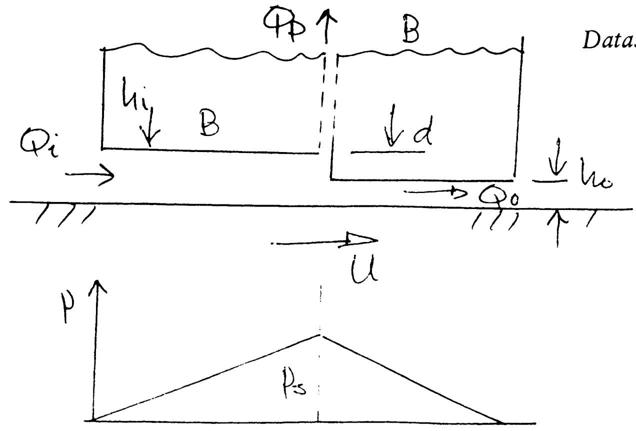
Part IIA - 2005 - 3C3 -

PART IIA 2005
3C3: Machine design – tribology
Principal Assessor: Dr J A Williams

Datasheet: 3C3/3C4 Data Sheet

1.



$$\frac{Q_i}{L} = -\frac{h_i^3}{12\eta}\,\frac{p_s}{B} + \frac{Uh_i}{2}$$

$$\frac{Q_o}{L} = -\frac{h_o^3}{12\eta}\,\frac{-p_s}{B} + \frac{Uh_o}{2}$$

But $\frac{Q_i}{L} = \frac{Q_p}{L} + \frac{Q_o}{L}$ and $Q_p = \beta Q_i$

$$\therefore (1-\beta)\left[-\frac{h_i^3}{12\eta}\,\frac{p_s}{B} + \frac{Uh_i}{2}\right] = \frac{h_o^3}{12\eta}\,\frac{p_s}{B} + \frac{Uh_o}{2}$$

i.e. $p_s\left[(1-\beta)h_i^3 + h_o^3\right] = 6B\eta U\left[(1-\beta)h_i - h_o\right]$

i.e. $p_s = \frac{6UB\eta}{h_o^2}\left\{\frac{(1-\beta)H - 1}{(1-\beta)H^3 + 1}\right\}$ where $H = \frac{h_i}{h_o}$

If $\beta = 0.25$ since $W = \frac{1}{2}p_s \cdot 2BL$

$$W = \left(\frac{6UB^2\eta L}{h_o^2}\right)\frac{0.75H - 1}{0.75H^3 + 1}$$

$$\frac{dW}{dH} = (\quad)\frac{(0.75H^3+1)\times 0.75 - (0.75H-1)\times 3 \times .75H^2}{(0.75H^3+1)^2}$$

$\Rightarrow 0$ when $0.75H^3 + 1 = 3(0.75H - 1)H^2$

i.e. $3H^3 + 4 = 9H^3 - 12H^2$

$$3H^3 - 6H^2 = 2$$

H	LHS
2	0
3	27
2.1	1.32
2.2	2.9
2.145	2.00

whence $H = 2.145$ i.e. $1 + \frac{d}{h_o} = 2.145$

$$d = 1.145h_o$$

2. (a) $u_{min} = (1-\varepsilon)c$

But $S = \frac{\eta\omega}{\bar{p}}\left\{\frac{R}{c}\right\}^2$ given —(1)

i.e. $c = \left(\frac{\eta\omega R^2}{\bar{p}}\right)^{1/2} / \sqrt{S}$

i.e. $u_{min} \propto \underbrace{\frac{(1-\varepsilon)}{\sqrt{S}}}$ So add this line to table

ε	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
S	102	47.6	28.2	17.7	11.2	6.72	3.64	1.65	0.46	.139
Q^*	0.0983	0.196	0.295	0.393	0.491	0.590	0.688	0.787	0.885	0.933
M^*	6.03	5.89	5.83	5.92	6.12	6.50	7.24	8.43	11.8	17.3
$(1-\varepsilon)/\sqrt{S}$					.149	.154	.157	.156	.147	
G_T					140	74	38.3	17.7	6.13	

$(1-\varepsilon)/\sqrt{S}$ has max when $\underbrace{\varepsilon \sim 0.7}$

(b) $M = \frac{\eta\omega L R^3}{c} \cdot M^*$ and $Q = LR\omega c Q^*$

Using adiabatic balance

$$M\omega = Q \times \rho \underbrace{s}_{\text{specific heat}} \times \Delta T$$

i.e. $\frac{\eta\omega^2 L R^3 M^*}{c} = 2R\omega c Q^* \times \rho s \times \Delta T$

$\therefore \Delta T = \eta\omega\frac{R^2}{c^2} \cdot \frac{M^*}{Q^*} \cdot \frac{1}{\rho s}$

i.e. $\Delta T = \frac{M^* S}{Q^*} \cdot \bar{p} \cdot \frac{1}{\rho s} = G_T\,\bar{p} \cdot \frac{1}{\rho s}$ —(2)

$R = 0.05\,m$, $D = 0.1\,m$, $L = 0.025\,m$ $\bar{p} = \frac{5\times10^3}{.1\times.025}$

$\Rightarrow 2\times10^6$ Pa

$\Delta T = 38.3 \times 2\times10^6 \times \frac{1}{880\times2\times10^3} \Rightarrow \underbrace{43.5°C}$

2.(a) $h_{min} = (1-\varepsilon)c$

But $\quad S = \frac{\eta\omega}{\bar{p}}\left\{\frac{R}{c}\right\}^2 \qquad$ —— (1)

ie. $\quad c = \left\{\frac{\eta\omega R^2}{\bar{p}}\right\}^{\frac{1}{2}} . \sqrt{S}$

ie. $\quad h_{min} \propto (1-\varepsilon)\sqrt{S}$

So add this line to the table.

ε	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
S	102	47.6	28.2	17.7	11.2	6.72	3.64	1.65	0.46	.139
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M^*	6.03	5.89	5.83	5.92	6.12	6.50	7.24	8.43	11.8	17.3
$(1-\varepsilon)\sqrt{S}$					·149	·154	·157	·156	·147	
G	6257	1430	557	267	140	74	38.3	17.7	6.13	2.58

This has a max when $\varepsilon \simeq 0.7$, (not very sensitive to ε)

(b) $\quad M = \frac{\eta\omega L R^3}{c} M^* \quad$ and $\quad Q = LR\omega c\, Q^*$

Work balance $\quad M.\omega = Q \times \rho s \times \Delta T$

ie. $\quad \frac{\eta\omega^2 L R^3}{c} M^* = LR\omega c \times \rho s \times \Delta T$

ie. $\quad \Delta T = \frac{\eta\omega R^2}{c^2} . \frac{M^*}{Q^*} . \frac{1}{\rho s}$

$$\Delta T = \frac{M^* S}{Q^*} \bar{p} . \frac{1}{\rho s} = G \bar{p} \frac{1}{\rho s} \qquad \text{—(2)}$$

$R = 0.05\,m \quad D = .1\,m \quad L = .025\,m \quad \therefore \bar{p} = \frac{5\times10^3}{.1 \times .025} = 2\times10^6\ Pa$

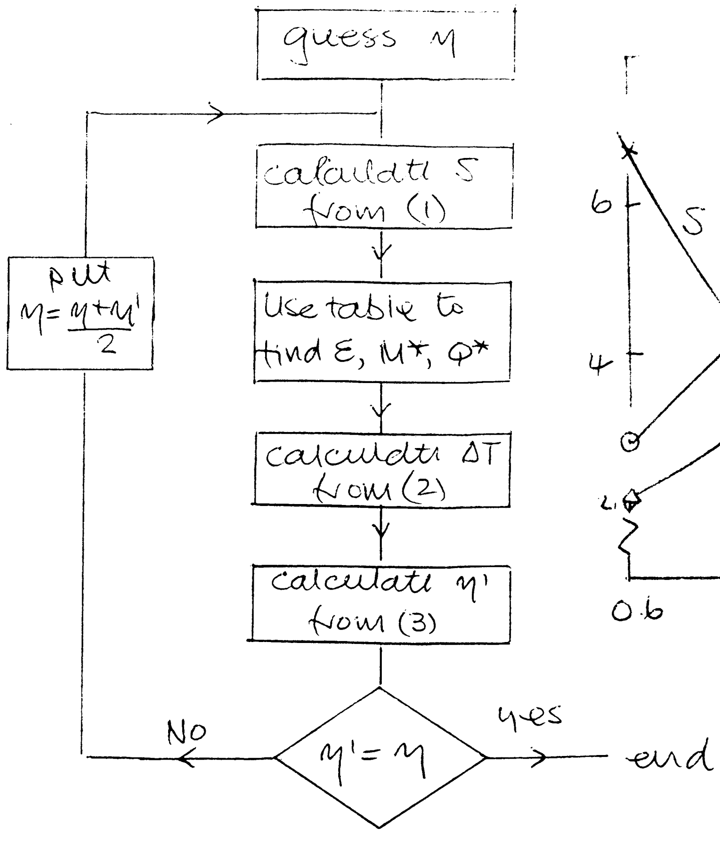
$$\Delta T = \frac{7.24 \times 3.64}{.688} \times 2\times10^6 \times \frac{1}{880 \times 2\times10^3} \Rightarrow 43.5°C$$

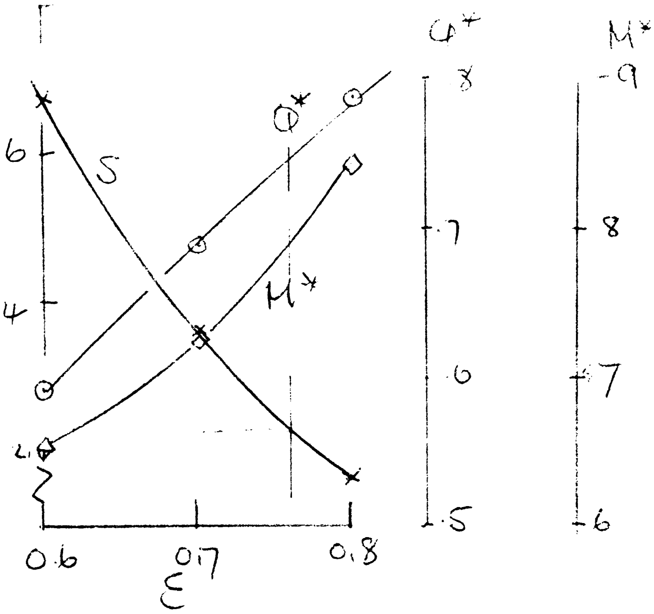
(c) When $S = 3.64$, $\eta = 3.64 \times \frac{P}{\omega} \times \left(\frac{C}{R}\right)^2 = 3.64 \times \frac{2\times10^6}{300} \times \left(\frac{75\times10^{-6}}{.05}\right)^2$

$= .0546$ Pas

Whereas at $\Delta T = 43.5°C$, $\eta' = .05 \exp(-.04\times43.5) = 0.0088$ Pa

A possible scheme could be to halve difference between η and η' until they converge.





So try $\eta = (.0546 + .0088)/2 \Rightarrow .032$ Pas

then $S = \frac{.032}{.0546} \times 3.64 = 2.13$

interpolating in table (or sketch) $\varepsilon = 0.76$, $\Phi^* = .745$, $M^* = 7.85$

then $\Delta T = \frac{7.85 \times 2.13}{.745} \times \frac{2.0\times10^6}{1.76\times10^6} \Rightarrow 25.5°C$

and $\eta = .05 \exp(-.04\times25.5) = 0.018$ Pas

Now repeat using $\eta = (0.032 + 0.018)/2 = .025$ Pas

$S = 1.67$

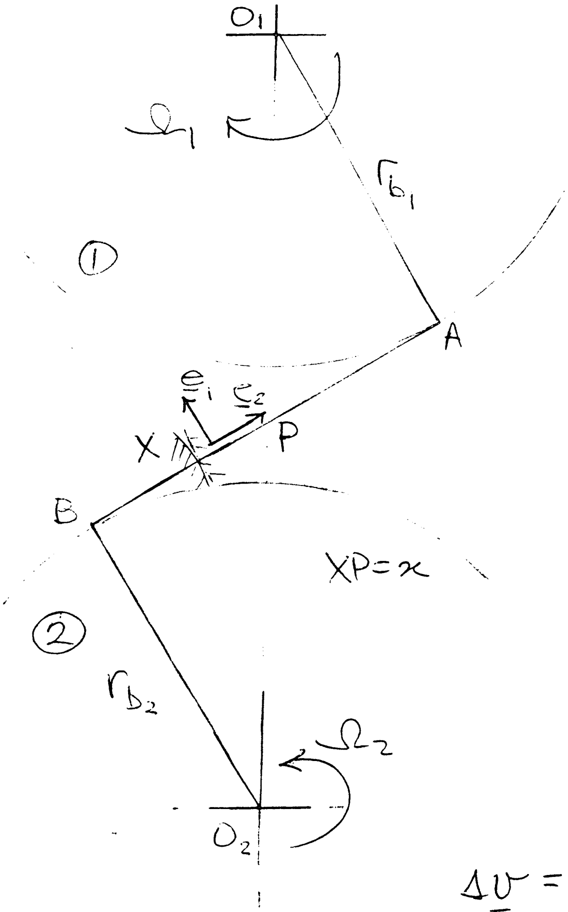
this leads to $\varepsilon \simeq 0.8$, $\Phi^* = .79$, $M^* = 8.43$ and

no real change in $(1-\varepsilon)\sqrt{S}$

$\Delta T \simeq 20.2°C$ $\eta = .0223$ Pas

close enough $\Delta T \sim 20°C$

3.(a) Involute tooth profile can be thought of as resulting from the unwinding of a string from base circle. Hence the normal to the tooth surface is the tangent to the base circle. At the contact point the two opposing teeth need to have a common normal which must be the line AB, the tangent to both base circles.



$$\underline{v}_{x_1} = -r_{b_1}\Omega_1 \underline{e}_2 + AX\Omega_1 \underline{e}_1$$

$$\underline{v}_{x_2} = -r_{b_2}\Omega_2 \underline{e}_2 + BX\Omega_2 \underline{e}_1$$

To maintain contact components of vely. in $\underline{e}_2$ dir'n must be same

$$\therefore \quad r_{b_1}\Omega_1 = r_{b_2}\Omega_2$$

$$\frac{\Omega_1}{\Omega_2} = \frac{r_{b_2}}{r_{b_1}}$$

Sliding velocity

$$\Delta \underline{v} = \underline{v}_{x_1} - \underline{v}_{x_2}$$

$$= (AX\Omega_1 - BX\Omega_2)\,\underline{e}_1$$

$$|\Delta v| \Rightarrow (AX - XP + XP)\Omega_1 - (BX + XP - XP)\Omega_2$$

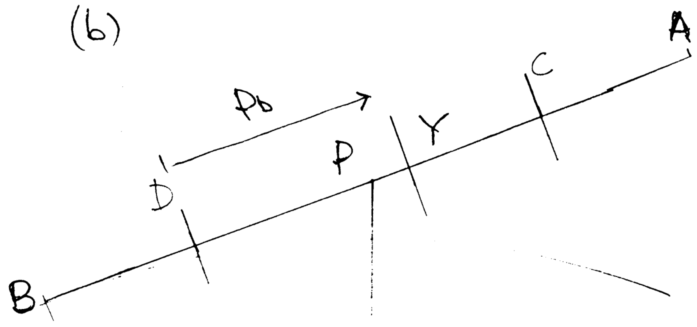
$$= AP\,\Omega_1 - BP\,\Omega_2 + XP(\Omega_1 + \Omega_2)$$

But from similar Δs $\quad \frac{AP}{BP} = \frac{r_{b_1}}{r_{b_2}}$

$$\therefore \quad AP\,\Omega_1 = BP\,\Omega_2$$

So that $|\Delta v| = x(\Omega_1 + \Omega_2)$

(b)



$PC = PD = l$

from data sheet

$$\frac{l}{m} = (0.2924\times 29^2 + 29+1)^{1/2} - 0.1710\times 29$$

$\therefore$ $PC = PD = 2.430\,m$

and $2l = 4.860\,m$

at critical point Y, when contact just moving from 2 teeth to 1

So radii are $BP - PD + DY - BY$

$$= \frac{Nm\sin\phi}{2} - 2.430m + 2.952m$$

and $AY = AP + PD - DY$

i.e. $5.481\,m$ and $4.437\,m$

$$\therefore R = \left[\frac{1}{5.481} + \frac{1}{4.437}\right]^{-1} m$$

$\Rightarrow 2.452\,m$

$$p_b = p\cos\phi = \frac{2\pi r}{N}\cos\phi$$

But $m = \dfrac{2r}{N}$

So $p_b = \pi m\cos\phi$

$p_b = 2.952\,m$

If contact force $\Rightarrow P$

then $p_0 = \left[\dfrac{P/w\,E^*}{\pi R}\right]^{1/2}$ w = tooth width

$$p_0^2 = [1200\times10^6]^2 = \frac{2T}{Nm\cos 20\times 5m}\times\frac{115\times10^9}{\pi\times 2.452m}$$

Torque $T = P\times r_b$

$$P = \frac{T}{r\cos\phi} = \frac{2T}{Nm\cos\phi}$$

$T = 5\,Nm$

$m^3 = 7.61\times10^{-10}$

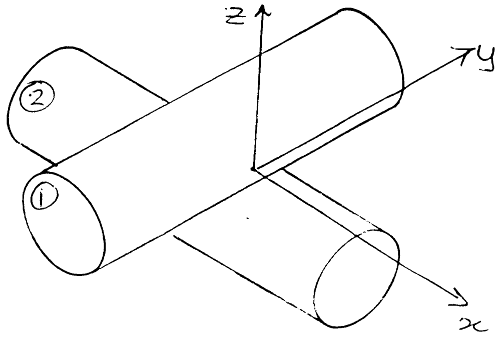
i.e. $m = 0.91$ mm

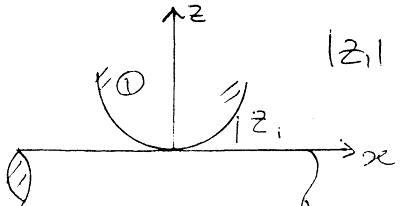
So take $m = 1$ mm

4. Hertzian idealisations

(a)
- smooth profiles - represent circles by parabolas
- small deformations
- no friction stress at surfaces
- linear elastic behaviour

(b)





$|z_1| \simeq \dfrac{x^2}{2R_1}$

Similarly $|z_2| \simeq \dfrac{y^2}{2R_2}$

So gap $z \simeq \dfrac{x^2}{2R_1} + \dfrac{y^2}{2R_2}$

If $R_1 = R_2$ then $z = \dfrac{1}{2R}(x^2+y^2)$ i.e. $\Rightarrow \dfrac{r^2}{2R}$

equivalent to sphere on flat.

(c) $R = .35\,m$ $E^* = 115\,GPa$ $W = 60\,kN$

from Data sheet $\delta = \dfrac{1}{2}\left\{\dfrac{9W^2}{2E^{*2}R}\right\}^{1/3}$ — (1)

i.e. $8\delta^3 = \dfrac{9W^2}{2E^{*2}R}$

Compliance is $\dfrac{d\delta}{dW}$ i.e. $24\delta^2 \dfrac{d\delta}{dW} = \dfrac{18W}{2E^{*2}R}$

(not $\dfrac{\delta_{total}}{W_{total}}$)

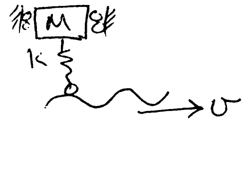
Substituting for δ from (1)

gives $\dfrac{d\delta}{dW} = (6RE^{*2}W)^{-1/3}$

$\Rightarrow [6 \times .35 \times (115\times10^9)^2 \times 60\times10^3]^{-1/3}$

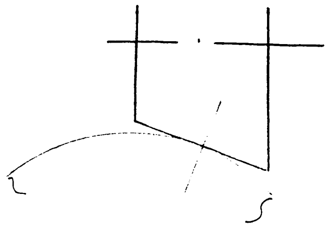
$\Rightarrow 8.437\times10^{-10}\ m/N$

$\omega_n = \sqrt{\dfrac{k}{m}} = (8.437\times10^{-10} \times 300)^{-1/2} = 1.99s^{-1} \Rightarrow 316.3\,Hz$



$\therefore\ v = 316.3 \times .05 = 15.8\,ms^{-1} = 56.9\,kph$

(c)



Conicity will lead to both non-circular contact patch geometry and to an element of spin between rail head and wheel.

Examiner's comments

Q.1 Popular and generally well done. Many candidates stumbled over finding a value of H which satisfies $3H^3 - 6H^2 = 2$

Q2 Parts (a) & (b) were very similar to an examples paper question & so well done. Good marks in (c) to any reasonable iterative procedure to reconcile conditions.

Q3 A very standard gear question. Not many demonstrations that sliding speed $\propto x$. A good number of correct numerical solutions.

Q4 Parts (a) & (b) generally well done. In (c) several candidates wanted compliance to mean total deflection ÷ total load rather than local gradient. Few mentions of spin in (c)(iii).

JAW
May 05