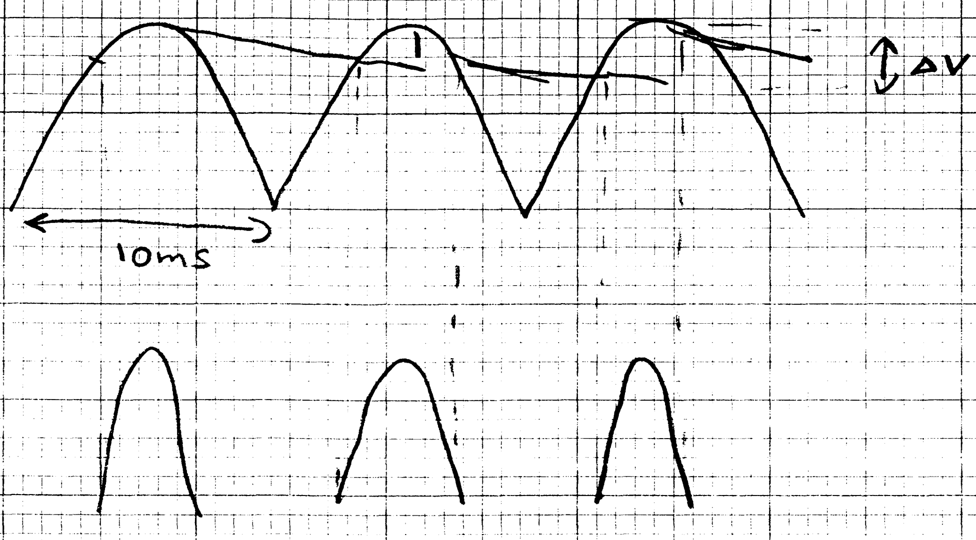
PART IIA 2005
3B3: Switch-mode electronics
Principal Assessor: Prof. A J Amaratunga

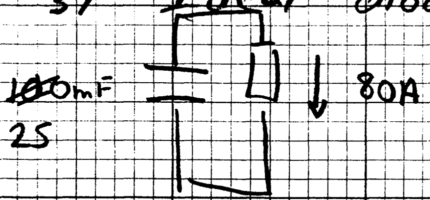
Datasheet: None

Question 1 CRIB



For Approximate Solution assume

1/ charge time negligable
2/ Linear capacitor discharge
3/ Ideal diodes



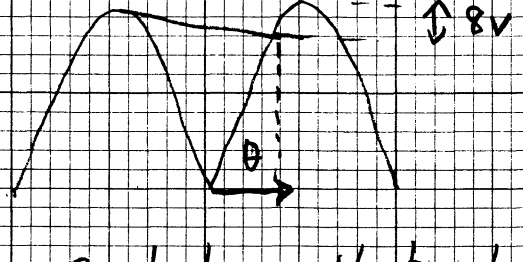
$\Delta T = 10\,mS$

$$I = C\frac{dv}{dt} = C\frac{\Delta V}{\Delta T}$$

$$\Rightarrow \Delta V = \frac{80 \times 10 \times 10^{-3}}{25\ \cancel{100} \times 10^{-3}}$$

$$= 32v$$

Conduction Angle :

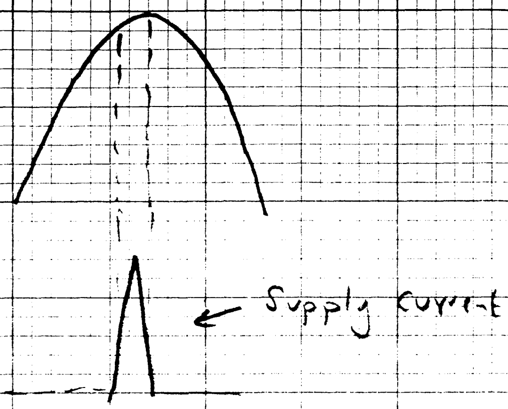


Assume conduction starts at $V = 240 \times \sqrt{2}\ \sin\omega t = 240\sqrt{2} - 32$

$$\Rightarrow \sin\omega t = \frac{240\sqrt{2} - 32}{240\sqrt{2}} = \frac{307}{339} = 0.904$$

Question 1 Crib

$\Rightarrow \omega t = 72{\cdot}1^\circ$



$$i = C\frac{dv_c}{dt} + i_{LOAD}$$

$$= 25\times10^{-3}\ \frac{d}{dt}\,339\sin\omega t + i_{LOAD}$$

Peak current

$$= 25\times10^{-3}\times 339\times\omega\cos\omega t + i_{LOAD}$$

$$= 25\times10^{-3}\times 339\times 100\pi\times\cos 83{\cdot}5 + 80$$

$$= 2662\times$$

$$= 301 + 80\,\text{A} = 381\ \text{A}$$

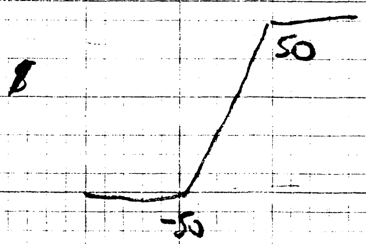
Question 1b Crib

Commutation Angle 0 and 30°

Commutation time

$$V = L\frac{di}{dt}$$

$$\Rightarrow i_1 - i_2 = \frac{1}{L}\int v\,dt$$



$$100 = \frac{50}{L}\int_{t_1}^{t_2} \sin\omega t\,dt$$

$$100 = \frac{50}{\omega L}\Big[-\cos\omega t\Big]_{t_1}^{t_2}$$

$$100 = \frac{50}{100\pi \times 2\times 10^{-4}}\left(\cos\omega t_1 - \cos\omega t_2\right)$$

$\alpha_1 = 0$

$$100 = \frac{50}{100\pi \times 2\times 10^{-4}}\left(1 - \cos\omega t_2\right)$$

$$\cos\omega t_2 = (1 - 0.04\pi) = 0.874$$

$$\omega t_2 = 29^\circ = 0.506 \text{ Rad}$$

$$\approx 30^\circ$$

$\alpha_1 = 30^\circ$ $\left(\omega t_1 = \frac{30}{180}\times\pi\right)$

$$100 = \frac{50}{\pi\, 0.02}\left(0.866 - \cos\omega t_2\right)$$

$$\cos\omega t_2 = (0.866 - 0.04\pi) = 0.740$$

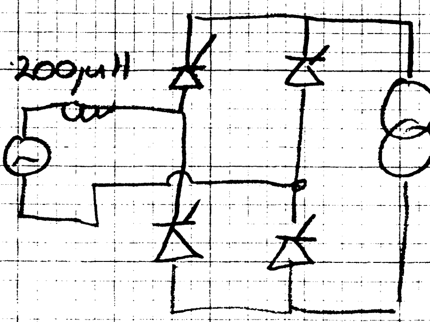
$$\omega t_2 = 42^\circ$$

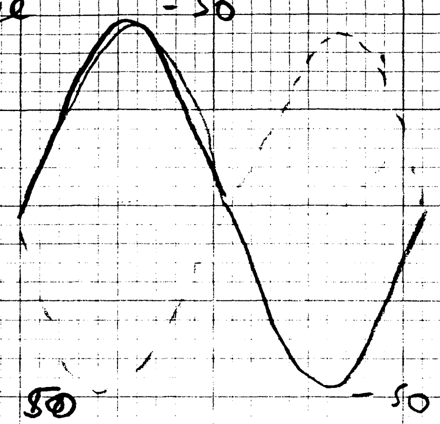
commutation angle = 12°

Question 1 CRIB

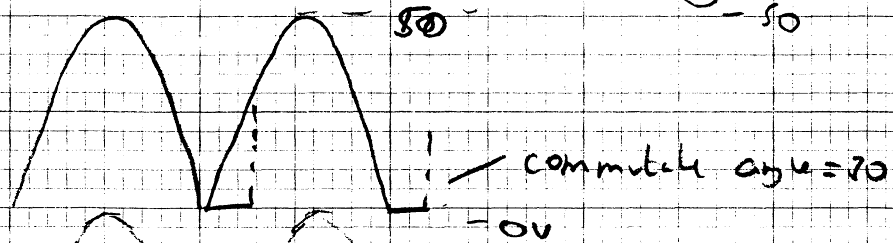
Question 1b

Two Phase Thyristor bridge

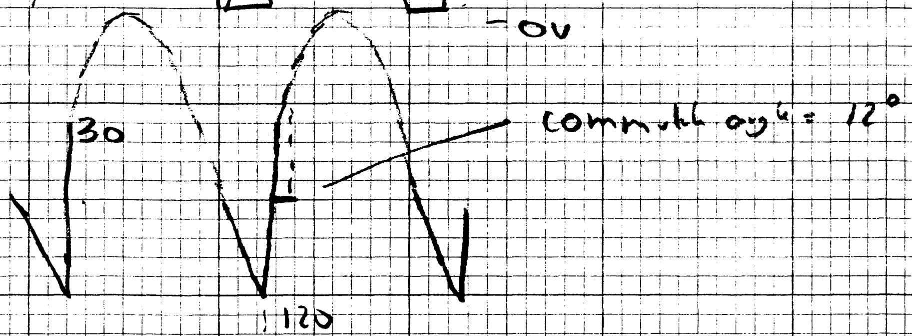




0° delay



30° delay



Output Voltage at $0° = \dfrac{1}{\pi}\displaystyle\int_0^{\pi} V_p \sin \omega t \, d(\omega t)$

$$= \frac{50}{\pi}\Big[\cos(\omega t)\Big]_0^{\pi} = \frac{100}{\pi} = 31.8V$$

Output Voltage at $30° =$

$$V_{ave} = \frac{50}{\pi}\int_{\cancel{30}\pi/6}^{\cancel{120}\,4\pi/6} \sin \omega t \, d\omega t$$

$$= \frac{50}{\pi}\Big[\cos \omega t\Big]_{\pi/6}^{4\pi/6}$$

$$= \frac{50}{\pi}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \cancel{68V}\ 21.7V$$

P1.6

In the commutation interval the output is zero.

Adjust for $\alpha = 0°$

$$\Rightarrow \quad V_0 = \frac{50}{\pi}\left[\cos \omega t\right]_{12}^{\pi}$$

0°

$$\frac{50}{\pi}\left(0.866 + 1\right) = 29.7V$$

Adjust for 30° Firing angle

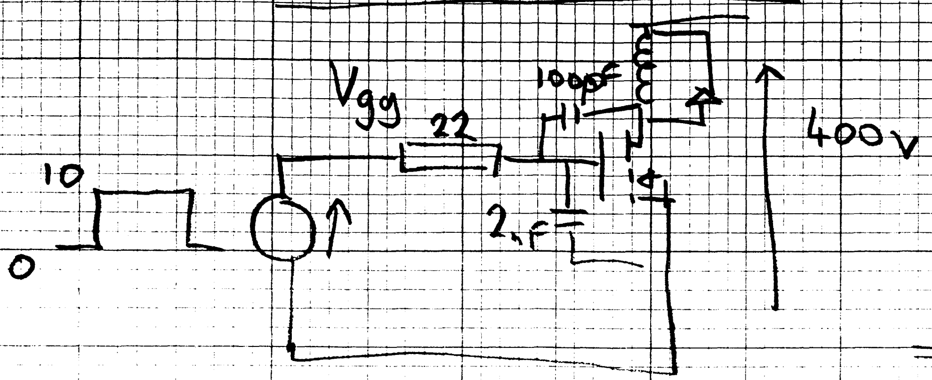
$$V_0 = \frac{50}{\pi}\left[\cos 42° - \cos 120°\right]$$

30°

$$= \frac{50}{\pi}\left[0.740 + 0.5\right] = 19.7V$$

Maximum Firing angle $\approx$ 150°

Question 2 Crib

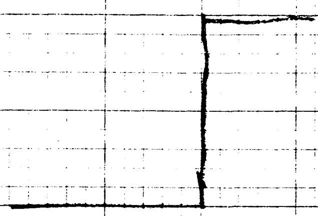


24 Ω load

$\Rightarrow$ load voltage $= 240$v

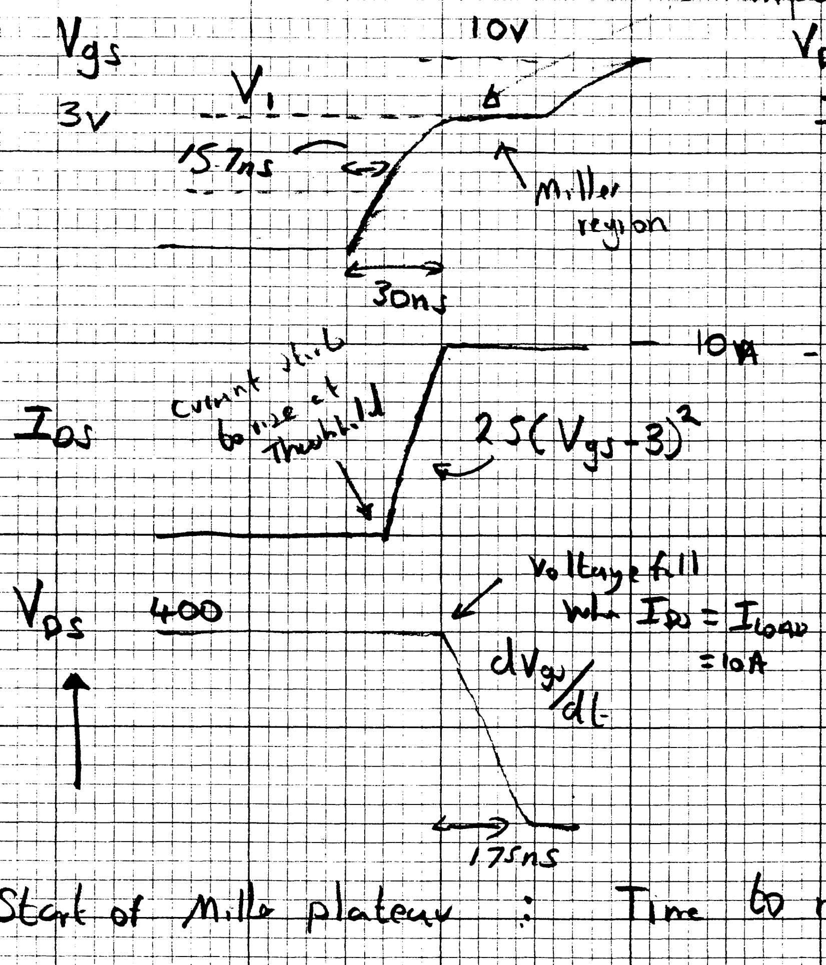
$\Rightarrow \frac{T_{ON}}{T} \cancel{400} = 24\cancel{0}$

$\frac{T_{ON}}{T} = 0.6$



input capacitance used

V_{DS} does not change until $I_D = 10$A



- Voltage can fall once $I_D = 10$A

$V_{DS} = 400$v

Start of Miller plateau :

$$2.5(V_{gs}-3)^2 = 10$$

$$\Rightarrow 2 = V_{gs}-3$$

$$V_{gs} = 5\text{V}$$

Time to reach threshold

$$10(1-e^{-t/CR}) = 3$$

$$e^{-t/RC} = 0.7$$

$$t = -RC\ln(0.7)$$

$$= 22 \times 2 \times 10^{-9} \ln 0.7 = 15.7\text{ns}$$

Question 2 Crib

Time for current to rise to 10A

$10(1-e^{-t/RC}) = 5 \qquad 2.5(V_{gs}-3)^2 = 10$

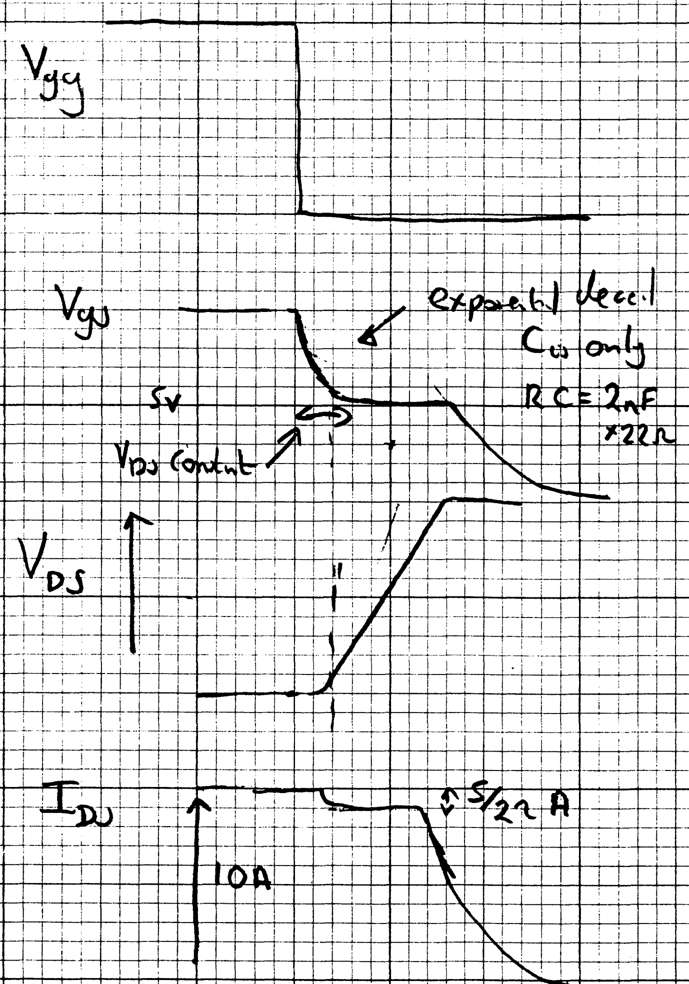
$\Rightarrow t = 44\times10^{-9} \ln 0.5$

$= \underline{30ns}$

Current rise time $= 30 - 15.7 = \underline{14.3ns}$

Current does not rise until $V_{gs} = 3V$

Turn-off



Plateau condition

$\frac{V_{gs}}{R} = I = \frac{V_{gs}}{22}$

$C_{gd}\frac{dv_c}{dt} = I$

$I = 10 - 2.5(V_{gs}-3)^2$

$\Rightarrow \frac{V_{gs}}{22} = 10 - 2.5V_{gs}^2 + 6V_{gs}\times2.5 - 9\times2.5$

$\frac{V_{gs}}{22} = -12.5 - 2.5V_{gs}^2 + 15V_{gs}$

$\Rightarrow 2.5V_{gs}^2 - \left(15+\frac{1}{22}\right)V_{gs} + 12.5$

$\Rightarrow V_{gs} \simeq \underline{5V}$

$\frac{dv_c}{dt} = \frac{5}{22\times C_{gs}}$

$= 2.3V/ns$

approx rise time $= 174ns$

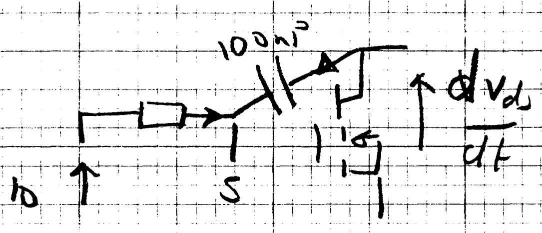
Question 2 Crib.

Voltage V_{DS} fall at turn on occurs once $I_D = 10A$

In this region the voltage fall is limited by the current injected into the gate via $C_{gd} = 100pF$

V_{gs} for $I_D = 10A$

$V_{gs} = 5V$



$$\frac{5}{22} = 100 \times 10^{-12} \frac{dV_{DS}}{dt}$$

Approx linear

Time to fall to 3V

$$\Delta t = \frac{22}{5} \times 100 \times 10^{-12} \times 397 = 175nS$$

Current to charge output capacitance at this rate

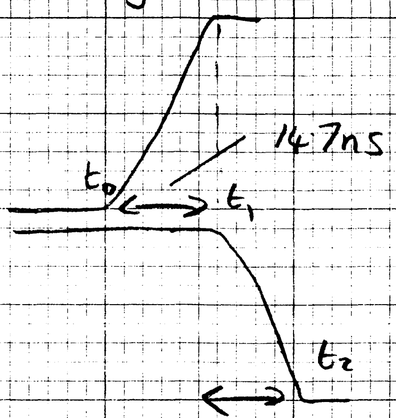
$$i = 400 \times 10^{-12} \times \frac{397}{175 \times 10^{-9}} = 0.9A$$

The current required to discharge the output capacitance is small in comparison to the available load current. The principal limit is therefore the miller capacitance.

Question 2 Crib

Turn-on Energy

$$E_{ON} = \int I_{DS} V_{DS}\, dt$$



t_0 : $t = 15.7ns$

Period $t_0 \rightarrow t_1$ $\quad V_{DS} = V_{DC} = 400$

$$I_{DS} = 25\,(V_{gs} - 3)^2$$

$$V_{gs} = 10(1 - e^{-t/RC}) \qquad RC = 44 \times 10^{-9}$$

$$I_{DS} = 25\,(10(1 - e^{-t/RC}) - 3)^2$$
$$25\,(7 - 10e^{-t/RC})^2$$
$$25\,(49 - 140e^{-t/RC} + 100\,e^{-2t/RC})$$

$$E_1 = \int_{15.7ns}^{30ns} 25 \times 400\,(49 - 140e^{-t/RC} + 100e^{-2t/RC})\,dt$$

$$= 1000\left[49t + 140e^{-t/RC} \times RC - 50RCe^{-2t/RC}\right]_{15.7ns}^{30ns}$$

$$= 1000\,(49 \times 30ns + 70.8 \times 44 \times 10^{-9} - 12.7 \times 44 \times 10^{-9})$$
$$- (49 \times 15.7 \times 10^{-9} + 98 \times 44 \times 10^{-9} - 24.5 \times 44 \times 10^{-9})$$
$$= 1000\,(4026 - 4003) \times 10^{-9}$$
$$= 23\mu J$$

Linear Approximation Also Valid

$$= 10 \times 400 \times \frac{14.7}{2} \times 10^{-9} = 29\mu J$$

Question 2 Crib

Second phase

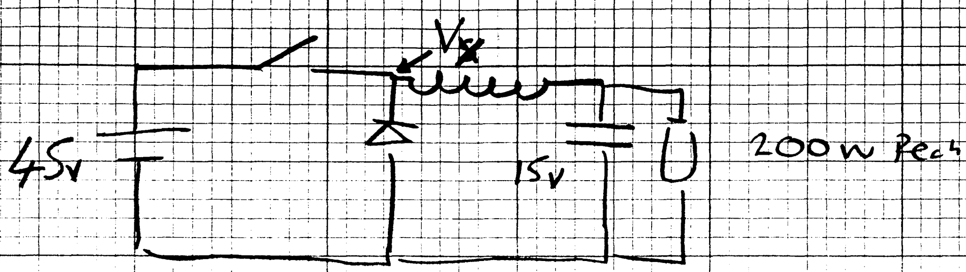
$I_D = 10A$

voltage can be approximated by a linear fall from $400 \rightarrow 0v$ in $175ns$

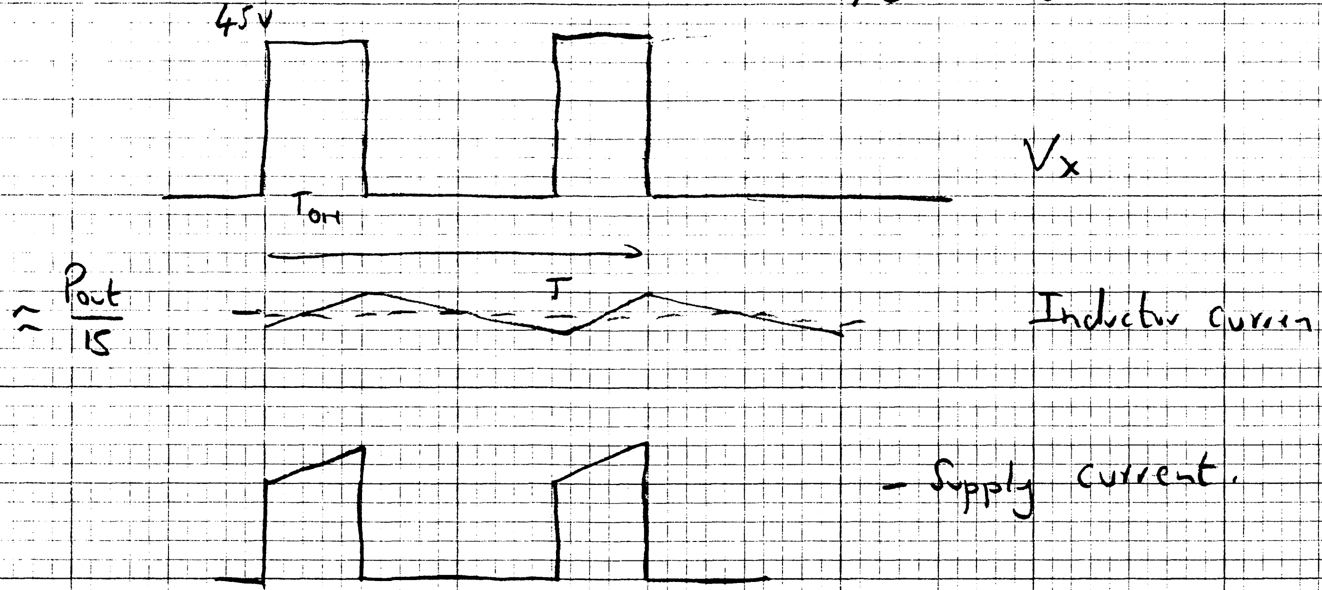
$$\Rightarrow E \quad \frac{400 \times 10 \times 175 \times 10^{-9}}{2}$$

$$= 350 \mu J$$

Question 3 Crib



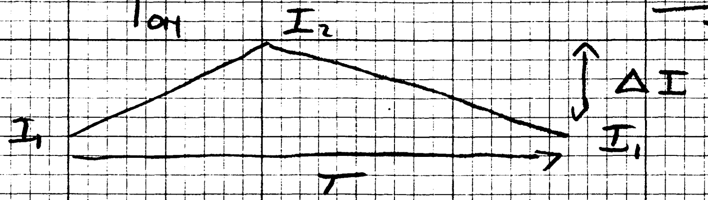
Duty cycle $\frac{T_{ON}}{T} = \frac{15}{45} = \frac{1}{3}$



Current Ripple

$V_o = 15v$

$$\frac{V_S\, T_{ON}}{T} = 15v$$



$$\Delta I = \frac{(V_s - V_o)\,T_{ON}}{L} = \left(\frac{V_s - 15}{L}\right)\frac{15\,T}{V_s}$$

$$\Delta I = \frac{15^2 T}{L}\left(\frac{1}{15} - \frac{1}{V_s}\right)$$

Question 3 Crib

Maximum current ripple occurs for maximum input voltage

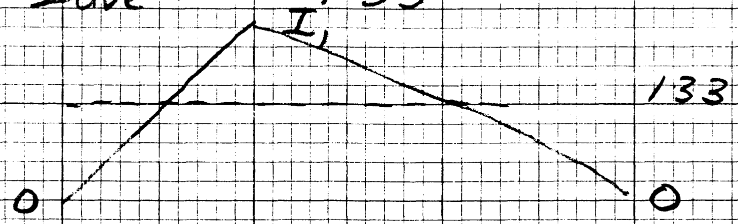
$$\Rightarrow \quad \Delta I = \frac{15^2 T}{L}\left(\frac{1}{15} - \frac{1}{50}\right)$$

$$= \frac{225}{100\times10^{-6}} T \,(0.0466)$$

$$= 104850\,T$$

At 10% of maximum output

$I_{ave} = 133$



For boundary of continuous conduction

$$\Delta I = 2 \times I_{ave}$$

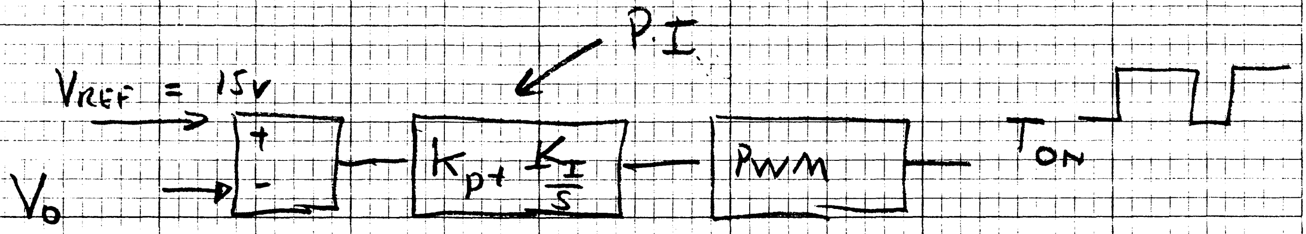
$$\Rightarrow \quad 2.66 = 104850\,T$$

$$T = \frac{2.66}{104850} = 25.4\,\mu s$$

$$f = \underline{394\ kHz}$$

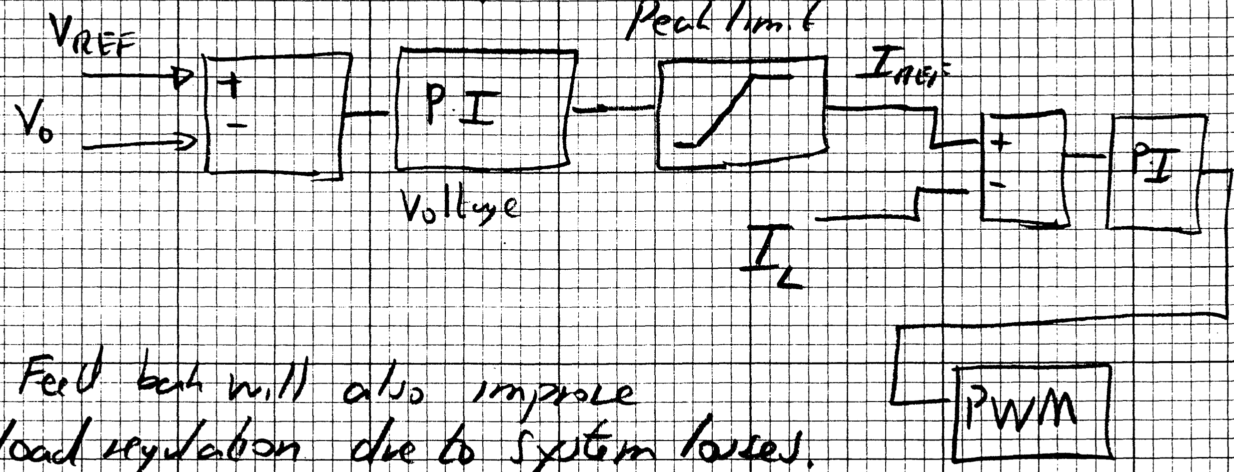
Question 3 Crib

Feedback is necessary to achieve a constant output voltage with variable input voltage. It can also act to improve regulation in response to changes in load current



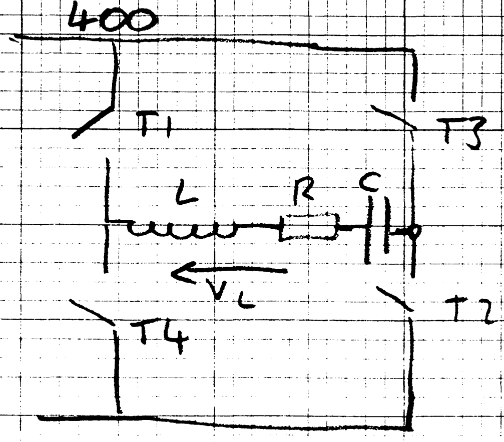
Duty cycle is varied to maintain the output voltage constant.

Inductor current feedback can improve load dynamics and limit the output current (in inrush startup

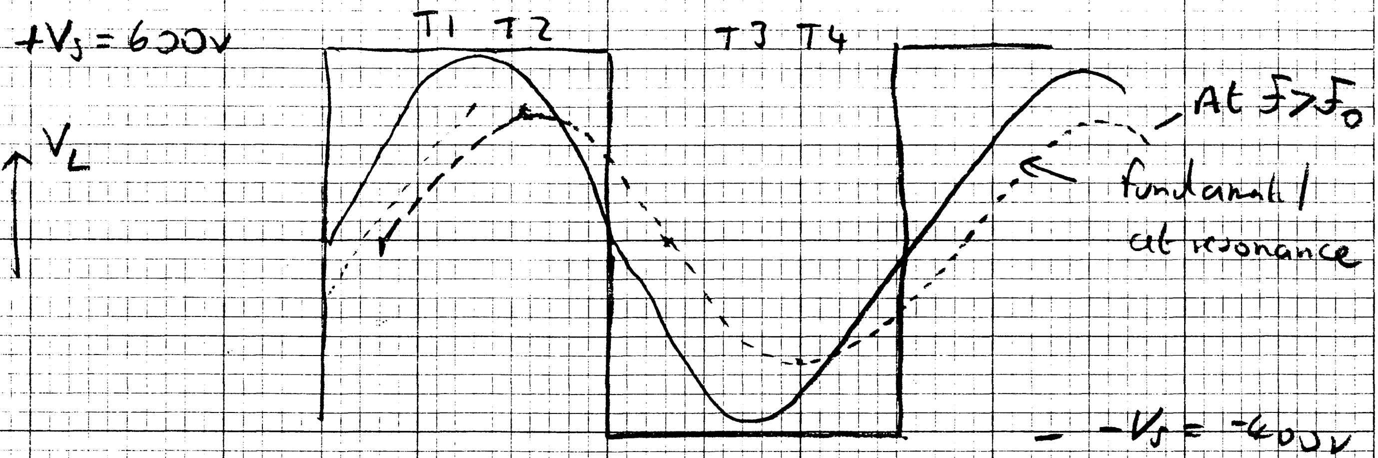


Feedback will also improve load regulation due to system losses. e.g inductor and switch resistance.

Question 4 Crib



$$Z_L = j\left(\omega L - \frac{1}{\omega C}\right) + R$$



V_L will contain Fundamental plus odd harmonics

when switching at the resonant frequency the current associated with the fundamental is inphase with the square wave.

Turn on and turn-off of the switches will occur at or close to zero current giving low switching loss

Higher order harmonics are attenuated by the RLC load

e.g. $3 \times \omega_0$

$$Q = \frac{\omega_0 L}{R}$$

$$I = \frac{V_3}{j\left(3\omega_0 L - \frac{1}{3\omega_0 C}\right) + R}$$

$$= \frac{V_3}{3\omega_0 L j\left(1 - \frac{1}{9\omega_0^2 LC}\right) + R} \simeq \frac{1}{3\omega_0 L j + R} = \frac{1}{R(1+3Qj)}$$

For high Q circuits the higher harmonics are significantly attenuated and may be neglected.

Fundamental component of the load voltage $= \frac{4}{\pi} V_{dc}$

For resonant operation

$$\omega_0 = \frac{1}{\sqrt{4 \times 10^{-9} \times 1 \times 10^{-3}}}$$

$$= 500000$$

check valid

$$F_0 = \cancel{79.6 kHz}$$

$$Q = \frac{500 \times 10^3 \times 1 \times 10^{-3}}{50} = 10$$

Power transferred to load at resonance.

For RMS

$$= \left(\frac{4 \times 600}{\pi}\right)^2 \frac{1}{50}\,\frac{1}{2} = 11.67\,kW/2$$

$$\hat{=} 5.83\,kW$$

As the frequency is changed from resonance the magnitude of the load current will decrease and likewise the power to the load $|I|^2R$ decreases

Question 4 Crib

For $f > f_0$ the load looks inductive and the current will lag the fundamental. The current in the switch is not zero and switching loss increases.

Switches turn off current but turn on while their anti parallel diodes are conducting.

For $f < f_0$ the load looks capacitive and current leads voltage. Turn-on loss appear in the switch but turn-off occurs with the anti parallel diodes conducting.

Question 4 Crib

Output power may be controlled by increasing frequency

Fundamental voltage component

$$P_{Resonance} \quad \left(\frac{4\times 400}{\pi}\right)^2 \quad \frac{1}{50} \quad \frac{1}{2}$$

$$P_{90\%} = |I|^2 R$$

$$I = \frac{4\times 400}{\pi\sqrt{2}} \quad \frac{1}{R+jX}$$

$$\Rightarrow |I|^2 = \left(\frac{4\times 400}{2\pi^2}\right)^2 \frac{1}{R^2+X^2}$$

$$\Rightarrow \cancel{\left(\frac{4\times 400}{\pi 2}\right)^2} \frac{1}{R^2+X^2} R = 0.9 \cancel{\left(\frac{4\times 400}{2\pi}\right)^2} \frac{1}{R}$$

$$\frac{R^2}{R^2+X^2} = 0.9$$

$$R^2(1-0.9) = X^2\, 0.9$$

$$X^2 = 0.111 R^2$$

$$X = 16.6$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} = 16.6 \quad \text{For increasing frequency.}$$

$$\omega^2 - \frac{16.6\omega}{L} - \frac{1}{LC} = 0$$

$$\omega^2 - \frac{16.6\,\omega}{1\times 10^{-3}} - 2.5\times 10^{11} = 0$$

$$\omega = \frac{16.6\times 10^3 \pm \sqrt{2.75\times 10^8 + 1\times 10^{12}}}{2}$$

$$\omega = 8.3\times 10^3 \pm 500\times 10^3$$

$$\omega = 8.3\times 10^3 + 500\times 10^3 = 508.3\times 10^3 \text{ rad s}^{-1}$$

Question 4 Crib

Current at $\omega = 508 \times 10^3$ rads^{-1}

$$\underline{I} = \frac{1600}{\pi} \Big/ 50 + j16.6$$

$$= \frac{1600}{\pi} \frac{1}{\quad}$$

$$\frac{1600}{\pi\, 52.6 \angle 18.3}$$

$$I = 9.68 \angle -18.3$$

Peak Values Not RMS

Current at turn-off = 3A

The device must break a current of 3A, around 30% of the peak current.

- Still more efficient than hard switching.