w}{V} - \frac{\dot{p}z}{V}\right)$$

(c) In terms of drag, for a general distribution of loading

$\Gamma(\phi) = 2bV\sum_{1}^{n} A_i \sin i\phi$ where $z = -\dfrac{b}{2}\cos\phi$, the induced drag is

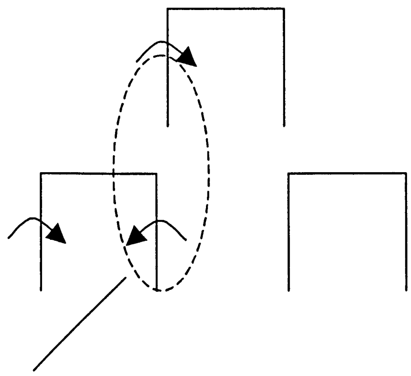
$$C_{D_i} = \frac{C_L^2}{\pi A_R}\left(1 + \sum_{2}^{n} i\frac{A_i^2}{A_1^2}\right)$$

For elliptic loading, i = 1 only, this drag is at a minimum. A rolling wing must have even order harmonics also present



Thus $\sum_{c}^{n} i\, A_i^2 / A_1^2$ must be non-zero $\Rightarrow$ more drag

8 (a)



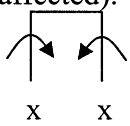
opposite sign vorticity might be expected to cancel; this would reduce the drag as the shed circulation represents secondary kinetic energy which dissipates into entropy $\Rightarrow$ drag.

(b) In practice there are a number of issues

- the vortex starts to dissipate the moment it leaves the wing tip so some potential drag will already be manifested.
- vorticity is actually shed across the whole Span rather than just at wing tips so there is less opportunity for beneficial interaction.
- opposite sign vortices do not necessarily combine and cancel; they can co-exist in a stable manner rotating around each other.

- aircraft control would need to be very accurate (and resilient to unknown atmospheric pertubations).

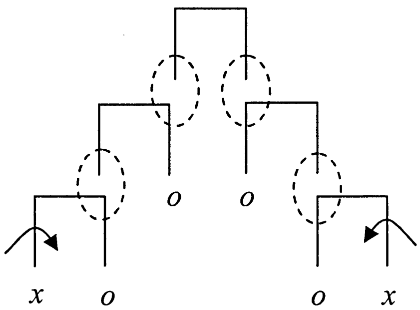
(c) The induced drag $\sim L^2 \sim \Gamma^2$ is likely to be half the total drag for an optimised aircraft; this might be reduced by formation flying (the other half, the friction drag will not be affected).



$$D = 2(X + F) = 4X$$

friction
induced

equal for optimised craft
$C_D = C_{D_o} + kC_L^2$



$$D = 5\text{x}2F + 2X = 12X \Rightarrow D/_{\text{craft}} = \frac{12}{5}X \; cf. \; 4X$$

$$\therefore \text{reduction} = 1 - \frac{12/5X}{4\text{X}} = 40\%$$

[Note: formation flying is the subject of current NASA funded work – aimed at long distance flight – and actual, real size flight tests show reductions of around 15% <u>may</u> be achievable!]

THE END