PART IIA 2005
3B4: Electric drive systems
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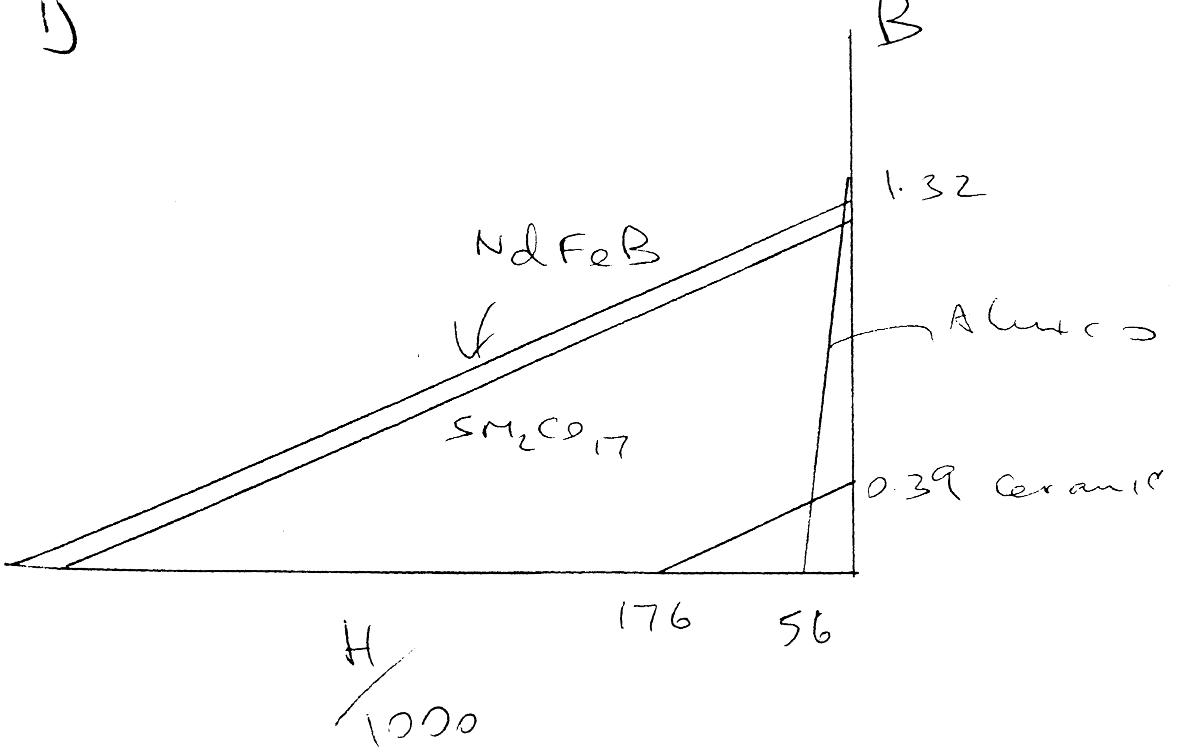
Datasheet: None

1)



a) $B = \mu_r \mu_0 H$

Alnico $\dfrac{1.32}{4\pi \times 10^{-7} \times 56 \times 10^3} = 18.75 = \mu_r$

Ceramic $\dfrac{0.39}{4\pi \times 10^{-7} \times 176 \times 10^3} = 1.76 = \mu_r$

SmCo $\dfrac{1.06}{4\pi \times 10^{-7} \times 815 \times 10^3} = 1.03 = \mu_r$

NdFeb $\dfrac{1.21}{840{,}000 \times 4\pi \times 10^{-7}} = 1.15 = \mu_r$

20%

d) ...

i) starting from

$$H_m l_m = -\frac{B_g l_g}{\mu_o}$$

area is halved since ϕ is conserved

$$B_g = B_m/2$$

$$\Rightarrow \quad H_m = -\frac{B_m}{2\mu_o}$$

substitute into

$$B_m = (H_m - H_c)\mu$$

$$\Rightarrow \quad -2\mu H_m = (H_m - H_c)\mu .$$

$$\Rightarrow \quad H_m = H_c/3 \text{ and } B_m = 2B_r/3 .$$

$$\Rightarrow (BH) = \frac{2B_r H_c}{9}$$

but this is per unit volume and volume is halved

$$\Rightarrow \quad \frac{(BH)_{tot\text{-}thin\ magnets}}{(BH)_{tot\text{-}part(b)}} = \frac{1/9}{1/4} = \frac{4}{9} \quad (10\%)$$

ii) Similarly (from part c)

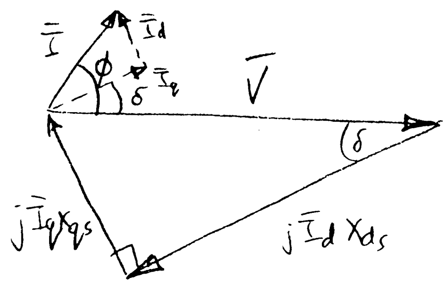
$$\frac{l_m}{l_g}\frac{H_m}{\pi} = \left(\frac{H_m}{\pi} - H_c\right)$$

$$\Rightarrow \quad -\frac{H_m}{2} = H_m - H_c$$

$$\Rightarrow \quad H_m = \frac{2H_c}{3} \quad \Rightarrow \quad B_m = B_r/3$$

$$\Rightarrow \quad (BH) = \frac{2B_r H_c}{9}$$ and again the volume is halved so again the ratio of the total BH products to the max one from part b) is $4/9$. (10%)

2. (b) ω_s rotor excitation. ... (10%)



$P_{out} = 3VI\cos\phi$

Now $I\cos\phi = I_q \cos\delta - I_d \sin\delta$

Also $I_d X_{ds} = V\cos\delta$

$I_q X_{qs} = V\sin\delta$

$$\Rightarrow P_{out} = 3V\left[I_q\cos\delta - I_d\sin\delta\right]$$

$$= 3V\left[\frac{V\sin\delta}{X_{qs}}\cos\delta - \frac{V\cos\delta}{X_{ds}}\sin\delta\right]$$

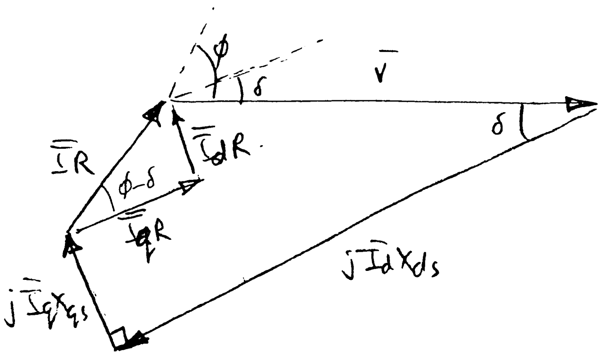
$$= \frac{3V^2}{2}\left[\frac{1}{X_{qs}} - \frac{1}{X_{ds}}\right]\sin 2\delta$$

$$\Rightarrow T = \frac{P_{out}}{\omega_s} = \underline{\frac{3V^2}{2\omega_s}\left[\frac{1}{X_{qs}} - \frac{1}{X_{ds}}\right]\sin 2\delta}$$

(35%)

(b) Now R not negligible.

New phasor diagram:



$I_d X_{ds} = V\cos\delta + I_q R$

$I_q X_{qs} = V\sin\delta - I_d R$

$I\cos\phi = I_q\cos\delta - I_d\sin\delta$ as before

$$\Rightarrow I_q X_{qs} = V\sin\delta - R\left(\frac{1}{X_{ds}}\right)\left(V\cos\delta + I_q R\right)$$

$$\Rightarrow I_q X_{qs} + \frac{R^2}{X_{ds}} I_q = V\sin\delta - \frac{R}{X_{ds}} V\cos\delta$$

$$\Rightarrow I_q = \frac{\left(V\sin\delta - \frac{R}{X_{ds}}V\cos\delta\right)}{X_{qs} + \frac{R^2}{X_{ds}}}$$

And $I_d X_{ds} = V\cos\delta + R\left(\frac{1}{X_{qs}}\right)\left(V\sin\delta - I_d R\right)$

$$\Rightarrow I_d\left[X_{ds} + \frac{R^2}{X_{qs}}\right] = V\cos\delta + \frac{RV\sin\delta}{X_{qs}}$$

$$\Rightarrow I\cos\phi = V\left[\frac{\sin\delta\cos\delta - R/X_{ds}\cos^2\delta}{X_{qs} + \frac{R^2}{X_{ds}}}\right] - V\left[\frac{\sin\delta\cos\delta + R/X_{qs}\sin^2\delta}{X_{ds} + R^2/X_{qs}}\right]$$

$$P = 3VI\cos\phi$$

$$\Rightarrow P = 3V^2\left[\frac{\sin 30\cos 30 - \frac{1}{2}\cos^2 30}{1+\frac{1}{2}} - \frac{\sin 30\cos 30 + \sin^2 30}{2 + 1/1}\right]$$

$$= 3V^2\left[\frac{\sqrt{3}/4 - \frac{1}{2}\times 3/4}{3/2} - \frac{\sqrt{3}/4 + 1/4}{3}\right]$$

$$= \frac{\cancel{3}V^2}{\cancel{3}}\left[2(\sqrt{3}/4 - 3/8) - (\sqrt{3}/4 + 1/4)\right]$$

$$= V_{phase}^2(\sqrt{3}/4 - 1)$$

(55%)

$$= \cancel{12\ kW\ (RMS)}$$

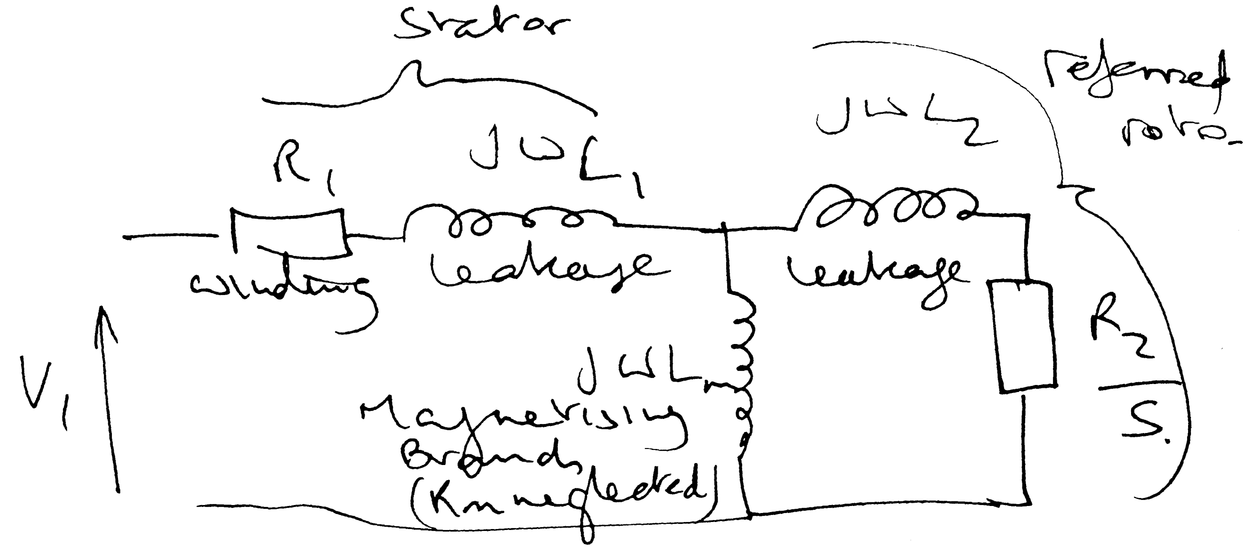
$$V_{phase} = V_{Line}/\sqrt{3}$$

$$= \frac{6.6^2}{3}(\sqrt{3}/4 - 1)\times 10^6$$

$$= 8.2\ kW$$

3.

a)



(30%)

b) $P_{gap} = P_{mech} + P_{losses}$

$$T\omega_s = T\omega_r + 3I_2^2 R_2$$

$$\Rightarrow \quad T(\omega_s - \omega_r) = 3I_2^2 R_2$$

$$\omega_s - \omega_r = s\omega_s$$

$$\Rightarrow \quad s\omega_s T = 3I_2^2 R_2$$

$$\Rightarrow \quad P_{gap} = \frac{3I_2^2 R_2}{s}.$$

$$= \frac{3R_2}{s} \times \frac{V_2^2}{(\omega L_2)^2 + (R_2/s)^2}$$

? As V_2 is the rotor voltage - neglecting $R_1 L_1$ we end up with

$$\cancel{RE} \; T\omega_s = \frac{3R_2}{S} \times \frac{V_1^2}{(\omega L_2)^2 + (R_2/S)^2}$$

differentiate with respect to s, and then place $\omega L_2 << R_2/S$.

$$\frac{dT}{ds} = -\frac{3V_1^2}{\omega_s} \times \frac{1}{R_2}$$

(30%)

c) $$\frac{ds}{d\omega_r} = -\frac{1}{\omega_s}$$

$$s\omega_s = \omega_s - \omega_r$$

$$\Rightarrow s = \frac{\omega_s - \omega_r}{\omega_s}$$

$$\frac{ds}{d\omega_r} = -\frac{1}{\omega_s}.$$

$$\Rightarrow \frac{dT}{d\omega_r} = -3\frac{V_1^2}{\omega_s^2} \cdot 1/R_2$$

ie if $\frac{V_1^2}{\omega_2^2}$ is constant then the torque speed curve has a negative gradient of $-1/R_2$ and is stable/linear operation.

(15%)

$\mathcal{S}$

d) field weakening

supplying frequency

$$T\omega_s = T\frac{\omega}{p} = \frac{3R_2V_1^2}{s\left((\omega L_2)^2+(R_2/s)^2\right)}$$

$V_1 = K\omega \Rightarrow$

$$\Rightarrow \quad \frac{T}{p} = \frac{3K^2\cancel{V_1^2}\,s\,\omega}{R_2}$$

$V_1 = K\omega.$

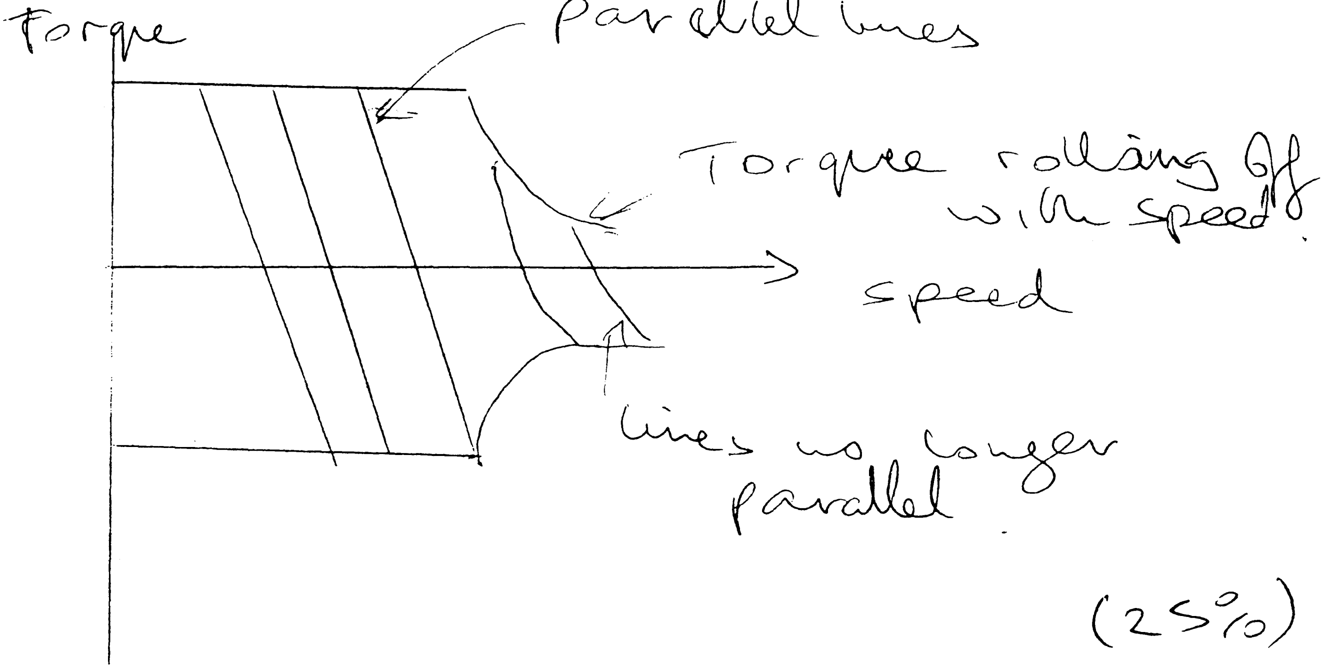
$$\Rightarrow \quad \frac{T_1}{p} = \frac{V_b^2 s}{\omega R_2}$$

but V_1 is fixed at V_b so sub back.

since the current is determined by supply voltage R_2 and slip there will be a max. supply current and hence a rated slip

$$I_2 = \frac{sV_1}{\sqrt{R_2^2+s^2\omega_s^2L_2^2}} \simeq \frac{sV_1}{R_2}$$

$\Rightarrow$ $s_{max} = \frac{I_b R_2}{V_b} \Rightarrow T = \left(\frac{3V_b^2 s_b}{R_2}\right) \times \frac{1}{\omega}$ all constants and the torque rolls off with speed.



(25%)

4. ∴ Mech power output = 1kW

240V single-phase, 50Hz, 4 pole (⇒ 2 pole pairs).

Duty cycle:

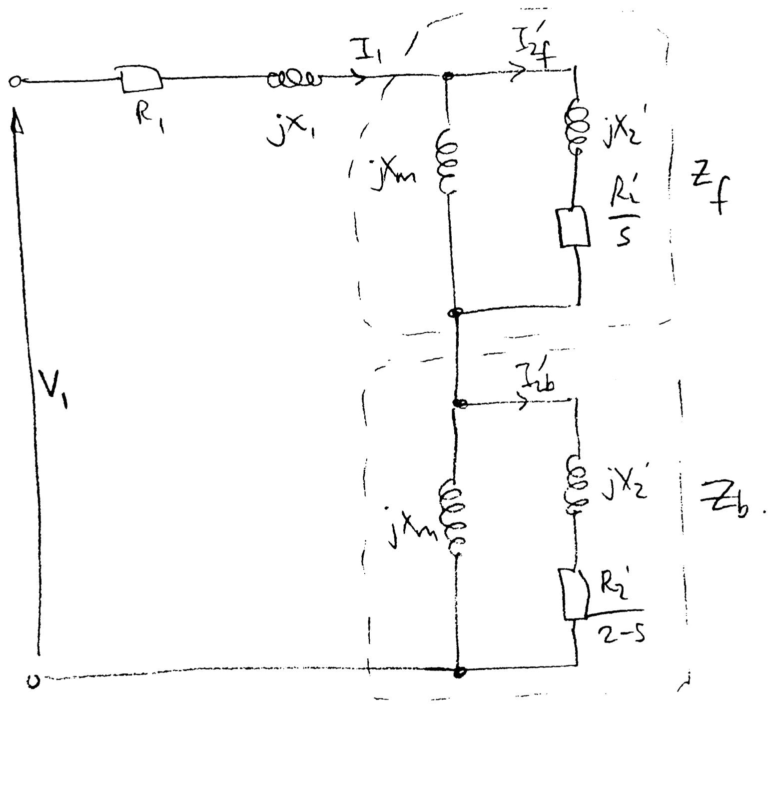
(a) Rest period: $P_{diss} = 0$ too.

$\Rightarrow$ Time const is: $\dfrac{40-20}{70-20} = \exp\left(-\dfrac{(12-10)}{\tau}\right) \Rightarrow \tau = \dfrac{2}{\ln 2.5} = 2.183 \text{ sec} \times 60$

$k = 1.1 \text{ WK}^{-1}$

Now $\tau = C/k \Rightarrow \quad C = \tau k = \underline{\underline{2.40 \text{ JK}^{-1}}} \times 60 = 144 \text{ J K}^{-1}$

(b)



$\therefore$ (c) R_1, X_1 negligible.

$$T_f = \frac{1}{\omega_s}|I_1|^2 \, Re\{Z_f\}$$

$$T_b = \frac{1}{\omega_s}|I_1|^2 \, Re\{Z_b\}$$

$$T_{out} = T_f - T_b = \frac{1}{\omega_s}|I_1|^2\left[Re\{Z_f\} - Re\{Z_b\}\right]$$

Assume $X_m >> \left|jX_2' + \frac{R_2'}{s}\right|$

$$X_m >> \left|jX_2' + \frac{R_2'}{2-s}\right|$$

$$X_2' << \frac{R_2'}{s}, \quad X_2' << \frac{R_2'}{2-s}$$

Now $P_{mech} = T_{out}\,\omega_r = \frac{\omega_r}{\omega_s}|I_1|^2\left(\frac{R_2'}{s} - \frac{R_2'}{2-s}\right) = (1-s)|I_1|^2 R_2'\left[\frac{2-s-s}{s(2-s)}\right]$

$$= 2|I_1|^2 R_2' \frac{(1-s)^2}{s(2-s)}$$

Power dissipated in windings $=$ $P_{diss} = 2|I_1|^2 R_2'$

Now steady-state power dissipation is:

$\frac{1}{2}$-power: $\frac{70-20}{T_1-20} = 1-\exp\left(-\frac{10}{2.183}\right) = 0.9898 \Rightarrow T_1 = 70.52°C$

$\Rightarrow P_{diss,1} = kT_1 = 1.1\times(70.52 - 20) = \quad 56W$

d) Full power: $\frac{80-40}{T_2-40} = 1-\exp\left(-\frac{3}{2.183}\right) = 0.7470 \Rightarrow T_2 = 93.55°C$

$\Rightarrow P_{diss,2} = kT_2 = 1.1\times(93.55 - 20) = \quad 81W$

Rated slip $=$ s at full power.

Now $P_{mech} = P_{diss}\frac{(1-s)^2}{s(2-s)} \Rightarrow 1000 = 81 \cdot \frac{(1-s)^2}{s(2-s)}$

$$\Rightarrow 12.35(2-s)s = (1-s)^2$$

$$\Rightarrow 13.35\, s^2 - 26.7\, s + 1 = 0$$

$$\Rightarrow s = 0.04 \text{ or } 1.96$$

$$\Rightarrow 13.35 s^2 - 26.75 s + 1 = 0$$