PART IIA 2005
3C4: Machine design – transmissions
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PART IIA 3C4 *Datasheet: 3C3/3C4 Data Sheet*

EXAMINER DR D J COLE

1

a Need to consider:

- contact forces associated with very large accelerations
- spring forces needed to maintain contact
- contact stiffness
- allow for early start and late finish to get best performance
- stress wear limitations for very high speeds
- minimise jerk – discontinuities in $\dddot{y}$ give high forces associated with changes in contact compression.

b

i Let $\theta = \omega t \dfrac{360}{2\pi}$, ω in rad/s

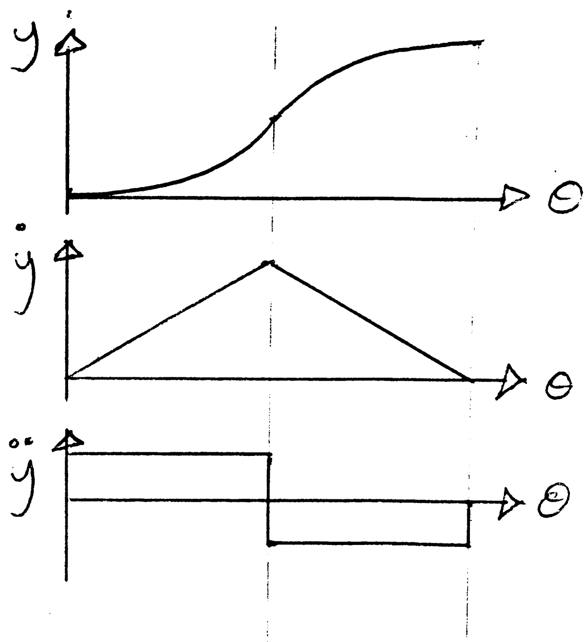
$$y = \frac{L}{2}\frac{\omega^2}{20^2}\left(\frac{360}{2\pi}\right)^2 t^2$$

$$= \frac{1}{2} L A^2 \omega^2 t^2 \qquad A = \frac{360}{20 \cdot 2\pi} = \frac{9}{\pi}$$

$$\dot{y} = L A^2 \omega^2 t$$

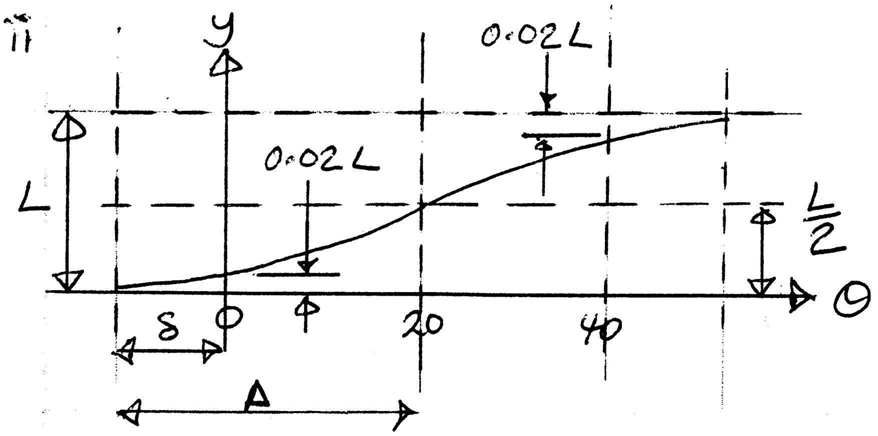
$$\ddot{y} = L A^2 \omega^2$$

$$= 81 \frac{L\omega^2}{\pi^2}$$



Any reduction in acceleration during the cycle leads to a reduction in the final displacement. Equal positive and negative acceleration phases are needed to ensure the follower is stationary at the end of the cycle.

ii



If acceleration is constant, during first half of lift, displacement is proportional to θ^2 or t^2.

Thus $\left(\frac{\delta}{\Delta}\right)^2 = \frac{0 \cdot 02L}{0 \cdot 5L}$

$\therefore \frac{\delta}{\Delta} = 0 \cdot 2$

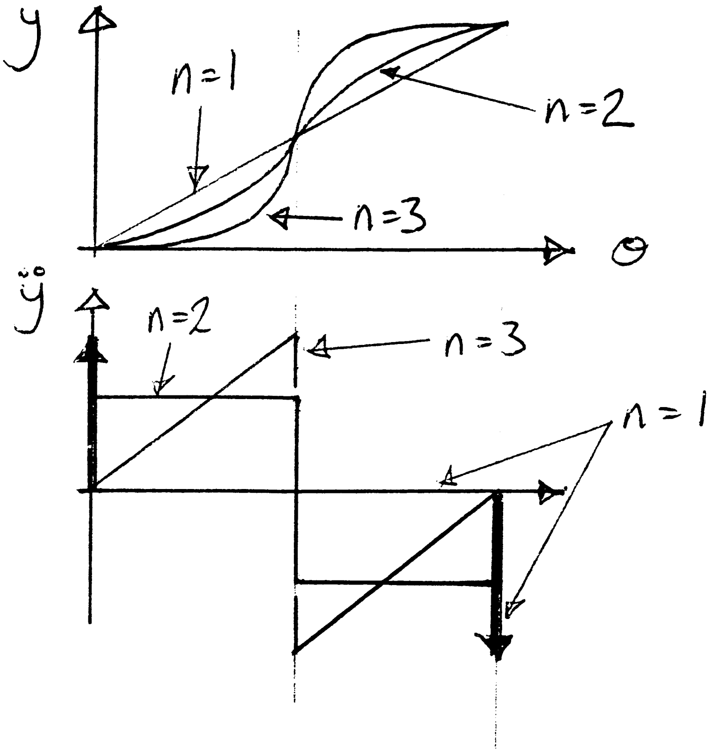
Now, $\delta - \Delta = 20^\circ$, so $\Delta = 25^\circ$

and

$$\ddot{y} = \left(\frac{360}{25.2}\right)^2 \frac{L\omega^2}{\pi^2}$$

$$= 51 \cdot 84 \frac{L\omega^2}{\pi^2}$$

iii Consider $n = 1, 2, 3$



$n=1$: $y = \frac{1}{2} L A \omega t$ $\quad 0^\circ < \theta \leq 20^\circ$

$\dot{y} = \frac{1}{2} L A \omega$

$\ddot{y} = 0$ $\quad$ but at $\theta = 0$, $\ddot{y} = \infty$ due to step in $\dot{y}$ $\Rightarrow$ large contact stresses

$n=3$: $y = \frac{1}{2} L A^3 \omega^3 t^3$ $\quad 0^\circ \leq \theta < 20^\circ$

$\dot{y} = \frac{3}{2} L A^3 \omega^3 t^2$

$\ddot{y} = 3 L A^3 \omega^3 t$

For $n=3$, peak acceleration is larger than for $n=2$.

Jerk is larger for $n=3$ than $n=2$, so $n=2$ appears to offer the best compromise.
A sinusoidal profile could provide better performance, with no discontinuity in acceleration.

(cont)

Many candidates had difficulty differentiating the lift equation. In part (b)(ii) a suitable solution method was often chosen but poor algebra led to an incorrect answer. Others used $s=0.5at^2$ to calculate the lift from the zero angle position, but neglected the velocity that the follower will have at this point when the cam is lifting early.

2

a

i Use virtual power

$$T_A \omega_A' + T_S \omega_S' + T_C \omega_C' = 0$$

put $\omega_C' = 0$

$$T_A \omega_A' = -T_S \omega_S'$$

$$\frac{T_A}{T_S} = -\left.\frac{\omega_S'}{\omega_A'}\right|_{\omega_C'=0} = +R = +\frac{A}{S}$$

(from the epicyclic speed equation on the data sheet)

Torque ratio is independent of speeds because torques depend only on static equilibrium of forces within the epicyclic.

ii epicyclic speed equation

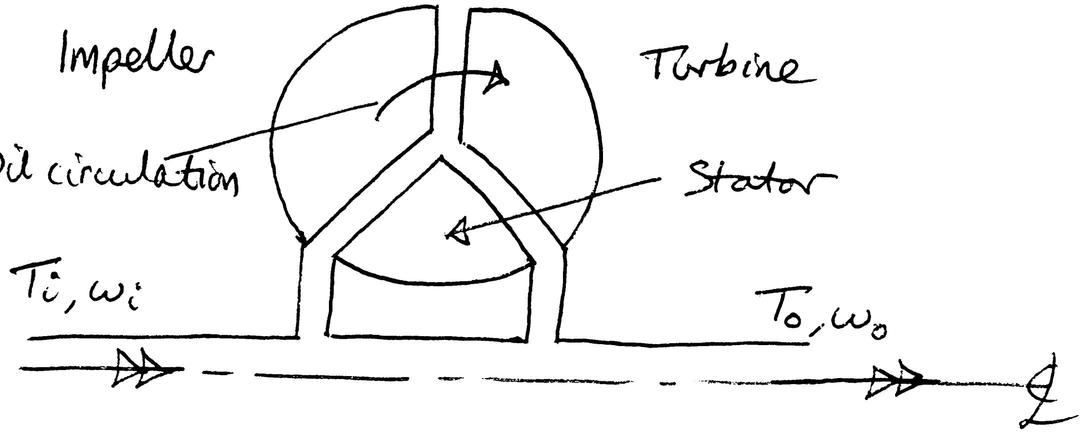
$$\omega_S = (1+R)\,\omega_C - R\omega_A \quad , \quad R = \frac{A}{S}$$

$$\omega_C = \frac{\omega_A}{2} \Rightarrow \omega_S = (1+R)\frac{\omega_A}{2} - R\omega_A$$

$$\therefore \frac{\omega_A}{\omega_S} = \frac{1}{\frac{1+R}{2} - R} = \frac{2}{1-R}$$

power ratio $\dfrac{T_A \omega_A}{T_S \omega_S} = \dfrac{2R}{1-R}$

b i



The converter-coupling works by hydrodynamic action. The impeller drives rotation of fluid, which circulates and drives the turbine. Vanes direct the motion of the fluid. In coupling mode ($\omega_o/\omega_i > 0.8$, here) the stator free wheels and the input and output torques are the same. In converter mode ($\omega_o/\omega_i < 0.8$) there is a reaction torque at the stator, so that the output torque exceeds the input torque.

ii In coupling mode $\left(\frac{\omega_o}{\omega_i} > 0.8\right)$, $T_i = T_o$ and so $\eta = \frac{\omega_o}{\omega_i}$

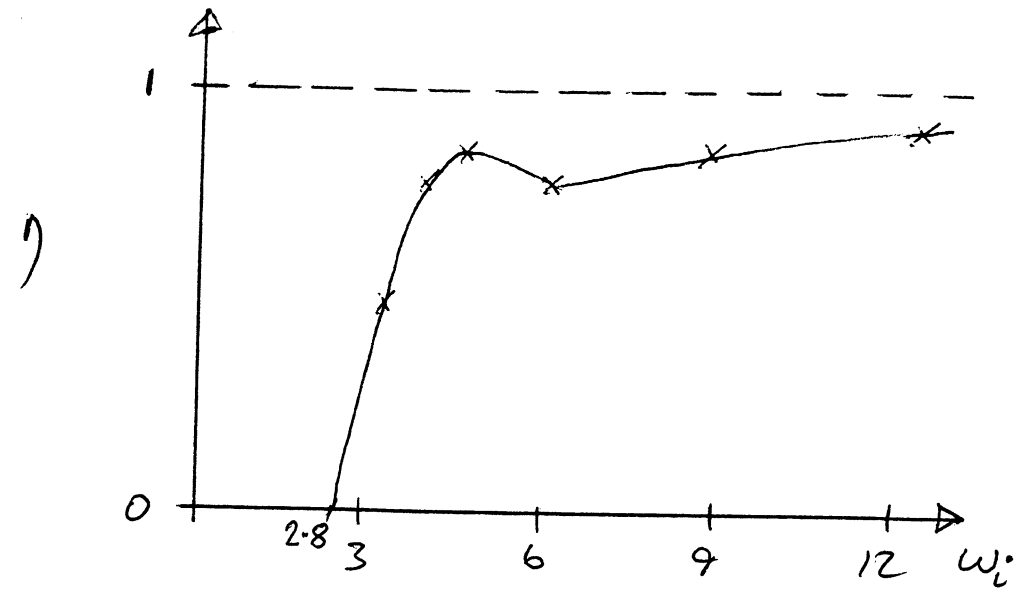
In converter mode, draw up a table

$$C_{T_i} = \frac{T_i}{\rho \omega_i^2 D^5} = \frac{0.2 \rho D^5}{\rho \omega_i^2 D^5} = \frac{0.2}{\omega_i^2}$$

$$\therefore \quad \omega_i = \sqrt{\frac{0.2}{C_{T_i}}} \qquad \text{——— (1)}$$

$$\text{and} \quad \eta = \left(\frac{\omega_o}{\omega_i}\right)\left(\frac{T_o}{T_i}\right) \qquad \text{——— (2)}$$

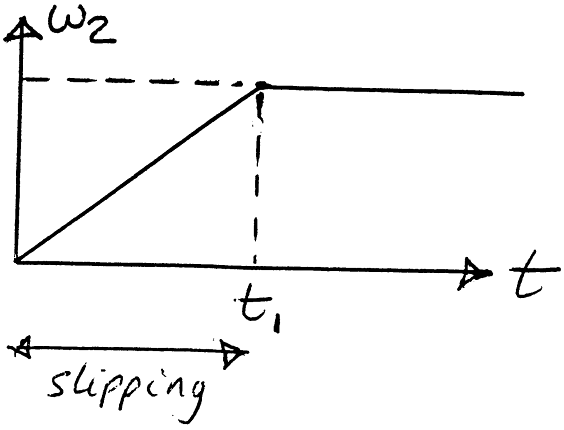
$\frac{\omega_o}{\omega_i}$	0	0.2	0.4	0.6	0.8	0.9	0.95	1.0
G_{T_i} (fig 1)	0.025	0.02	0.015	0.01	0.005	0.0025	0.00125	0
ω_i (eq 1)	2.8	3.2	3.7	4.5	6.3	8.9	12.6	–
T_o/T_i (fig 1)	3	2.5	2	1.5	1.0	1.0	1.0	1.0
η (eq 2)	0	0.5	0.8	0.9	0.8	0.9	0.95	1.0



Part (a) on the epicyclic gear was generally answered well. In part (b)(i) most candidates were able to explain the operation and characteristic of a converter coupling. In part (b)(ii) the efficiency was often sketched as a function of the speed ratio, not as a function of the input speed as requested.

3

a



$T_2 \omega_2 = T_3 \omega_3$

$G = \frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$

When clutch slips, $T_1 = T_2 = T_N$

Newton's 2nd Law on inertia: $J \dot{\omega}_3 = T_3 = T_2 G$

$\therefore J \frac{\dot{\omega}_2}{G} = T_N G$

$\therefore \dot{\omega}_2 = \frac{T_N G^2}{J}$

Time for ω_2 to reach ω_1 from 0.0:

$\dot{\omega}_2 t_1 = \omega_1 - 0$

$\therefore t_1 = \frac{\omega_1}{\dot{\omega}_2} = \underline{\underline{\frac{\omega_1 J}{T_N G^2}}}$

Energy from power source $= \int_0^{t_1} \omega_1 T_N \, dt = \omega_1 T_N t_1$

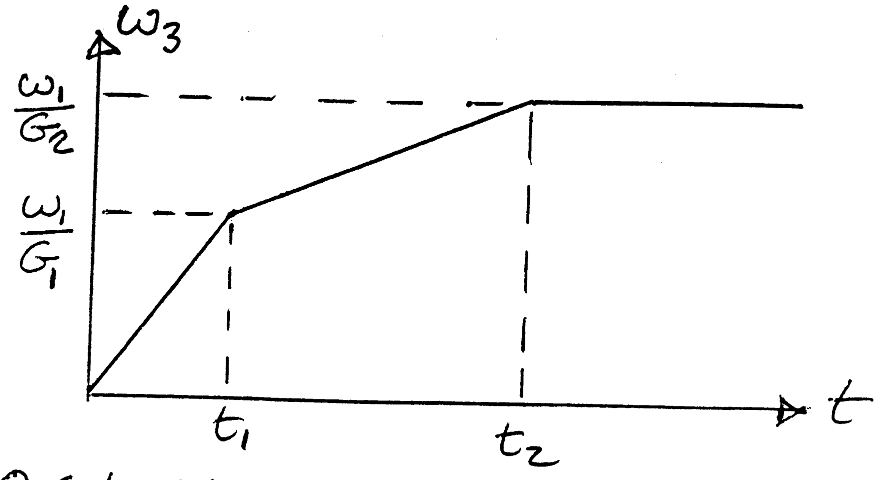
$= \omega_1 T_N \frac{\omega_1 J}{T_N G^2}$

$= \frac{\omega_1^2 J}{G^2}$

Energy into inertia $= \frac{1}{2} J \left(\frac{\omega_1}{G} \right)^2$

$\therefore$ Fraction lost $= \underline{\underline{\frac{1}{2}}}$

b i



$0 < t < t_1$

$$J\dot{\omega}_3 = T_3 = T_2 G$$

$$J\frac{\dot{\omega}_2}{G_1} = T_N G_1$$

$$\therefore \dot{\omega}_2 = \frac{T_N G_1^2}{J}$$

$$\dot{\omega}_3 t_1 = \frac{\dot{\omega}_2}{G_1} t_1 = \frac{\omega_1}{G_1} - 0$$

$$\therefore t_1 = \frac{\omega_1 J}{T_N G_1^2}$$

$t_1 < t < t_2$

$$J\dot{\omega}_3 = T_3 = T_2 G$$

$$J\frac{\dot{\omega}_2}{G_2} = T_N G_2$$

$$\therefore \dot{\omega}_2 = \frac{T_N G_2^2}{J}$$

$$\dot{\omega}_3(t_2 - t_1) = \frac{\dot{\omega}_2}{G_2}(t_2 - t_1) = \frac{\omega_1}{G_2} - \frac{\omega_1}{G_1}$$

$$\therefore t_2 - t_1 = \frac{J\omega_1}{T_N G_2}\left(\frac{1}{G_2} - \frac{1}{G_1}\right)$$

thus $$t_2 = \frac{\omega_1 J}{T_N}\left(\frac{1}{G_1^2} - \frac{1}{G_1 G_2} + \frac{1}{G_2^2}\right)$$

ii

Energy into inertia $= \frac{1}{2} J \left(\frac{\omega_1}{G_2}\right)^2$

Energy from power source $= \int_0^{t_2} \omega_1 T_N \, dt$

$$= \omega_1 T_N t_2$$

$$= J\omega_1^2\left(\frac{1}{G_1^2} - \frac{1}{G_1 G_2} + \frac{1}{G_2^2}\right)$$

$\therefore$ Energy dissipated $= J\omega_1^2\left(\frac{1}{G_1^2} - \frac{1}{G_1 G_2} + \frac{1}{G_2^2}\right) - \frac{1}{2} J \left(\frac{\omega_1}{G_2}\right)^2$

$$= J\omega_1^2\left(\frac{1}{G_1^2} - \frac{1}{G_1 G_2} + \frac{1}{2G_2^2}\right)$$

iii If $G_1 = 2$ and $G_2 = 1$, energy dissipated is

$$J\omega_1^2\left(\frac{1}{4} - \frac{1}{2} + \frac{1}{2}\right) = \frac{J\omega_1^2}{4}.$$

Fraction of energy from power source:

$$\frac{\frac{J\omega_1^2}{4}}{J\omega_1^2\left(1 - \frac{1}{2} + \frac{1}{4}\right)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \underline{\underline{\frac{1}{3}}}$$

compared to $\frac{1}{2}$ in part (a).

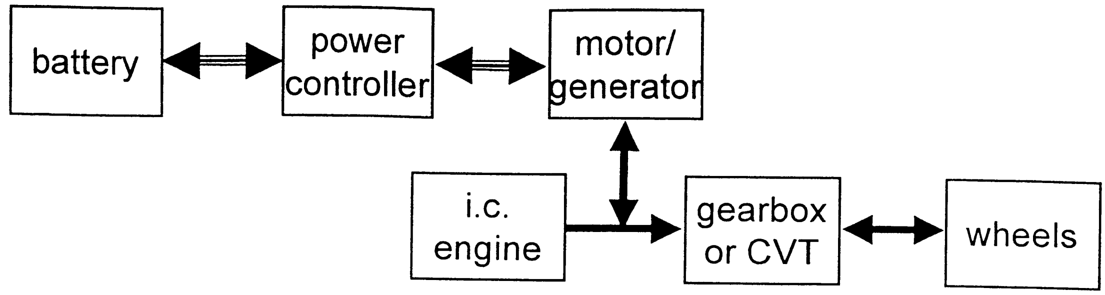
The energy dissipation is reduced because the two gear ratios allow closer match of the source and load speeds.

The dissipation can be further reduced by increasing the number of speed ratios. In the limit, the dissipation in the clutch can be eliminated with an infinite number of speed ratios (continuously variable transmission).

This was by far the least popular question and the average mark was low, although there were many good solutions. The question concerned a part of the syllabus that has not been examined regularly. Candidates that scored low marks demonstrated poor understanding of the relationships between torque, speed, energy, power and time.

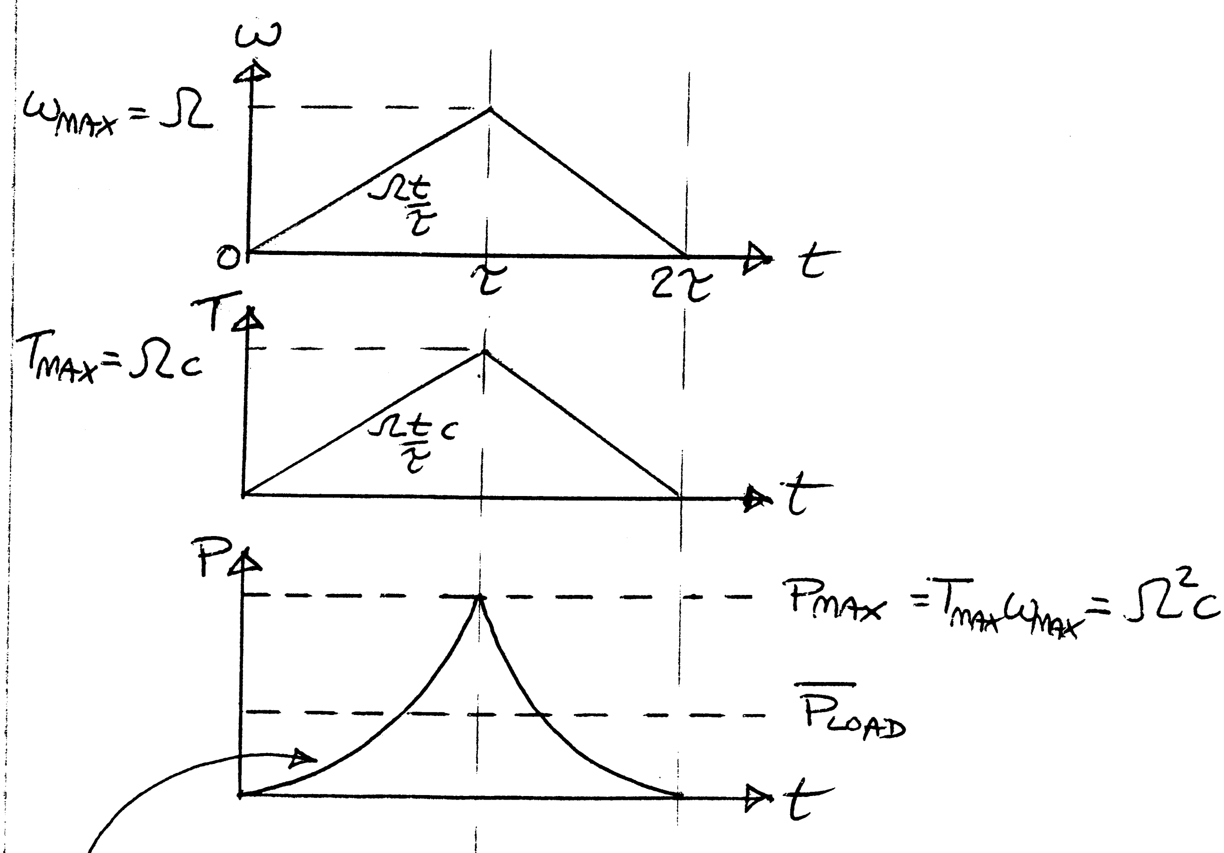
4

a



- Electric motor/generator directly coupled to engine crankshaft.
- Output of engine and motor add together (hence 'parallel').
- Engine provides the mean power requirement.
- Battery stores energy from engine or from braking.
- Alternative energy storage device is a flywheel, either mechanically coupled to the i.c. engine/gearbox, or electrically coupled via a second motor/generator.

b



$$P_{LOAD} = \omega T = \frac{\Omega t}{\tau} \cdot \frac{\Omega t}{\tau} c = \frac{\Omega^2 t^2}{\tau^2} c \qquad 0 < t < \tau$$

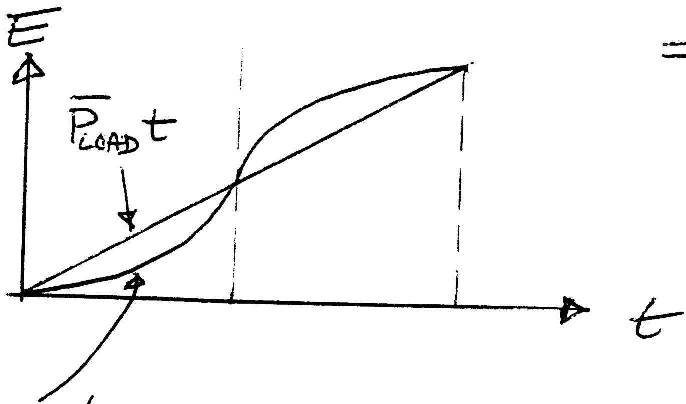
$$\overline{P_{LOAD}} = \frac{1}{\tau}\int_0^\tau \frac{\Omega^2 t^2 c}{\tau^2}\, dt = \frac{1}{\tau}\left[\frac{\Omega^2 t^3 c}{3\tau^2}\right]_0^\tau = \underline{\underline{\frac{1}{3}\Omega^2 c}}$$

c
i max power into store $= \overline{P}_{LOAD} - 0 = \underline{\underline{\frac{1}{3}\Omega^2 c}}$

max power out of store $= P_{MAX} - \overline{P}_{LOAD} = \Omega^2 c - \frac{1}{3}\Omega^2 c$

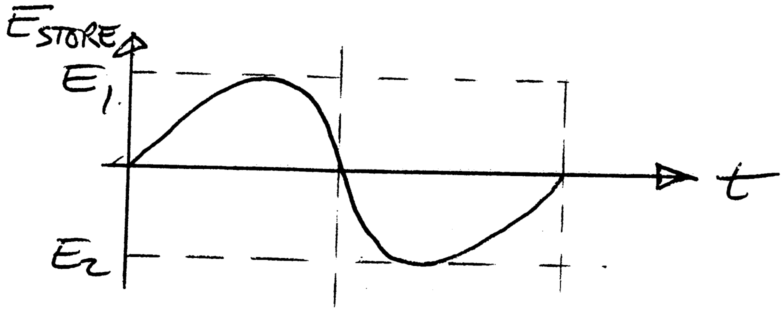
$$= \underline{\underline{\frac{2}{3}\Omega^2 c}}$$

ii



$$E_{LOAD} = \int_0^t P_{LOAD}\, dt = \int_0^t \frac{\Omega^2 t^2 c}{\tau^2}\, dt = \frac{\Omega^2 t^3 c}{3\tau^2}$$

$0 < t < \tau$



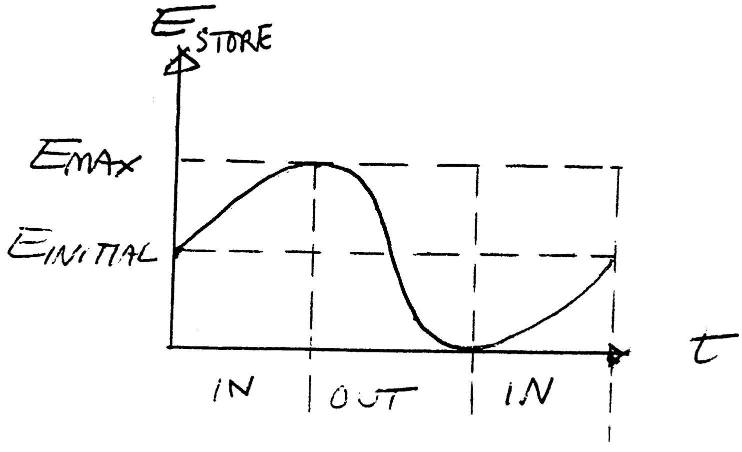
$E_{STORE} = \overline{P}_{LOAD}\, t - E_{LOAD}$ ——— ①

E_{STORE} is a maximum when $P_{LOAD} = \overline{P}_{LOAD}$

$$\frac{\Omega^2 t^2 c}{\tau^2} = \frac{1}{3}\Omega^2 c \implies t = \frac{\tau}{\sqrt{3}}$$

hence using ① $E_1 = -E_2 = \frac{1}{3}\Omega^2 c \frac{\tau}{\sqrt{3}} - \frac{\Omega^2 c}{3\tau^2}\frac{\tau^3}{3\sqrt{3}}$

$$= \Omega^2 c\, \tau \, \frac{2}{9\sqrt{3}}$$



Cannot have negative energy storage,
hence $E_{INITIAL} = E_1 = \Omega^2 c \tau \dfrac{2}{9\sqrt{3}}$

iii $\quad E_{MAX} = E_1 - E_2 = 2E_1 = \Omega^2 c \tau \dfrac{4}{9\sqrt{3}}$

d $\quad$ Total energy flow into store is $2E_1$
" " " out of store is $2E_1$

Hence energy lost $E_{LOST} = 4E_1(1-\eta)$

Hence mean power must increase by $\dfrac{E_{LOST}}{2\tau}$

$$= \dfrac{\dfrac{4\Omega^2 c \tau\, 2}{9\sqrt{3}}(1-\eta)}{2\tau}$$

$$\overline{P}_{INCREASE} = \frac{4}{9\sqrt{3}}\Omega^2 c(1-\eta)$$

assuming small $(1-\eta)$

Parts (a) and (b) were straightforward and answered well. In parts (c)(ii) and (iii) many candidates failed to consider the second half of the duty cycle, where the minimum in stored energy occurs. Few sketched the stored energy as a function of time, and although this was not asked for in the question, it would have helped in answering the question correctly. A worrying number of candidates stated that power = energy x time.