PART IIA 2005
3B5: Semiconductor engineering
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Datasheet: None

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Q.1

a) Photoelectric effect: e.m wave as a particle

Light impinging on a metal result in extraction of electrons. The energy of the electrons increases linearly with the frequency of the light but is independent of the ligh intensity. The effect can only be explained by assuming the e.m. waves behave as particles "photons", the energy of the photon being $E = h\nu$, where ν is the frequency of the e.m. wave.

Electron diffraction

Particles behaving like waves

Electrons are diffracted by a crystal lattice. Maxima of the diffraction pattern satisfy Bragg's equation:

$$2d\sin\theta = m\lambda \quad m = 1, 2, \ldots$$

d = lattice planes distance

$\lambda = \frac{h}{p}$ — electron momentum — De Broglie relationship

1b

r, ϑ, ϕ are polar coordinates, r is the distance from the origin, in this case the centre of the nucleus, which is a single proton; ϑ is the angle between the projection of the position vector ~~and~~ on the xy plane ~~with~~ and the x-axis; ϕ is the angle between the position vector and the z-axis. $R(r)$ is the part of the wavefunction depending from the distance between the electron and the nucleus; $Y(\vartheta,\varphi)$ is the part depending from the angles.

1c

$$R(r) = c\,e^{-\alpha r}$$

$$\frac{dR}{dr} = -c\alpha e^{-\alpha r}$$

$$\frac{d^2R}{dr^2} = c\alpha^2 e^{-\alpha r}$$

Substituting the above in equation 2 in question text:

$$\cancel{c}\alpha^2 \cancel{e^{-\alpha r}} - \frac{2}{r}\cancel{c}\alpha\cancel{e^{-\alpha r}} + \frac{2m}{\hbar^2}E\cancel{c}\cancel{e^{-\alpha r}}$$

$$+ \frac{2me^2\cancel{c}\cancel{e^{-\alpha r}}}{\hbar^2 4\pi\varepsilon_0 r} = 0$$

For this equation to be satisfied:

Sum of terms containing $\frac{1}{r} = 0$
and
Sum of terms not containing $\frac{1}{r} = 0$

From first condition:

$$\alpha = \frac{me^2}{\hbar^2 4\pi\epsilon_0}$$

From 2nd condition:

$$E = -\frac{me^4}{8\epsilon_0^2 h^2} = -\frac{me^4}{32\epsilon_0^2\pi^2\hbar^2}$$

d)

Probability density $|\psi|^2 = C^2 e^{-2\alpha r}$
Probability particle being within r and $r+dr$:

$$P(r)dr = 4\pi r^2 C^2 e^{-2\alpha r} dr$$

Probability particle being between 0 and ∞ is equal $= 1$. Therefore

$$4\pi C^2 \int_0^\infty r^2 e^{-2\alpha r} dr = 1$$

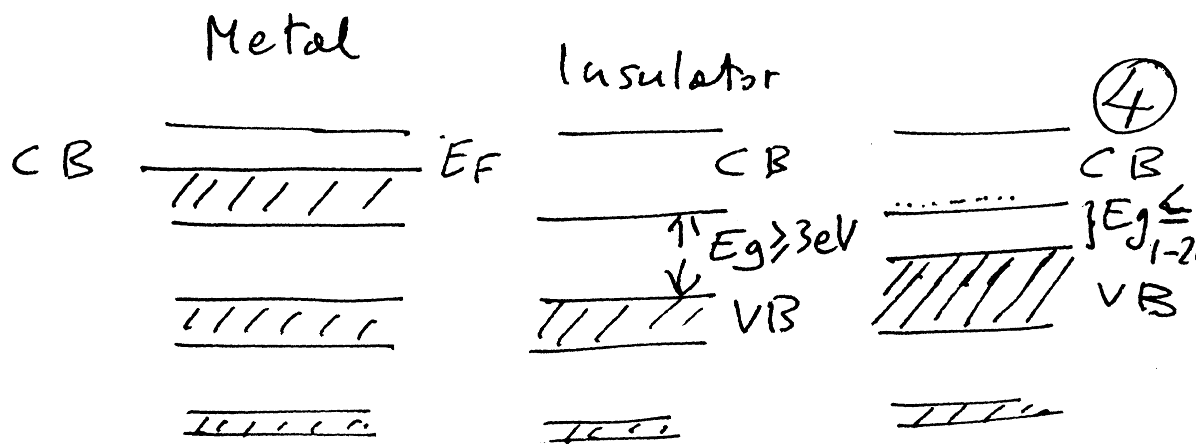
Integration by parts gives:

$$\int_0^\infty r^2 e^{-2\alpha r} dr = 4\alpha^3$$

Hence $$C = \frac{\alpha^{3/2}}{\sqrt{\pi}}$$

Q. 2

(a)



For an electron to be able to move in a material, it must gain energy. Since energy bands result from discrete energy levels very close together in energy and the occupation of these levels is dictated by Pauli principle – not two electrons can have the same 4 quantum numbers –, an electron can only gain energy by moving from one energy level to another which is not completely full.

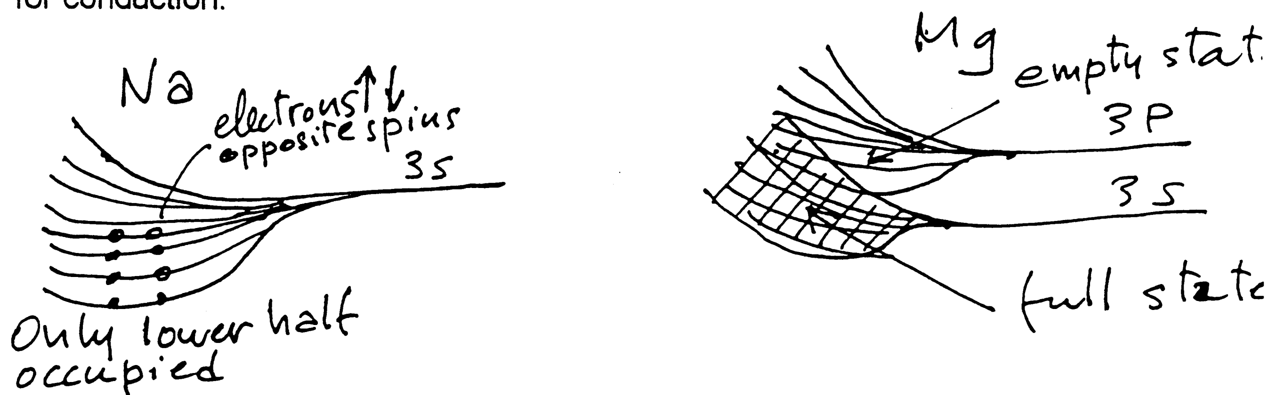
In metals the uppermost band is only partially full, so electrons near the Fermi Energy E_F are able to move to unoccupied levels. Metals are therefore good conductors.

In insulators the uppermost band containing electrons (the Valence Band) is completely full. Therefore electrons can only gain energy by jumping to the next empty band. To do this they have to gain an energy larger the the gap ($E_g \gtrsim 3eV$), which

is possible only at very high electric fields ($F > 10^5 - 10^6\ V cm^{-1}$). When these fields are applied the insulator breaks down and loses its insulating properties.

In semiconductors E_g is small enough for electrons to be thermally excited from the Valence (VB) to the Conduction Band (CB). So they can conduct electricity.

2b. In sodium the 3s orbital has only one electron. So the band originating from this level contains N level and N electrons. Since the electrons occupie the lowest energy level compatible with Pauli Exclusion Principle, they will end up two per level filling the lower half of the uppermost band. In the case of Mg the band originating from the 3s atomic states is completely full (2 electrons per energy level), so one would expect it to be an insulator. However the 3s and 3p bands overlap, so the 3s band is full but the empty 3p band levels are available for conduction.



2c

n-type Silicon

Each Group V atom will replace a silicon atom

–This is known as *substitutional doping*

–In doing so, the Gp. V atom will use 4 of its 5 valence electrons to form covalent bonds with 4 neighbouring Si atoms

–There will now be one spare electron on the Gp. V atom

–The electron is only weakly bound

–In energy terms, it will form an energy level E_D only ~0.03eV below E_C which will be occupied at 0°K

–Electrons may be easily excited into the CB from the energy level E_D as temperature rises

–So, the addition of each Gp. V impurity leads to an extra available conduction electron *without* the creation of a hole in the VB to produce a *n-type semiconductor*. The Gp. V impurity is called a '*donor*'–*In a n-type semiconductor c*onduction is predominantly due to electrons in the conduction band which have been excited from the donor levels.

p-type Silicon

To dope a Group IV semiconductor p-type, we add a Group III element to the material, such as Boron

–Boron $1s^2 2s^2 2p$ has only 3 valence electrons, but the surrounding silicon atoms provide 4 bonds

–3 of these form covalent bonds with the 3 electrons of the Boron

–The 4th has only 1 electron, while we know it can take 2 electrons with opposite spin.

–The associated energy level E_A is located ~0.030 eV above E_V

–At 0°K this level is empty

–At higher temperatures (300°K) an electron can jump into the empty space from other bonds; another electron can jump into the newly vacated place and so on•This is can be viewed more easily in terms of *hole* rather than *electron*:

–a *hole* is bound to the Boron atom at 0°K , but can move about *at higher temperature*– Now there is an excess *–ve* charge *–e* on the Boron site due to the captured electron

–The captured electron stays on the Boron site while the hole moves about

–Such Gp.III impurity is called an '*acceptor*

To dope III-V semiconductors such as GaAs which already contains atoms from Groups III and V we use Group II elements (e.g. Zn) as acceptors and Group VI elements (e.g. S) as donors. Group IV elements can act as p-type or n-type dopants depending on their chemical preference for substituting for the Ga or the As

- C acceptor
- Si donor
- Ge amphoteric
- Sn usually a donor, but occasionally acts as an acceptor

2d

From the data book:

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

By analogy:

$$p = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

For an intrinsic semiconductor:

$$n = p$$

$$N_C \exp\left(\frac{E_F}{kT}\right) \exp\left(-\frac{E_C}{kT}\right) =$$

$$= N_V \exp\left(\frac{E_V}{kT}\right) \exp\left(-\frac{E_F}{kT}\right)$$

Set $E_V = 0 \Rightarrow E_C = E_g$ (energy gap)

$$\exp\left(\frac{2E_F}{kT}\right) = \frac{N_V}{N_C} \exp\left(\frac{E_g}{kT}\right)$$

$$E_F = \frac{kT}{2} \ln \frac{N_V}{N_C} + \frac{E_g}{2}$$

$$= 0.547 \text{ eV}$$

2e)

$$p = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$p \simeq N_A$$

$$E_F - E_V = kT \ln\frac{N_V}{N_A} = 0.178\,\text{eV} \quad *$$

2f)

p-side: from * $\Rightarrow$

$$\Rightarrow E_{F_p} = E_V + kT \ln\frac{N_V}{N_A} \qquad (1)$$

n-side

$$n \simeq N_D \qquad N_D = N_c \exp\left(\frac{E_{F_n} - E_C}{kT}\right)$$

$$E_{F_n} = E_C + kT \ln\frac{N_D}{N_C} \qquad (2)$$

Subtract (1) from (2)

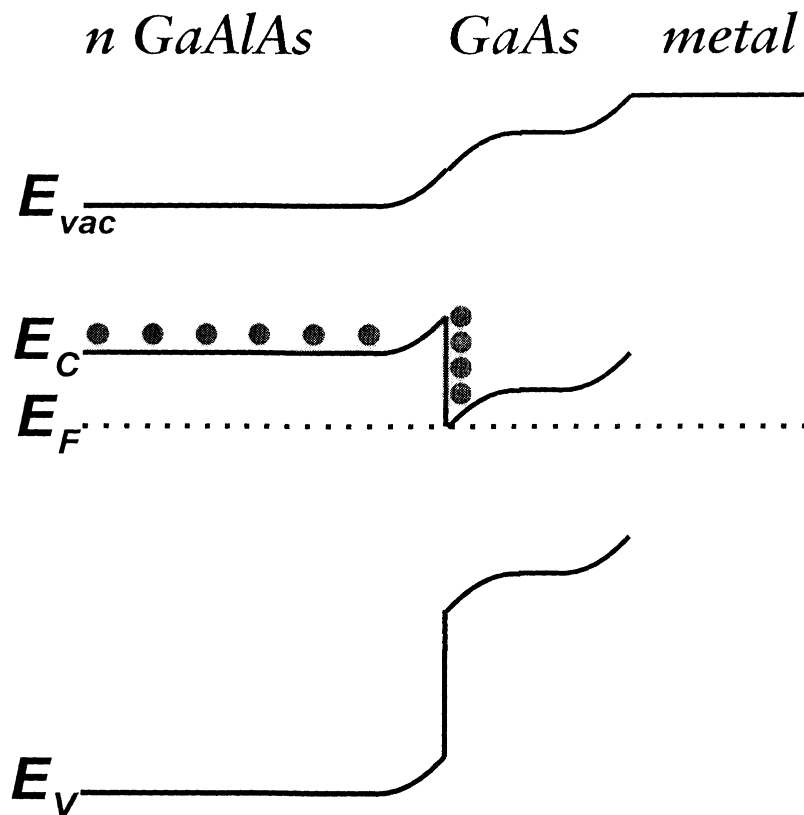
$$E_{F_n} - E_{F_p} = E_g + kT \ln\frac{N_D N_A}{N_V N_C} =$$

$$= 0.675\,\text{eV}$$

Built-in potential

$$V_{bi} = 0.675\,\text{V}$$

3 (a) The High Electron Mobility Transistor (HEMT) is a variant on the MESFET. If we wish to increase conductivity of a MESFET, we could consider raising the doping level, but the increased number of impurity atoms causes more electron scattering and reduces mobility. Therefore, we use a wide bandgap doped semiconductor with a thin, low doped semiconductor to form a heterojunction with an accumulation of electrons in a thin layer. The doped, wide bandgap material provides a source of electrons which accumulate in a two dimensional layer in the GaAs. We therefore have a large carrier density in a region of low doping density. These carriers therefore have a high mobility, and a very high speed device is the result.



(b) (i) Calculating the work function of the GaAs assuming that all donors are ionized:

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right)$$

$$= 0\cdot 862 \times 10^{-4} . 298 \ln\left(\frac{3\cdot 3 \times 10^{20}}{4\cdot 0 \times 10^{23}}\right)$$

$$E_F - E_C = -0\cdot 18\,\text{eV}$$

The work function is then

$$\phi_{\text{GaAs}} = \chi_{\text{GaAs}} + E_C - E_F$$
$$= 4 \cdot 07 + 0 \cdot 18$$
$$\phi_{\text{GaAs}} = 4 \cdot 25 \text{ eV}$$

Calculating the work function of GaAlAs:

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right)$$
$$= 0 \cdot 862 \times 10^{-4} .298 \ln\left(\frac{3 \cdot 3 \times 10^{20}}{8 \cdot 0 \times 10^{23}}\right)$$
$$E_F - E_C = -0 \cdot 20 \text{ eV}$$

The work function is then

$$\phi_{\text{GaAlAs}} = \chi_{\text{GaAlAs}} + E_C - E_F$$
$$= 3 \cdot 75 + 0 \cdot 20$$
$$\phi_{\text{GaAlAs}} = 3 \cdot 95 \text{ eV}$$

(ii) The band bending is the difference between the two work functions, which is 0.30 eV. Therefore, assuming band bending is equally distributed between the two sides, then 0.15 eV is taken up on each side of the junction, and so the conduction band in the GaAs at the interface is 0.15 eV closer to the Fermi level, so E_F - E_F is reduced by this amount to –0.03 eV. Hence,

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$
$$= 4 \cdot 0 \times 10^{23} \exp\left(\frac{-0 \cdot 03}{0 \cdot 862 \times 10^{-4} .298}\right)$$
$$n = 1 \cdot 24 \times 10^{23} \text{ m}^{-3}$$

(iii) The width of the depleted region is

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r V_0}{eN_D}\right)^{1/2}$$
$$= \left(\frac{2.8 \cdot 854 \times 10^{-12} .12 \cdot 2.0 \cdot 15}{1 \cdot 602 \times 10^{-19} .3 \cdot 3 \times 10^{20}}\right)^{1/2}$$
$$w = 7 \cdot 83 \times 10^{-7} \text{ m}$$

The amount of charge in the depleted region is then

$$\sigma = eN_D w$$
$$= 1 \cdot 602 \times 10^{-19} . 3 \cdot 3 \times 10^{20} . 7 \cdot 83 \times 10^{-7}$$
$$\sigma = 4 \cdot 14 \times 10^{-5} \text{ C m}^{-2}$$

Assuming that the charge density calculated in part (ii) is uniformly distributed across a thickness t in the GaAs, then

$$t = \frac{\sigma}{ne}$$
$$= \frac{4.14 \times 10^{-5}}{1.24 \times 10^{23} . 1.602 \times 10^{-19}}$$
$$t = 2 \cdot 08 \text{ nm}$$

4 (a) ϕ_m and ϕ_{sc} are the work functions of the metal and semiconductor respectively, and arise from the filling of the electron density of states for each material. Given that the vacuum level must be continuous, the difference between the two gives the energy difference between the vacuum levels on either side of the MOS structure. C_i is the capacitance per unit area of the gate insulator. Q_i is the charge per unit area introduced into the system by the oxide, and most notably the oxide-semiconductor interface, and is made up from interface trapped charges, fixed oxide charges, oxide trapped charges and mobile ionic charges. Q_d is the charge per unit area in the depleted region of the semiconductor. eV_f is the energy difference between the Fermi level in the bulk of the semiconductor and the middle of the band gap.

(b) (i) Rearranging the equation given to make Q_n the subject gives

$$Q_f = -C_i\left[V_{GS} - \left(V_{FB} - \frac{Q_d}{C_i} - V_s\right)\right]$$

However, it is only the first two terms from the equation in part (a) that contribute to the flat band voltage,

$$V_T = \underbrace{\frac{\phi_m - \phi_{sc}}{e} - \frac{Q_i}{C_i}}_{V_{FB}} - \frac{Q_d}{C_i} - 2V_f$$

Hence, substituting the flat band voltage gives

$$Q_f = -C_i\left[V_{GS} - \left(V_T + 2V_f - V_s\right)\right]$$

The surface potential V_s difference causing band bending is therefore the voltage required to induce strong inversion, $2V_f$ less the potential in the channel, so

$$V_s = 2V_f - V(x)$$

and hence,

$$\underline{Q_f = -C_i\left[V_{GS} - V_T - V(x)\right]}$$

(ii) The resistance of a small length of the channel δx is

$$\delta R = \frac{\rho\delta x}{t_{inv}W}$$

where t_{inv} is the thickness of the inversion layer and if the density of holes in the inversion layer is p_{inv}, then