

Datasheet: None

Q1 (i)
(a)

$$h = h(T, p)$$

$$\delta h = \left. \frac{\partial h}{\partial T} \right|_p \delta T + \left. \frac{\partial h}{\partial p} \right|_T \delta p \quad (i)$$

Also

$$dh = Tds + vdp$$

$$\left. \frac{\partial h}{\partial p} \right|_T = T \left. \frac{\partial s}{\partial p} \right|_T + v$$

Now

$$\left. \frac{\partial s}{\partial p} \right|_T = - \left. \frac{\partial v}{\partial T} \right|_p$$

$$\therefore \left. \frac{\partial h}{\partial p} \right|_T = - T \left. \frac{\partial v}{\partial T} \right|_p + v$$

From (i) for $\delta T = 0$

$$dh = \left. \frac{\partial h}{\partial p} \right|_T dp = dp \left[v - T \left. \frac{\partial v}{\partial T} \right|_p \right]$$

(ii)

$$v = \frac{RT}{P} - \frac{C}{T^3}$$

$$\left. \frac{\partial v}{\partial T} \right|_p = \frac{R}{P} + \frac{3C}{T^4}$$

$$\Rightarrow \left. T \frac{\partial v}{\partial T} \right|_p = \frac{RT}{P} + \frac{3C}{T^3}$$

$$\Rightarrow v - T \left. \frac{\partial v}{\partial T} \right|_p = - \frac{4C}{T^3}$$

$$\therefore dh = \frac{-4C}{T^3} dp \quad \therefore h_2 - h_1 = \frac{-4C}{T^3} (P_2 - P_1)$$

(2)

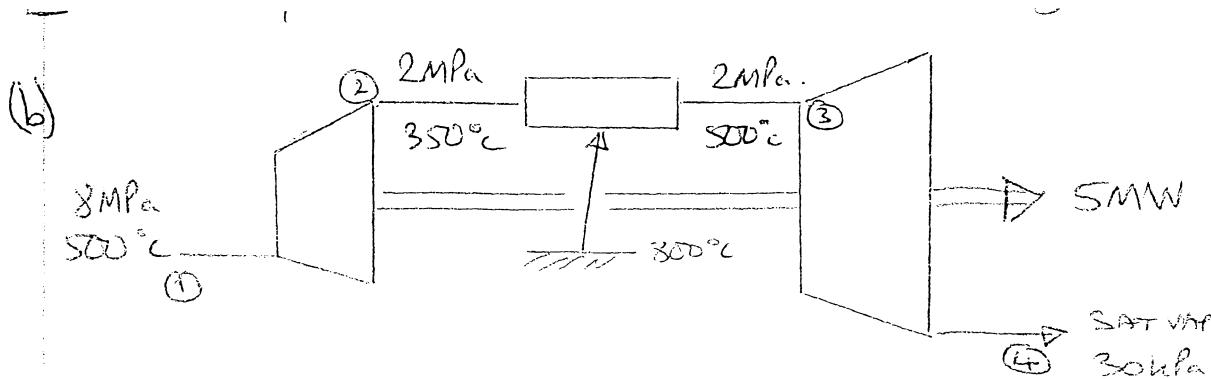
$dT = 0 \therefore$ from Maxwell

$$ds = -\left.\frac{\partial v}{\partial T}\right|_P dp$$

$$= \left[\frac{R}{P} - \frac{3C}{T^4} \right] dp$$

$$\Rightarrow s_2 - s_1 = -R \ln \frac{p_2}{p_1} - \frac{3C}{T^4} (p_2 - p_1)$$

~~Ans~~



(i) SFEE for 2 turbines.

$$\dot{m}[(h_1 - h_2) + (h_3 - h_4)] = \dot{W}_n = 5 \text{ MW}$$

$$\dot{m} = \frac{5 \times 10^3}{(3398.3 - 3137.1) + \left(\frac{(3462.6 - 2628.2)}{8.2} \right) \frac{4.3}{9.81}} = \frac{4.5385}{223} \text{ kg/s}$$

(3)

~~(i)~~ Max power given by

$$\dot{W}_{X_{REV_1}} = m(E_1 - E_2) = m(B_1 - B_2)$$

$$\dot{W}_{X_{REV_2}} = m(E_3 - E_4) = m(B_3 - B_4)$$

where

$$B_1 = h_1 - T_0 s_1 = 3399.5 - 298 \times 6.727 = 1394.85 \\ 1064.22$$

$$B_2 = h_2 - T_0 s_2 = 3137.7 - 298 \times 6.958 = \cancel{1063.52}$$

$$B_3 = h_3 - T_0 s_3 = 3468.2 - 298 \times 7.434 = 1252.87$$

$$B_4 = h_4 - T_0 s_4 = 2624.5 - 298 \times 7.767 = 309.93 \\ 1064.22 \quad 1495.4$$

$$\therefore \dot{W}_{X_{REV_1}} = 4.5228 (1394.85 - \cancel{1063.52}) = \cancel{4478.5} \text{ kW}$$

$$\dot{W}_{X_{REV_2}} = 4.5228 (1252.87 - 309.93) = 4264.7 \text{ kW}$$

$$\text{TOTAL} = \underline{\underline{5763.25}} \text{ kW} \quad \underline{\underline{5760.1}} \text{ kW}$$

(ii) Lost Power in turbines = $\underline{\underline{760.1}}$ ~~1063.25~~ kW

~~(iii)~~ $\dot{E}_2 = \dot{W}_{max} = \left(1 - \frac{T_0}{T_\infty}\right) \dot{Q}$

SFEE across heater gives

$$\dot{Q} = m(h_3 - h_2)$$

$$= 4.5228 (3468.2 - 3137.7) = 1494.78 \text{ kW}$$

$$\therefore \dot{E}_2 = \left(1 - \frac{298}{1073}\right) 1494.78 = \underline{\underline{1079.64}} \text{ kW} \quad (B_1 - B_4)$$

(iii) ~~(iv)~~ $\dot{W}_{max} = \dot{E}_1 + \dot{E}_2 - \dot{E}_4 = m(E_1 - E_4) + \dot{E}_2$

$$= 4.5228 (1394.85 - 309.93) + 1079.64 = \underline{\underline{5986.524}}$$

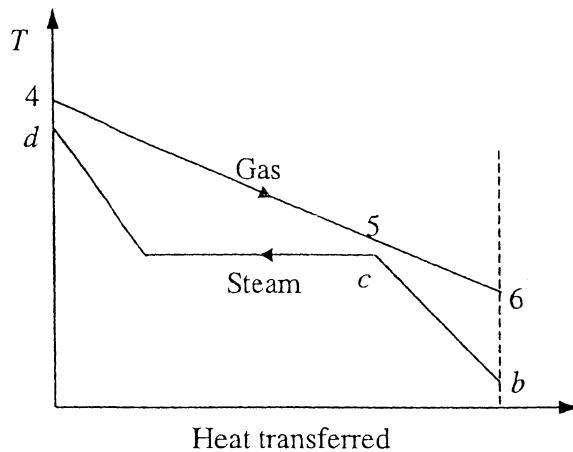
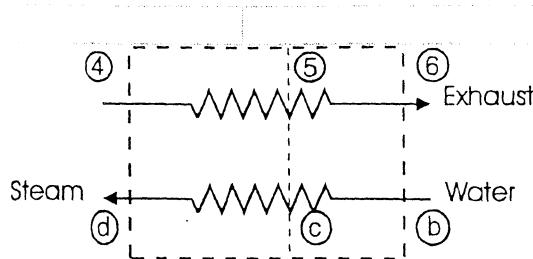
(iv) ~~(v)~~ $\therefore \text{Lost Power} = 9.86 \text{ kW}$

~~(vi)~~ This is higher because of irrev. heat exchange [i.e. excess loss during heat addition] probably due to leak.

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(2)

(a)



- the 'pinch-point' temperature difference $\Delta T_p = (T_5 - T_c)$ must be maintained positive - for a given heat transfer coefficient, ΔT_p controls the physical size of the HRSG and cannot be less than about 10 °C

(b) To find the ratio of mass flows \dot{m}_g / \dot{m}_s in the cycle, we apply the SFE to the hotter part of the HRSG

(i)

$$\dot{m}_s(h_d - h_c) = \dot{m}_g(h_4 - h_5) = \dot{m}_g c_{pg}(T_4 - T_5)$$

where c_{pg} is an average value of c_p for the gas stream. Thus,

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{(h_4 - h_5)}{(h_d - h_c)} = \frac{c_{pg}(T_4 - T_5)}{(h_d - h_c)}$$

From tables:

$$h_c = h_f(40\text{ bar}) = 1087.5 \quad [T_c = 250 - 35^\circ\text{C}]$$

$$h_d = h(550^\circ\text{C}, 40\text{ bar}) = 3560.3$$

$$T_5 = T_c + 25^\circ\text{C} = 275.35^\circ\text{C}$$

$$\therefore \frac{\dot{m}_s}{\dot{m}_g} = 1.15(550 - 275.35) / (3560.3 - 1087.5) = 0.1417$$

The stack temperature can then be obtained by a further application of the SFE to the cooler part of the HRSG,

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$$m_s(h_c - h_b) = m_g(h_5 - h_6) = m_g c_{pg} (T_5 - T_6)$$

giving,

$$T_6 = T_5 - \frac{m_s}{m_g} \frac{(h_c - h_b)}{c_{pg}}$$

From values $h_b \approx h_{sat\text{ liquid}} = 125.7 \text{ kJ/kg}$

$$\therefore T_6 = 275.35 - 0.1416 \left[\frac{1087.5 - 125.7}{1.15} \right] = \underline{\underline{156.86^\circ\text{C}}} //$$

Ideally, the stack temperature T_6 should be as low as possible to reduce

- the exhaust exergy loss
- but also T_6 is limited by
- the 'pinch-point' temperature difference
- the risk of acidic corrosion in the HRSG
 - stack temperature > dew point temperature
 - 80°C for sulphur-free fuels
 - 135°C fuels with sulphur (the dew-point of H₂SO₄ is around 130 °C).
 - typical boiler efficiencies are around 95%.

\therefore this value of T_6 is ok but could be lower

(e) Per kg gas:

(c) Max pos work from steam = $W_{s\max}$

$$\begin{aligned} W_{s\max} &= (E_d - E_b) = (B_d - B_b) \text{ kJ/kg} \\ &= [(h_d - h_b) - T_0(s_d - s_b)] \text{ kJ/kg} \\ &= [(3560.3 - 125.7) - (303)(7.235 - 0.437)] 0.1417 \\ &= \underline{\underline{194.8 \text{ kJ/kg}}} \end{aligned}$$

On gas side:

$$E_4 - E_6 = B_4 - B_6 = (h_4 - h_6) - T_0(s_4 - s_6)$$

But $P_4 = P_6 = P_{\text{atm}}$ (no p-loss) ∴

$$E_4 - E_6 = c_p(T_4 - T_6) - T_0 c_p \ln\left(\frac{T_4}{T_6}\right)$$

$$\begin{aligned} &= 1.15(580 - 156.86) - 303 \cdot 1.15 \ln\left(\frac{580 + 303}{156.86 + 303}\right) \\ &= \underline{\underline{247.81 \text{ kJ/kg}}} \end{aligned}$$

This is equal to max possible work from HRSG

∴ Lost-work in HRSG given by

$$(E_4 - E_6) - (E_d - E_b) = \underline{\underline{53.18 \text{ kJ/kg}}}$$

Reduce this by using multiple pressure levels in steam cycle.

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- 3 (a) The choice of fuel directly affects the amount of CO₂ produced. Natural gas produces about half the CO₂ of coal for the same heat transferred. Oil lies about halfway between these two. Therefore, switching to natural gas reduces emissions

Renewable fuels (e.g. wood or bio fuels) produce no net CO₂. Therefore, using these fuels reduces emissions.

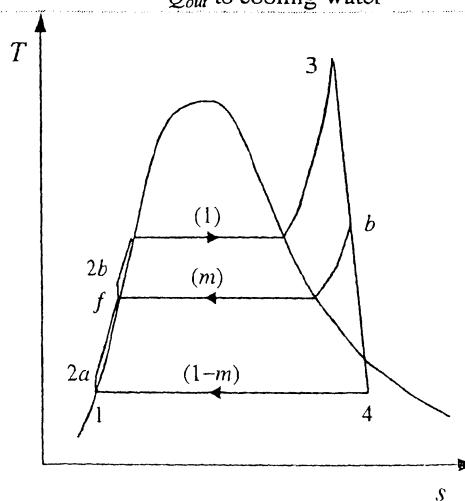
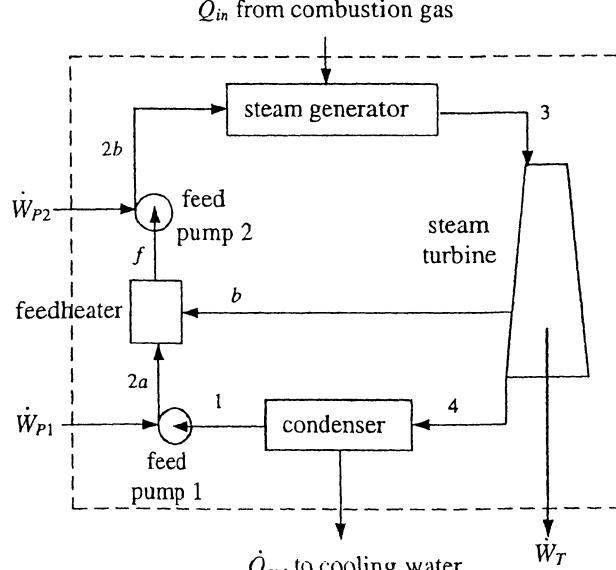
Natural gas (& oil) can be used in CCGTs which have a higher efficiency than gas turbines or steam cycles. This has been a major contribution to the reduction in CO₂ emissions in the UK during the 1990s.

The efficiency of basic gas turbines can be improved using water/steam injection in such as the HAT and STIG cycles. Steam cycles can be improved if supercritical pressures are employed. In both cases, the efficiency will not reach that of existing CCGTs.

Nuclear Power, wind power, hydroelectric power do not produce CO₂ but there are other environmental costs.

CO₂ can be captured and sequestered. This adds cost to the process but is technically viable.

(b) (i)



(i) Neglecting feed pump work & for ~~neg~~ kg bleed steam per kg circulating through boiler SFEF gives

$$m h_b + (1-m) h_1 = h_f \quad h_b = 2685 \text{ kJ/kg K}$$

$$\therefore m = \frac{h_f - h_1}{h_b - h_1} = \underline{0.1495}$$

(ii) Per kg circulating thro' boiler, SFEF gives

$$W_{hp} + W_{lp} = (h_3 - h_b) + (1-m)(h_b - h_4)$$

$$\text{Now, } h_4 = (x h_g + (1-x) h_f)_4 = 0.85(2553.7) + 0.15(121.4) \\ = 2188.86 \text{ kJ/kg}$$

$$S_f = (x s_g + (1-x) s_f)_4 = 0.85(8.473) + 0.15(0.422) \\ = 7.265 \text{ kJ/kg K}$$

From chart (Turbine expansion = straight line)

$$h_b = 2685 \text{ kJ/kg K} \quad s_b = 7.08 \text{ kJ/kg K}$$

$$\therefore W_{hp} + W_{lp} = (3502 - 2685) + (1-0.1495)(2685 - 2189) \\ = 1238.8 \text{ kJ/kg}$$

$$\therefore \text{in boiler} = \frac{500 \times 10^3}{1238.8} = \underline{\underline{404 \text{ kg/s}}}$$

$$(iv) \eta_{\text{turb}} = \frac{\dot{W}_{NET}}{\dot{Q}_{IN}} = \frac{500 \text{ MW}}{\text{in boiler}(h_3 - h_f)} \\ = \frac{500 \times 10^3}{404(3502 - 2685)} = \underline{\underline{41.3\%}} \quad \left\{ \begin{array}{l} \text{with feed} \\ \text{heating} \end{array} \right.$$

Without feed heating,

$$\gamma = \frac{W_{NET}}{Q_{IN}} \approx \frac{h_3 - h_4}{h_3 - h_1} = \frac{3502 - 2188.86}{3502 - 121.4}$$

$$= \underline{\underline{38.8\%}}$$

(f) ~~(e)~~ lost power given by $m T_o \Delta S$ created since no external α or W_A . \therefore

$$\text{Lost Power} = 404 \cdot \frac{(273 + 28.96)}{T_o} (S_f - [(1-m)S_1 + mS_b])$$

$$= 404 \cdot 302 (1.530 - [(0.1495) 0.422 + 0.1495 \cdot 7.08])$$

$$= \underline{\underline{13741 \text{ kW}}}$$

$$\{ \approx 1.10\% Q_{IN} \}.$$

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4 (a) (i) The half-life $T_{1/2}$ is the time at which the number of radioactive atoms present (or equivalently the source activity or RTG power) drops to half its initial value. Hence when $t = T_{1/2}$

$$P = \frac{1}{2} P_0 = P_0 \exp(-\lambda T_{1/2})$$

$$\therefore \ln(\frac{1}{2}) = -\lambda T_{1/2} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda}$$

$$(ii) P_{\text{mean}} = \frac{1}{NT_{1/2}} \int_0^{NT_{1/2}} P dt = \frac{1}{NT_{1/2}} \int_0^{NT_{1/2}} P_0 \exp(-\lambda t) dt$$

$$\therefore P_{\text{mean}} = \frac{P_0}{\lambda NT_{1/2}} [1 - \exp(-\lambda NT_{1/2})] = \frac{P_0}{N \ln 2} [1 - \exp(-N \ln 2)] = \frac{P_0}{N \ln 2} \left[1 - \frac{1}{2^N} \right]$$

(b) The energy in MeV released is 931Δ where Δ is the change in mass (in u). Hence

$$E = 931 \times [209.9829 - 205.9744 - 4.0026] = 931 \times 0.0059 = 5.493 \text{ MeV}$$

$$\therefore E = 5.493 \times 1.6 \times 10^{-13} = 8.789 \times 10^{-13} \text{ J}$$

(c) An α -emitter is a good choice because α -particles are very easy to stop and therefore their energy (which is the majority of that released in the decay reaction) is readily recovered.

Other considerations:

- suitable half-life – too long gives low activity and therefore low power density, too short and the source may not last the mission;
- decay should not produce too much γ -radiation – represents energy loss and, depending on the application, a potential health hazard;
- cost should be reasonable.

(d) (i) If the power after three half-lives is P , the initial power is $2^3 \times P = 8P$, i.e. $8 \times 250 = 2000 \text{ W} = 2 \text{ kW}$.

(ii) The recoverable energy per decay = $5.493 - 0.802 = 4.691 \text{ MeV}$

$$= 4.691 \times 1.6 \times 10^{-13} = 7.506 \times 10^{-13} \text{ J}$$

$$\therefore \text{Initial activity for required power } A_0 = \frac{2 \times 10^3}{7.506 \times 10^{-13}} = 2.665 \times 10^{15} \text{ Bq}$$

$A_0 = \lambda N_0$ where N_0 is the number of ^{210}Po atoms initially present.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{138.4 \times 24 \times 3600} = 5.797 \times 10^{-8} \text{ s}^{-1}$$

$$\therefore N_0 = \frac{A_0}{\lambda} = \frac{2.665 \times 10^{15}}{5.797 \times 10^{-8}} = 4.597 \times 10^{22} \text{ atoms}$$

\therefore The mass of ^{210}Po initially present

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$$M_0 = \frac{4.597 \times 10^{22} \text{ atoms}}{6.022 \times 10^{26} \text{ atoms kmol}^{-1}} \times 210 \text{ kg kmol}^{-1} = 0.016 \text{ kg}$$

(e) Over the mission power drops to $1/8^{\text{th}}$ its initial value $\Rightarrow 7/8^{\text{th}}$ of the ^{210}Po atoms initially present decay.

$$\therefore \text{Number of decays} = \frac{7}{8} N_0 = \frac{7}{8} \times 4.597 \times 10^{22} = 4.022 \times 10^{22}$$

This is the number of ^4He atoms released.

$$\therefore \text{Number of mols of } ^4\text{He released} = \frac{4.022 \times 10^{22} \text{ atoms}}{6.022 \times 10^{23} \text{ atoms mol}^{-1}} = 0.0668 \text{ mols}$$

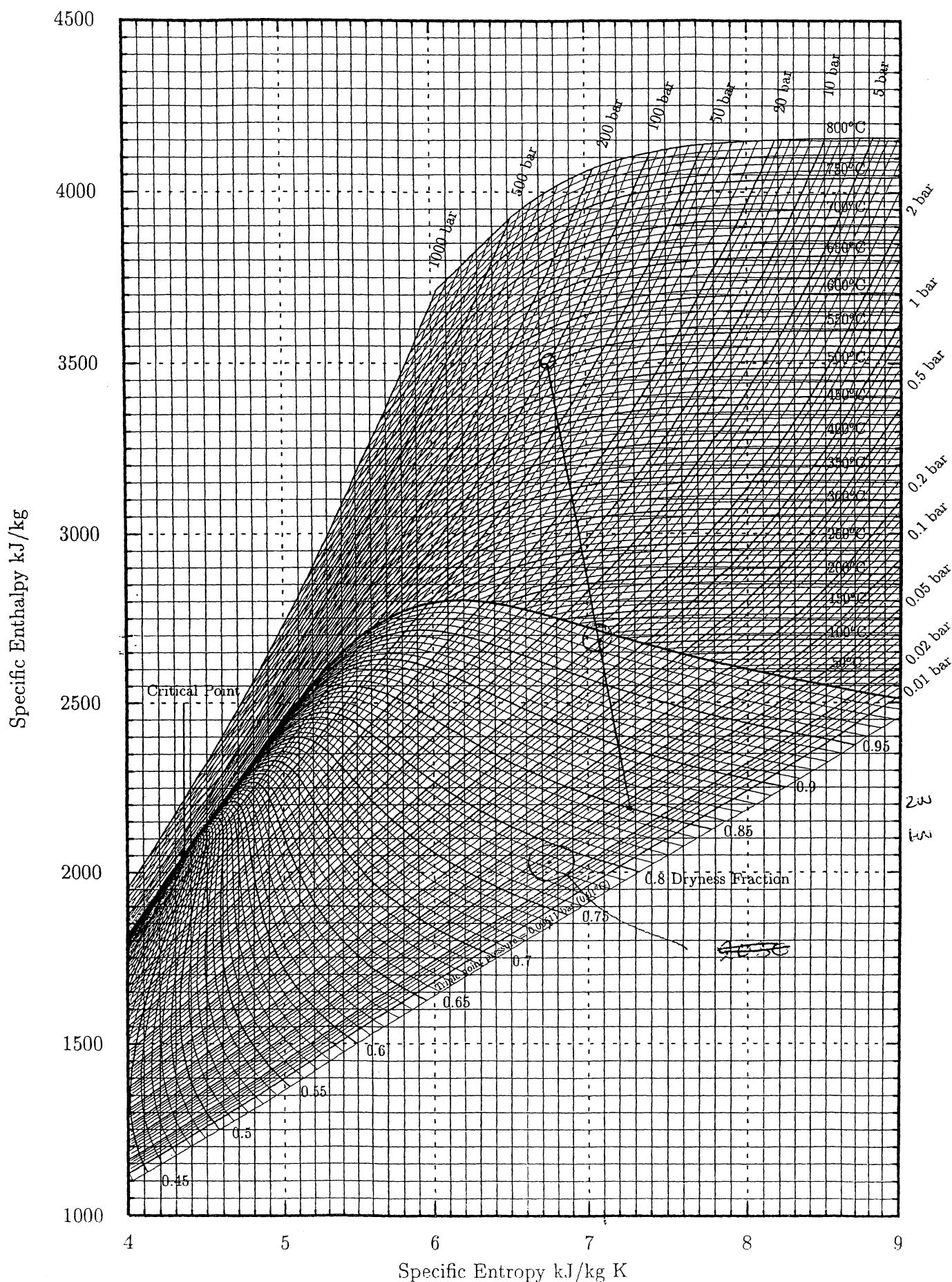
Using the perfect gas equation $pV = n\bar{R}T$

$$\therefore V = \frac{n\bar{R}T}{p} = \frac{0.067 \times 8.314 \times 523}{100 \times 10^5} = 2.904 \times 10^{-5} \text{ m}^3$$

Enthalpy-Entropy Diagram for Steam

Plotted from the IAPWS equations <http://www.iapws.org> (Duncan A. Simpson (2002))

(11)



Part 3A5- Energy and Power Generation

5 (a) Flammability limits are the limits in equivalence ratio (given also in terms of fuel to air ratio) between which a mixture can sustain a self-propagating flame. The lean limit is the smallest possible fuel to air ratio. The existence of the flammability limit is due to the low temperature as the mixture becomes leaner, which leads to a dominance of chain-terminating reactions over chain-propagating ones. A rule of thumb is that this happens at around 1500 K for hydrocarbon flames at atmospheric pressure.

See Lecture Notes, Chapter 2 (for a discussion on the reactions) and Lecture 6 (for a definition of flammability). See also experimental handout (which I tell the students is examinable material).

(b) Energy balance for the reactants from the inlet to just before the flame, and for the flame products from the flame to the exit, gives:

$$(1) \quad \dot{m}c_p(T_R - T_{in}) = \dot{m}c_p(T_f - T_{out})$$

Energy balance across the flame is

$$(2) \quad \dot{m}c_p(T_f - T_R) = \dot{m}Y_{fuel}Q$$

With $T_{out}=1000$ K and $T_f=1600$ K, Eq. (1) gives that $T_R=900$ K. Then, Eq. (2) gives that the fuel mass fraction in the reactants is $1.2 \times (1600-900)/50000 = 0.0168$.

Equivalence ratio $\phi = \frac{Y_{fuel}/(1-Y_{fuel})}{Y_{fuel,st}/(1-Y_{fuel})_{st}}$. From the complete combustion equation for methane $CH_4 + \frac{2}{0.21}(0.21O_2 + 0.79N_2) = CO_2 + 2H_2O + \frac{2 \times 0.79}{0.21}N_2$, we get that $Y_{fuel,st} = \frac{16}{2 \times 32 + 2 \times 79/21 \times 28} = 0.0583 = \frac{1}{17.167}$.

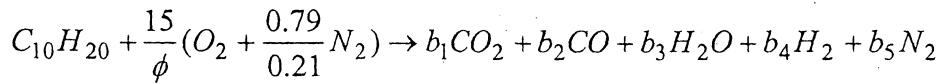
Hence, the leanest equivalence ratio of operation before the flame temperature drops below 1600 K is 0.293.

(c) At these low temperatures, we do not expect thermal NO to be created. Since we are burning methane, we do not expect fuel-bound NO. Hence the only NO that can appear is the prompt NO (Lecture 2).

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6 (a) In the flame, chemical kinetics dominates so there is no equilibrium. After the flame, equilibrium will be approached relatively slowly.

(b) The combustion reaction is



The atom conservation equations become:

$$\text{For O: } 30/\phi = 2b_1 + b_2 + b_3$$

$$\text{For C: } 10 = b_1 + b_2$$

$$\text{For H: } 20 = 2b_3 + 2b_4$$

$$\text{For N: } \frac{2 \cdot 0.79}{\phi \cdot 0.21} = b_5$$

We have five unknowns and four equations, so we need one equilibrium reaction. As this is rich combustion, we take the water gas shift reaction $CO + H_2O \rightleftharpoons CO_2 + H_2$ (Reaction 8 in Thermofluids Tables). Hence

$$K_p = \frac{(P_{CO_2}/P^0)(P_{H_2}/P^0)}{(P_{CO}/P^0)(P_{H_2O}/P^0)} = \frac{(b_1/n)(b_4/n)}{(b_2/n)(b_3/n)} \frac{(P/P^0)(P/P^0)}{(P/P^0)(P/P^0)} = \frac{b_1 b_4}{b_2 b_3}$$

From Tables, $\ln(K_p) = -1.091$ (at 1600 K), hence $K_p = 0.33588$. Algebraic manipulation of the conservation equations for $\phi=2.5$ and substitution in the K_p formula gives:

$$K_p = \frac{(b_4 - 8)b_4}{(18 - b_4)(10 - b_4)}$$

The final solutions are:

$$b_1 = 0.5421$$

$$b_2 = 9.4579$$

$$b_3 = 1.4579$$

$$b_4 = 8.5421$$

$$b_5 = 22.5714$$

Using $n = b_1 + b_2 + b_3 + b_4 + b_5$, the mole fractions ($X_i = b_i/n$) can be calculated:

$$X_{CO_2} = 0.0127$$

$$X_{CO} = 0.2221$$

$$X_{H_2O} = 0.0342$$

$$X_{H_2} = 0.2006$$

$$X_{N_2} = 0.5302$$

Which are equal to the volume fractions

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- (c) With steam addition, the equilibrium of the water-gas shift reaction probably shifts towards the right hand side (Le Chatelier rule), hence more H₂ is expected to be produced.
- (d) No. Since $\sum v_i = 0$ for the reaction yielding the K_p relation, there is no impact of the pressure in both cases.

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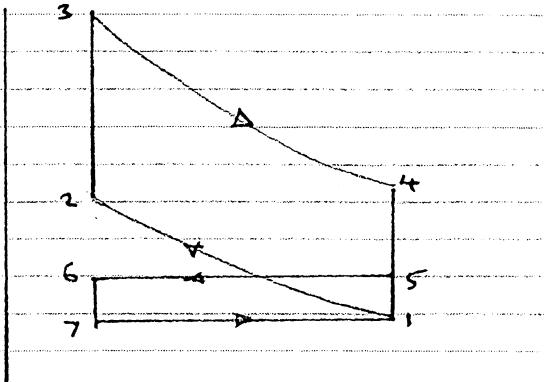
(a) & (b)

2

~~(iii)~~

$$\eta = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Neglecting pumping loop:



$$Q_{in} = mc_v(T_3 - T_2)$$

$$Q_{out} = mc_v(T_4 - T_1)$$

$$\therefore \eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4(1 - \frac{T_1}{T_4})}{T_3(1 - \frac{T_2}{T_3})}$$

$$T^{r-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\underline{\underline{\eta = 1 - \frac{1}{T^{r-1}}}}$$

(c) ~~(i)~~ Let $i_{imp} = \bar{P}$ where $\bar{P} V_{swept} = W_{net}$

$$\begin{aligned} W_{net} &= Q_{in} - Q_{out} = m_f Q_f - m_c v(T_4 - T_1) \\ &= \bar{P}(V_1 - V_2) \end{aligned}$$

$$\therefore \bar{P} = \frac{m_f Q_f - m_c v(T_4 - T_1)}{(V_1 - V_2)}$$

Now,

$$(V_1 - V_2) = V_1(1 - \frac{V_2}{V_1}) = V_1(1 - \frac{V_r}{V_1}) = V_1 \underline{\underline{r-1}}$$

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$$\therefore \bar{P} = \frac{m_f Q_f - m_c v T_1 \left(\frac{T_4}{T_1} - 1 \right)}{v_1} (r)$$

$$v_1 = \frac{m R T_1}{P_1}$$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_2} \frac{T_2}{T_1} = \frac{1}{r^{y-1}} \frac{T_3}{T_2} r^{y-1} = \frac{T_3}{T_2}$$

Now

$$m_f Q_f = m c v (T_3 - T_2)$$

$$\therefore \frac{m_f Q_f}{m c v T_2} = \left(\frac{T_3}{T_2} - 1 \right) = \frac{m_f Q_f}{m c v T_1 r^{y-1}} \left(\frac{T_2}{T_1} = r^y \right)$$

$$\therefore \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1 = \frac{m_f Q_f}{m c v T_1 r^{y-1}}$$

$$\therefore \bar{P} = \frac{m_f Q_f - m c v T_1 \left[\frac{m_f Q_f}{m c v T_1 r^{y-1}} \right]}{m R T_1} (r)$$

$$R = (r-1) c_v$$

$$= P_1 \left(\frac{r}{r-1} \right) \frac{1}{y-1} \left[\frac{m_f Q_f}{m c v T_1} - \left[\frac{m_f Q_f}{m c v T_1 r^{y-1}} \right] \right]$$

$$= P_1 \left(\frac{r}{r-1} \right) \frac{1}{y-1} \frac{m_f Q_f}{m c v T_1} \left[1 - \frac{1}{r^{y-1}} \right]$$

$$(d) \quad r = 9 \quad P_1 = P_i = 0.9 \text{ bar} \quad T_1 = 310 \text{ K}$$

$$\frac{m_f}{m} = \frac{1}{14.7} \quad Q_f = 414 \times 10^6 \text{ J/kg} \quad Y = 1.4$$

$$c_v = 0.75$$

(17)

$$\eta = 1 - \frac{1}{\gamma^{f-1}} = 1 - \frac{1}{9^{0.4}} = 58.5\%$$

$$\bar{P} = \text{imep} = 0.9 \times 10^5 \cdot \frac{9}{8} \cdot \frac{1}{0.4} \cdot \frac{44 \times 10^6}{15.7 \times 0.75 \times 10^3 \times 310} \times 0.58 \\ = 17.84 \text{ MPa. //}$$

Part 3A5- Energy and Power Generation

(c) ENGINE SPEED AFFECTS HEAT TRANSFER AND ENGINE BREATHING
PRIMARILY.

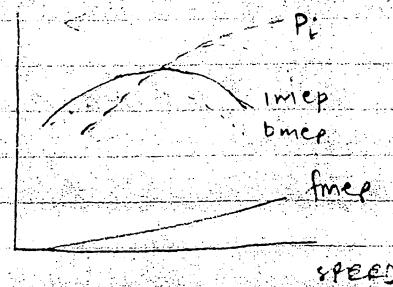
FOR SI ENGINES, HEAT TRANSFER PER UNIT CHARGE DECREASES AS SPEED INCREASES, THIS INCREASING IMEP. HOWEVER, ENGINE BREATHING IS AFFECTED, AND THE VOLUMETRIC EFFICIENCY IS LOWERED AS SPEED INCREASES, THIS DECREASING THE EFFECTIVE CHARGE. THE NET EFFECT IS TO CREATE A PEAK IMEP AT SOME SPEED.

FOR CI ENGINES, ESPECIALLY TURBOCHARGED ENGINES, HEAT TRANSFER PER UNIT CHARGE ALSO DECREASES, BUT THE EFFECT IS NOT AS ENHANCED AS AVENUE. EXHAUST TEMPERATURES ARE LOWERED. BREATHING IS ALSO NOT SO AFFECTED, AS THE ENGINE IS NOT TURBED. THE NET RESULT IS A FLATTER IMEP CURVE WITH SPEED, BUT ALONG A PEAK.

FRICION INCREASES WITH SPEED, SO THAT f_{mech} IS A MONOTONICALLY INCREASING FUNCTION, IN BOTH CASES.

ENGINE POWER IS PROPORTIONAL TO $IMEP \times SPEED$, THE CORRESPONDING EFFECT OF SPEED ON POWER IS TO INCREASE THE INDICATED + BRAKE POWER UP TO A PEAK ATTAINABLE GREATER THAN FOR PEAK IMEP AND BMEP.

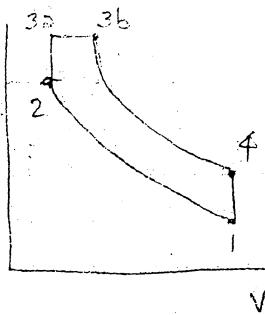
FRICION POWER INCREASES WITH SPEED, AND SO DOES b_{fric} .



Part 3A5- Energy and Power Generation

8 (i)

(a)



$$W_C = mC_V(T_1 - T_2)$$

$$W_E = m [C_V(T_{3b} - T_4) + p_3(u_{3b} - u_{3a})]$$

$$Q = m[C_V(T_{3a} - T_2) + C_p(T_{3b} - T_{3a})]$$

$$\underline{W = m \{ C_V [(T_1 - T_2) + (T_{3b} - T_4)] + R(T_{3b} - T_{3a}) \}}$$

$$\underline{W = m \{ C_V [(T_1 - T_2) + (T_{3b} - T_4)] + C_p(T_{3b} - T_{3a}) - C_V(T_{3b} - T_{3a}) \}}$$

$$= m [C_V(T_1 - T_2) + \underbrace{C_V(T_{3a} - T_2)}_{\textcircled{1}} + C_p(T_{3b} - T_{3a})]$$

$$\frac{W}{Q} = 1 - \frac{(T_4 - T_1)}{T_{3a} - T_2 + \gamma(T_{3b} - T_{3a})}$$

$$\frac{T_2}{T_1} = r^{\gamma-1}$$

$$\frac{T_{3b}}{T_{3a}} = \frac{u_{3b}}{u_{3a}} = \beta$$

$$\frac{T_{3a}}{T_2} = \frac{p_3}{p_2} = \alpha$$

$$\frac{T_4}{T_{3b}} = \left(\frac{u_{3b}}{u_4} \right)^{\gamma-1} = \left[\left(\frac{u_{3b}}{u_{3a}} \right) \left(\frac{u_{3a}}{u_4} \right) \right]^{\gamma-1} = \left(\frac{\beta}{r} \right)^{\gamma-1}$$

$$\frac{W}{Q} = 1 - (\beta/r)^{\gamma-1} (\beta)(\alpha)^{\gamma-1} / \gamma$$

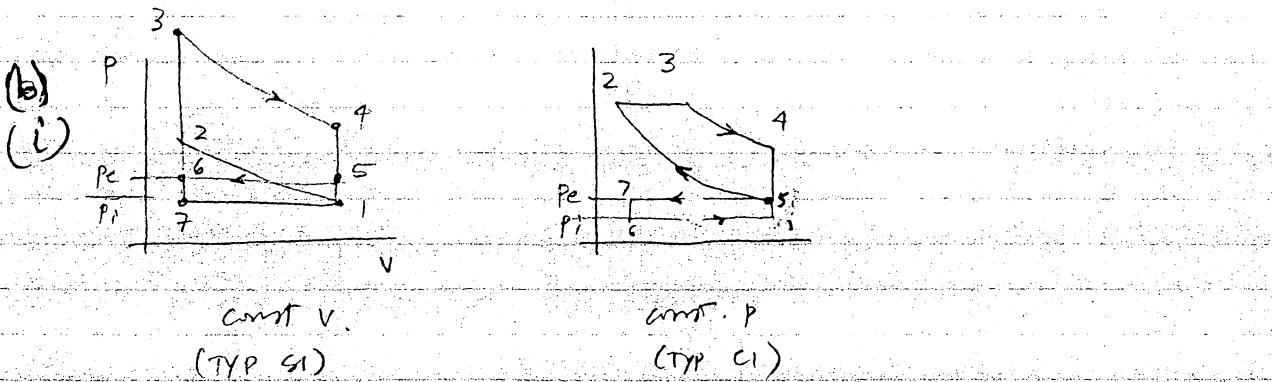
$$\frac{W}{Q} = \frac{1 - (\beta/r)^{\gamma-1} (\beta)(\alpha)^{\gamma-1}}{\gamma^{\gamma-1}(\alpha-1) + \gamma^{\gamma-1}\alpha(\beta-1)}$$

$$\boxed{\eta = 1 - \frac{1 - (\beta/r)^{\gamma-1} (\beta)(\alpha)^{\gamma-1}}{\gamma^{\gamma-1}(\alpha-1) + \gamma^{\gamma-1}\alpha(\beta-1)}}$$

Part 3A5- Energy and Power Generation

(ii)

C.I. ENGINES OPERATE AT HIGHER COMPRESSION RATIOS THAN SI ENGINES. (TYPICALLY 20 VS. 10 FOR SI ENGINES). THEY ALSO OPERATE UNTHROTTLED, THIS AVOIDING PUMPING LOSSES. A SMALLER ADVANTAGE COMES FROM OPERATING FILTER USED, WHICH INCREASES T.



$$(i) \quad x_r = \frac{m_r}{m} = \frac{v_c/v_5}{v_c/v_2} = \frac{v_2}{v_5} = \frac{v_4}{v_5} \frac{1}{r}$$

$$\frac{v_4}{v_5} = \left(\frac{p_5}{p_4}\right)^{1/r} = \left(\frac{p_e}{p_4}\right)^{1/r} \quad (\text{ISENTROPIC EXPANSION OF TRAPPED MASS})$$

$$x_r = \frac{1}{r} \left(\frac{p_e}{p_4}\right)^{1/r} = \frac{1}{r} \left(\frac{p_e}{p_1}\right)^{1/r} \left(\frac{p_1}{p_4}\right)^{1/r}$$

$$\frac{p_4}{p_1} = \frac{p_e}{p_1} = \frac{p_4}{p_3} \frac{p_3}{p_2} \frac{p_2}{p_1}$$

$$\text{CONST. VOLUME: } \frac{p_4}{p_1} = \frac{1}{r^{n-1}} \left(\frac{T_3}{T_2}\right)^r = \frac{T_3}{T_2}$$

$$\text{CONST. PRESSURE: } \frac{p_4}{p_1} = \left(\frac{v_4}{v_3}\right)^{-r} (1)^{r-n} = \left(\frac{v_4}{v_2} \frac{v_2}{v_3}\right)^{-r} = r^{-r} \left(\frac{T_2}{T_3}\right)^{r-n} = \left(\frac{T_3}{T_2}\right)^{r-n}$$

$$\text{CONST. VOLUME HEAT RELEASE: } m_C_V (T_3 - T_2) = Q$$

$$\left(\frac{T_3}{T_2} - 1\right) = \frac{Q}{m C_V T_2} = \frac{Q}{m C_V T_1 r^{n-1}} = \frac{q^*}{r^{n-1}}$$

$$\text{CONST. PRESSURE HEAT RELEASE: } m C_p (T_3 - T_2) = Q$$

$$\left(\frac{T_3}{T_2} - 1\right) = \frac{Q}{m C_p T_2} = \frac{Q}{m C_V T_1 r^{n-1} \frac{C_V}{C_p}} = \frac{q^*}{r^{n-1}}$$

Part 3A5- Energy and Power Generation

Finally:

$$\text{ad. volume: } x_r = \frac{1}{r} \left(\frac{p_e}{p_i} \right)^{\frac{1}{k}} \left(1 + \frac{q^*}{r^{k-1}} \right)^{-\frac{1}{k}}$$

$$\text{corr. pressure: } x_r = \frac{1}{r} \left(\frac{p_e}{p_i} \right)^{\frac{1}{k}} \left(1 + \frac{q^*}{r^{k-1}} \right)^{-1}$$

((b) (ii)) REACTOR GAS FRACTION INCREASES WITH THE RATIO OF EXHAUST AND INLET PRESSURES, AND DECREASES WITH INCREASING COMPRESSION RATIO, WITH A WEAKER DEPENDENCE ON THEIR RELATIVE RATE.

Therefore, SI ENGINES, OPERATING AT HIGH AND LOWER COMPRESSION RATIOS (p_e/p_i can be of the order of 3 and r of the order of 10) TYPICALLY YIELD THE HIGHEST RADIUM,

CI ENGINES OPERATE UNTHROTLED, WITH $p_e = p_i$; FOR NM-TURBO CHARGED ENGINES, OR $p_e > p_i$ FOR TURBOCHARGED ENGINES, THE COMPRESSION RATIOS ARE VERY HIGH (~ 20) AND RADIUMS ARE VERY SMALL.

THE HEAT RELEASED MULTIPLIER DOES NOT AFFECT RATE CONCLUSIONS MUCH, EVEN THOUGH THE HEAT RELEASE PER UNIT MASS IN THE CONSTANT PRESSURE CI ENGINE IS LOWER THAN IN SI ENGINES, THIS LEADING TO A LOWER FINAL TEMPERATURE AND HIGHER DENSITY.

RADIUMS ADD INERT CHARGE, AFFECTING THE HEAT RELEASE PER UNIT MASS, THIS LOWERING PEAK TEMPERATURES AND INCREASING THE P AND POWER DENSITY.

Part 3A5- Energy and Power Generation

~~(c)~~ THE MAIN DIFFERENCES BETWEEN THE IDEAL AND ACTUAL CYCLES ARE :

- HEAT TRANSFER - INCREASES TEMPERATURES AND PRESSURES REPEATED, PARTICULARLY AFTER END OF COMBUSTION
- FINITE COMBUSTION TIME - INSTEAD OF INSTANTANEOUS COMBUSTION, (STAY AT CONSTANT VOLUME), COMBUSTION RATE LIMITS RELEASE TO A RANG OF CRANKANGLE DEGREES, OVER A RANGE OF PRESSURES AND VOLUMES,
- FINITE RATE OF GAS EXCHANGE - WHILE THE IDEALIZED INSTANTANEOUS VALVE OPENING AND CLOSING EVENTS, CUTTING PREBURN AND EXHAUST RELEASE PHASES PLACE OVER A RANGE OF CRANKANGLE DEGREES, IN ADDITION, VOLUMETRIC EFFICIENCY IS A FUNCTION OF OPERATING CONDITIONS.
- REAL GAS PROPERTIES - REAL GASES HAVE SPECIFIC HEATS WHICH VARY WITH TEMPERATURE AND COMPOSITION RATHER THAN A CONSTANT
- CLEARANCE EFFECT AND CHARGE SWASHAGE - GENERALLY UP RECIPROCATING ENGINE IS SUBJECT TO CHARGE SWASH, OR RECIPROCATING IN CRANKCASES.

*Principal Assessor: Prof H P Hodson/ Dr N
Answers Swaminathan*

1. (a)(ii) $h_2 - h_1 = -\frac{4C}{T^3} (p_2 - p_1), \quad s_2 - s_1 = -R \ln \left(\frac{p_2}{p_1} \right) - \frac{3C}{T^4} (p_2 - p_1)$

(b)(i) 4.523 kg/s, 1495.4 kw, 4264.7 kw

(ii) 760.1 kw, 1079.6 kw

(iii) 5.986 Mw

(iv) 986 kw - high because of exergy loss during heat addition

2 (b)(i) 0.142

(ii) 156.85°C

(c) 194.8 kJ/kg 53.18 kJ/kg - use multiple pressure levels

3 (b)(ii) 0.1495

(iii) 404 kg/s

(iv) 41.3% with feed heating and 38.8% without feed heating

(v) 1.374 Mw 1.1%

4 (a)(i) $T_{1/2} = \frac{\ln 2}{\lambda}$

(b) 8.789×10^{-13} J

(d)(i) 2.0 kw

(ii) 0.016 kg

(e) 2.904×10^{-5} m³

5 (b) 0.293

6 (b) $X_{CO_2} = 0.0127, X_{CO} = 0.2221, X_{H_2O} = 0.0342, X_{H_2} = 0.2006, X_{N_2} = 0.5302$

7 (d) 58.5%, 17.84 MPa

*Swaminathan
26 May 2005*