PART IIA 2005
3D1: Soil mechanics
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Datasheet: Soil Mechanics Data Book

Engineering Tripos
Soil Mechanics

1 a) $G_s = 2.70$, $S_r = 1$, final $w = 0.\cancel{442}\ 208$

$\therefore$ final $e = 2.70 \times \cancel{0.442}\ 0.208 = 0.562$

$\therefore$ final $V = 1 + e = 1.562$

There ~~are a fixed number~~ is a fixed volume of soil solids in the oedometer.

$\therefore V \propto h$

So the sequence of V values can be deduced,

σ' kPa	20	40	80	40	20
V	1.711	1.612	1.513	1.538	1.562

Note that when the stress doubles V reduces by 0.099 and when it halves V increases by 0.0245.

For normal compression:

$V = N - \lambda \ln \sigma'$ so $\underline{\lambda = 0.099/0.693 = 0.143}$

$V = 1.711$ at $\sigma' = 20$ so $\underline{N = 2.139}$

On rebound:

$V = V_\kappa - \kappa \ln \sigma'$ so $\underline{\kappa = 0.025/0.693 = 0.035}$

According to Cam Clay $\Gamma = N - (\lambda - \kappa) = 2.031$ 6

[Marking: many solutions will be based on a graph, and full marks should be given for λ and κ values within 10% of the "right" answer.

Some students may simply quote the N value, as the intercept of the NCL on the $\ln \sigma' = 0$ axis. This should be accepted in lieu of Γ. The only reason for requesting Γ is that students will be more familiar with it, following Cam Clay.]

b) The full data of the C_v test is:

t min	0	1	2	4	8	16	60
h mm	17.90	17.62	17.51	17.34	17.12	16.93	16.81
$\sqrt{t}$ min	0	1	1.414	2	2.828	4	7.746
Δh mm	0	0.28	0.39	0.56	0.78	0.97	1.09
$\Delta h/\Delta h_{ult}$	0	0.257	0.358	0.514	0.716	0.890	1

1 b cont.) Note linearity of early data of $R_v = \frac{\Delta h}{\Delta h_{ult}}$ versus $\sqrt{t}$. According to parabolic isochrones:

$$R_v = \sqrt{\frac{4}{3}T_v} = \sqrt{\frac{4}{3}\frac{c_v t}{d^2}}$$

$$\therefore c_v = \frac{3d^2}{4}\left(\frac{R_v}{\sqrt{t}}\right)^2$$

Take d as $\frac{(17.90 + 16.81)}{2 \times 2} = 8.68\ \text{mm}$

Find early $\frac{R_v}{\sqrt{t}} = \frac{0.514}{2} = 0.257\ \text{min}^{-1/2}$

$$\therefore c_v = 0.75 \times 8.68^2 \times 0.257^2\ \text{mm}^2/\text{min}$$

$$\therefore c_v = 3.73\ \text{mm}^2/\text{min} = \underline{6.2 \times 10^{-8}\ \text{m}^2/\text{s}}$$ 6

[Marking: most solutions will be based on a graph, but some will use a single R_v, T_v pair from the listed Fourier solutions. Full marks can be given for c_v values within 10% of the "right answer". Many students will take a simpler view of the drainage distance d; this can be forgiven.]

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c) (i) The definition of c_v from consolidation theory is $(E_o k / \gamma_w)$. The change of v from 80 kPa to 40 kPa is very small (1.7%) so the change of permeability k will be negligible. On the other hand

$$E_o \text{ loading} = \sigma' v / \lambda \ ; \quad E_o \text{ unloading} = \sigma' v / \kappa$$

So c_v unloading $= c_v$ loading $\times \lambda/\kappa \simeq \underline{25 \times 10^{-8}\ \text{m}^2/\text{s}}$ 4

(ii) If $c_v(ncl)$ = constant

Then $\frac{\sigma' v}{\lambda}\frac{k}{\gamma_w}$ = constant

But λ and γ_w are supposed to be constant, v does not change much, so it implies $\underline{k_{ncl} \propto \frac{1}{\sigma'}}$

This supports the idea of clay agglomerates crushing, collapsing the macropores, and strongly reducing the diameters of flow channels. 4

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1 c)

[Marking: students have encountered the use of λ and κ, as appropriate, to calculate E_o values in consolidation questions in Examples Paper 2 Questions 6 & 7.

However, part (c) clearly does call for conceptual thinking whereas parts (a) and (b) are routine applications of basic ideas. That is why part (c) is judged to be worth 8 marks.]

2. a) Cone volume $= \pi \frac{0.1^2}{4} \times 0.3 = 2.356 \times 10^{-3} m^3$

$$\therefore \rho = 4.845/2.356 = \underline{2056 \text{ kg/m}^3}$$

$$w = \frac{20.32 - 16.93}{16.93} = \underline{0.200}$$

$$G_s = 15.41/5.749 = \underline{2.68}$$

$$\frac{\rho}{\rho_w} = \frac{G_s + eS_r}{1+e} \quad \text{and} \quad w = \frac{eS_r}{G_s}$$

$$\therefore \frac{G_s + eS_r}{1+e} = 2.056 \quad \text{and} \quad eS_r = 0.2\,G_s$$

$$\therefore e = \cancel{0.624/1.056 = 0.591} \; \underline{0.564}\,; \quad S_r = \cancel{0.907} \; \underline{0.95}$$ 6

Optimum conditions for compaction of fine grained soils leads to saturation ratios of 0.9 to 1.0, as here. The density is also high, and typical.

[Marking: formulae are given in the Data Book, but students have to organise the unknowns in the later part. It is essential to keep high precision in the arithmetic. Marks will be lost if S_r does not fall in the range 0.94 to 0.96.]

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b) Since $\rho = 2056 \text{ kg/m}^3$, $\gamma = 20.2 \text{ kN/m}^3$

So at $Z = 1\text{m}$: $\sigma_v = 20.2 \text{ kPa} \simeq 20 \text{ kPa}$

at t_1, $\sigma_v' = 20 - (-40) = 60 \text{ kPa}$

at t_2, $\sigma_v' = 20 - 0 = 20 \text{ kPa}$

at t_3, $\sigma_v' = 20 - 10 = 10 \text{ kPa}$

And at $Z = 3\text{m}$: $\sigma_v \simeq 60 \text{ kPa}$

at t_1, $\sigma_v' = 60 - (-20) = 80 \text{ kPa}$

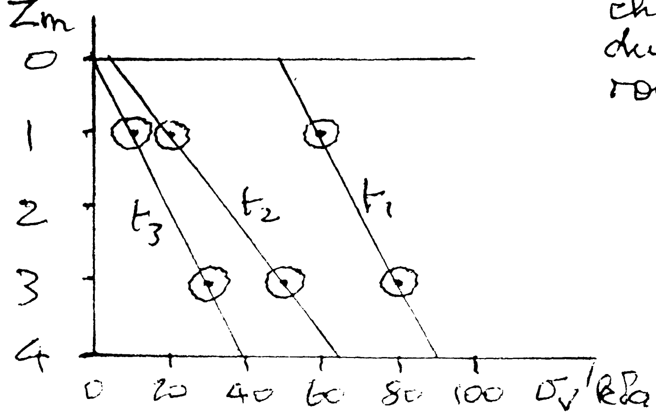
at t_2, $\sigma_v' = 60 - 10 = 50 \text{ kPa}$

at t_3, $\sigma_v' = 60 - 30 = 30 \text{ kPa}$

We have ignored the effect of the slopes and used 1D stress assumption. We have assumed that the embankment flooded to its crest, and ignored flow.

2 b cont.)

We have also ignored changes of soil unit weight due to swelling, and have rounded γ and γ_w values.



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[Marking: piezometer data is only accurate to nearest kPa (if that) so it is excusable to round $\gamma = 20$ and γ_w to 10 kN/m³. We are looking for correct use of Terzaghi's principle with pore pressures positive or negative]

c) Let us define excess pore pressure

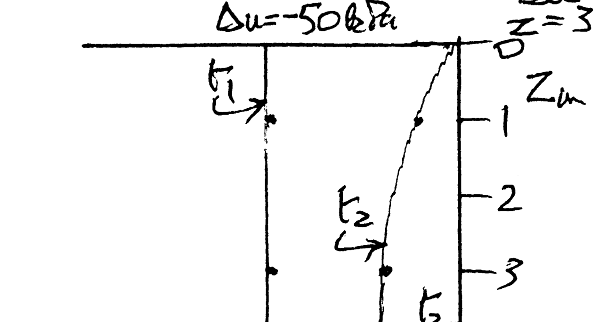
$$\Delta u = u_{t1} - u_{t3}$$

So at time t_1, $\Delta u_{z=1m} = -50\,kPa$

$\Delta u_{z=3m} = -50\,kPa$

And at time t_2, $\Delta u_{z=1m} = -10\,kPa$

$\Delta u_{z=3m} = -20\,kPa$



At t_3 all $\Delta u = 0$ by definition.

Sketch an assumed parabola for t_2 consistent with inflow at the embankment crest, and no flow at the base.

Rv at t_2 is swept area / total area

$$\therefore R_v = \left(50\times4 - \tfrac{2}{3}\times20\times4\right)/(50\times4) = \underline{0.73}$$

2 c cont.) Using $R_v = 1 - \frac{2}{3}\exp(0.25 - 3T_v)$

$\exp(0.25 - 3T_v) = 0.4$

$\therefore \underline{T_v = 0.39}$

Taking $d = 4\,m$, $t_2 = 3$ days

$$C_v = \frac{0.39 \times 4^2}{3} = 2.1\ m^2/day$$

$\underline{(\text{or } 2.4 \times 10^{-5}\ m^2/s)}$

For $R_v = 0.90$, $0.9 = 1 - \frac{2}{3}\exp(0.25 - 3T_v)$

$\therefore T_v = 0.72$

and $t = 3 \times 0.72/0.39 = \underline{5\tfrac{1}{2}\text{ days}}$ 6

[Marking: it is acceptable to take T_v, R_v values from the Table in the Data Book.]

d) The embankment was apparently well compacted.

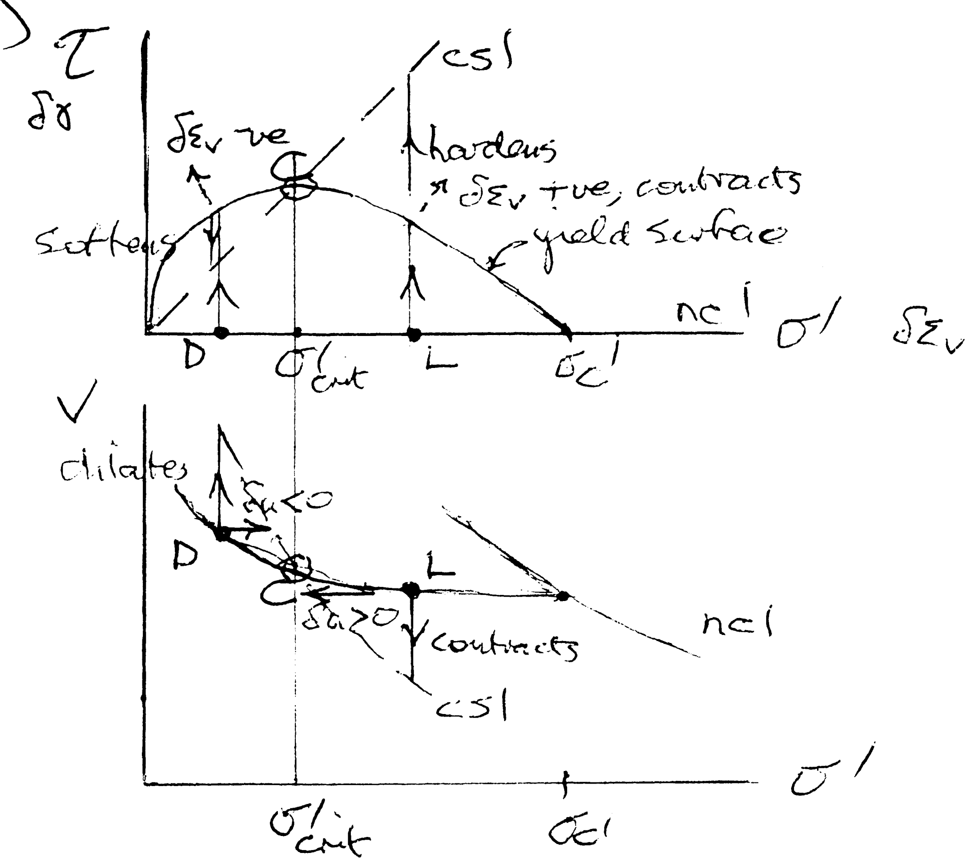
But the purpose of a flood embankment is to retain water, albeit temporarily. The rate of swelling when the flood arrives is too fast. Within a week, all suctions are lost. Water will arrive at the landward slope which stands at 1V : 2H which is 26.6°. This must be close to the angle of friction of the clay. Any positive pore pressures under the slope could lead to slippage and the opening of tension cracks, as observed.

The soil is too permeable. 2

[Marking: the tricky aspect of (c) and (d) is that the soil is swelling instead of consolidating. Students have encountered this before in Examples Paper 2 Question 6 but negative excess pore pressures may nevertheless cause conceptual difficulty.

Some credit must be given for showing knowledge of isochrones, R_v and T_v even if the actual isochrones are drawn incorrectly.]

3. a)



Yielding with $\sigma' > \sigma'_{crit}$ e.g. from L undrained shear test, leads to contraction & hardening.

yielding with $\sigma' < \sigma'_{crit}$ eg. from D undrained shear test, leads to dilation & softening.

The key is that soil must find a critical state after yielding. If undrained it must loose σ' from L or gain σ' from D. Since it is shearing at constant σ, this can only be achieved by generating $\delta u > 0$ from L, $\delta u < 0$ from D. 6

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b) (i) In an undrained test $\tau_{u,max} = \tau_{u,ult}$ if $\sigma' > \sigma'_{crit}$, i.e if $\sigma_c'/\sigma_0' < E$ (i.e 2.72)

If $\sigma_c'/\sigma_0' > 2.72$, the yield stress τ_y might possibly exceed $\tau_{u,ult}$.

3 b cont.) (i) $v_o = \Gamma + \lambda - \kappa - \lambda \ln \sigma_c' + \kappa \ln \frac{\sigma_c'}{\sigma_o'}$

But $v_o = v_{u,crit} = \Gamma - \lambda \ln \sigma_{crit}'$

And $\tau_{u,ult} = \tan \phi_{crit} \, \sigma_{crit}'$

So $\frac{\tau_{u,ult}}{\sigma_o'} = \tan \phi_{crit} \frac{\sigma_{crit}'}{\sigma_o'}$

where $\ln \sigma_{crit}' = \ln \sigma_c' - \frac{\kappa}{\lambda} \ln \sigma_c' + \frac{\kappa}{\lambda} \ln \sigma_o' - \frac{(\lambda - \kappa)}{\lambda}$

so $\sigma_{crit}' = (\sigma_c')^{1-\frac{\kappa}{\lambda}} (\sigma_o')^{\frac{\kappa}{\lambda}} E^{-(1-\frac{\kappa}{\lambda})}$

$\therefore \frac{\tau_{u,ult}}{\sigma_o'} = \tan \phi_{crit} \left(\frac{\sigma_c'}{E\sigma_o'}\right)^{1-\frac{\kappa}{\lambda}}$ where E is 2.72

(ii) Find τ_y on the "dry" side and ask if $\tau_y > \tau_{u,ult}$.

The equation of the yield surface is:

$$\frac{\tau_y}{\sigma_o'} = \tan \phi_{crit} \ln \frac{\sigma_c'}{\sigma_o'}$$

Evaluate this, compare with (i), and if greater select this for (ii). Otherwise (ii) $\equiv$ (i)

(iii) $\frac{\tau_{d,ult}}{\sigma_o'} = \tan \phi_{crit}$ by definition

(iv) $\frac{\tau_{d,max}}{\sigma_o'} = \tan \phi_{crit} \ln \frac{\sigma_c'}{\sigma_o'}$ as per (ii)

for yield on the "dry" side with $\frac{\sigma_c'}{\sigma_o'} > 2.72$.

Otherwise (iv) $\equiv$ (iii)

At OCR $= 1$, $\sigma_c' = \sigma_o'$

$$\frac{\tau_{u,ult}}{\sigma_o'} = \frac{\tau_{u,max}}{\sigma_o'} = \frac{\tan \phi_{crit}}{E^{1-\kappa/\lambda}} = \frac{\tan \phi_{crit}}{2.72^{0.62}}$$

$$\therefore \frac{\tau_{u,ult}}{\sigma_o'} = \frac{\tan \phi_{crit}}{1.86} = \underline{0.23}$$

$$\frac{\tau_{d,ult}}{\sigma_o'} = \frac{\tau_{d,max}}{\sigma_o'} = \tan \phi_{crit} = \underline{0.42}$$

3 b cont.) At OCR = 10, $\sigma_c' = 10\,\sigma_o'$

$$\frac{\tau_{u,ult}}{\sigma_o'} = \tan\phi_{crit}\left(\frac{10}{2.72}\right)^{0.62} = 2.24\tan\phi_{crit}$$

$$\therefore \frac{\tau_{u,ult}}{\sigma_o'} = 0.95$$

$$\frac{\tau_y}{\sigma_o'} = \tan\phi_{crit}\ \ln 10 = 2.30\tan\phi_{crit}$$

$$\therefore \frac{\tau_{u,max}}{\sigma_o'} = 0.98$$

$$\frac{\tau_{d,ult}}{\sigma_o'} = \tan\phi_{crit} = 0.42$$

$$\frac{\tau_{d,max}}{\sigma_o'} = \frac{\tau_y}{\sigma_o'} = 0.98$$ 10

Whenever $\tau_{max} > \tau_{ult,crit}$, τ may fall further as $\phi_{crit} \rightarrow \phi_{resid}$.
This is known as the "residual strength" on "slickensides".

[Marking: the crucial (and difficult) question is whether the student understands the potential fall from peak strength to critical state strength in the case of soils yielding on the "dry" side.

The ultimate undrained and drained strengths is a much more routine question

Students who miss the algebra but compute all 4 ratios for both OCR 1 and 10 might get 6/10]

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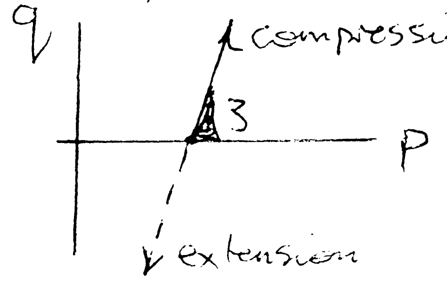
c) Mud has $\tau_u < \tau_d$ and can not stand at its "angle of repose". It slumps with temporary excess pore pressures (+ve).

Stiff clay has $\tau_u > \tau_d$ and is strain softening as well as softening due to water ingress satisfying negative excess pore pressures if it is stressed beyond yield. It seems very stable, but this is only temporary. If rupture bands form, ϕ_{crit} drops to ϕ_{res}. 4

4 a) $q = \sigma_a - \sigma_r$; $p = \frac{1}{3}(\sigma_a + 2\sigma_r)$

If $\delta\sigma_r = 0$, $\delta q = \delta\sigma_a$ and $\delta p = \frac{1}{3}\delta\sigma_a$

$\therefore \frac{\delta q}{\delta p} = 3$



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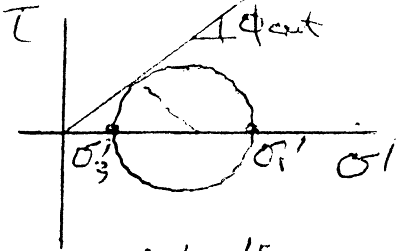
b) $M = \left(\frac{q}{p'}\right)_{crit}$



$$\sin\phi_{crit} = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2} \qquad \therefore \left(\frac{\sigma_1'}{\sigma_3'}\right)_{crit} = \frac{1+\sin\phi_{crit}}{1-\sin\phi_{crit}}$$

In a compression test, $\sigma_a' = \sigma_1'$, $\sigma_r' = \sigma_3'$

$$M_{comp} = \frac{(\sigma_1' - \sigma_3')}{\frac{1}{3}(\sigma_1' + 2\sigma_3')} = 3 \cdot \frac{2\sin\phi_{crit}}{3 - \sin\phi_{crit}}$$

$$\therefore M_{comp} = \frac{6\sin\phi_{crit}}{3 - \sin\phi_{crit}}$$

In an extension test, $\sigma_a' = \sigma_3'$, $\sigma_r' = \sigma_1'$

$$M_{extn} = \frac{(\sigma_1' - \sigma_3')}{\frac{1}{3}(2\sigma_1' + \sigma_3')} = 3 \cdot \frac{2\sin\phi_{crit}}{3 + \sin\phi_{crit}}$$

$$\therefore M_{extn} = \frac{6\sin\phi_{crit}}{3 + \sin\phi_{crit}}$$

4

[Marking: since this was an exercise set in Examples Paper 4 Question 2, it is not appropriate to award more marks]

4 c) Ham River Sand has $\sigma_c' = 15$ MPa when dense. It is tested at $I_D = 1$. $\phi_{crit} = 32°$.

For a triaxial test, $\phi_{max} - \phi_{crit} = 3\,I_R$
where $I_R = I_D\,I_C - 1$ and $I_C = \ln(\sigma_c'/p')$

First stab: take $p' = \sigma_{cell} = 1$ MPa

$\therefore I_R = \ln 15 - 1 = 1.71$

$\therefore \underline{\phi_{max} = 32° + 5° = 37°}$

Compression test: $\sigma_a' = \dfrac{1 + \sin 37°}{1 - \sin 37°} \cdot 1$ MPa

$\therefore \sigma_a' = 4.02$ MPa

$\therefore p' = (4 + 1 + 1)/3 = 2$ MPa (better value)

$\therefore I_R = \ln 7.5 - 1 = 1.01$

$\therefore \phi_{max} = 32° + 3° = 35°$

$\therefore \underline{\sigma_a' = 3.69 \text{ MPa}}$ (enough iteration)

Extension test: $\sigma_a' = \dfrac{1 - \sin 37°}{1 + \sin 37°} \cdot 1$ MPa

$\therefore \sigma_a' = 0.25$ MPa

$\therefore p' = (1 + 1 + 0.25)/3 = 0.75$ MPa

$\therefore I_R = \ln 20 - 1 = 2.0$

$\therefore \phi_{max} = 32° + 6° = 38°$

$\therefore \underline{\sigma_a' = 0.24 \text{ MPa}}$

$\therefore \underline{q_{comp} \cong 2.69 \text{ MPa}, \; q_{extn} \cong -0.76 \text{ MPa}}$ 6

d) In a critical state $I_R = 0$, so $I_C = 1/I_D$

$\therefore \ln 15/p'_{crit} = 1$, $\underline{p'_{crit} = 5.5 \text{ MPa}}$

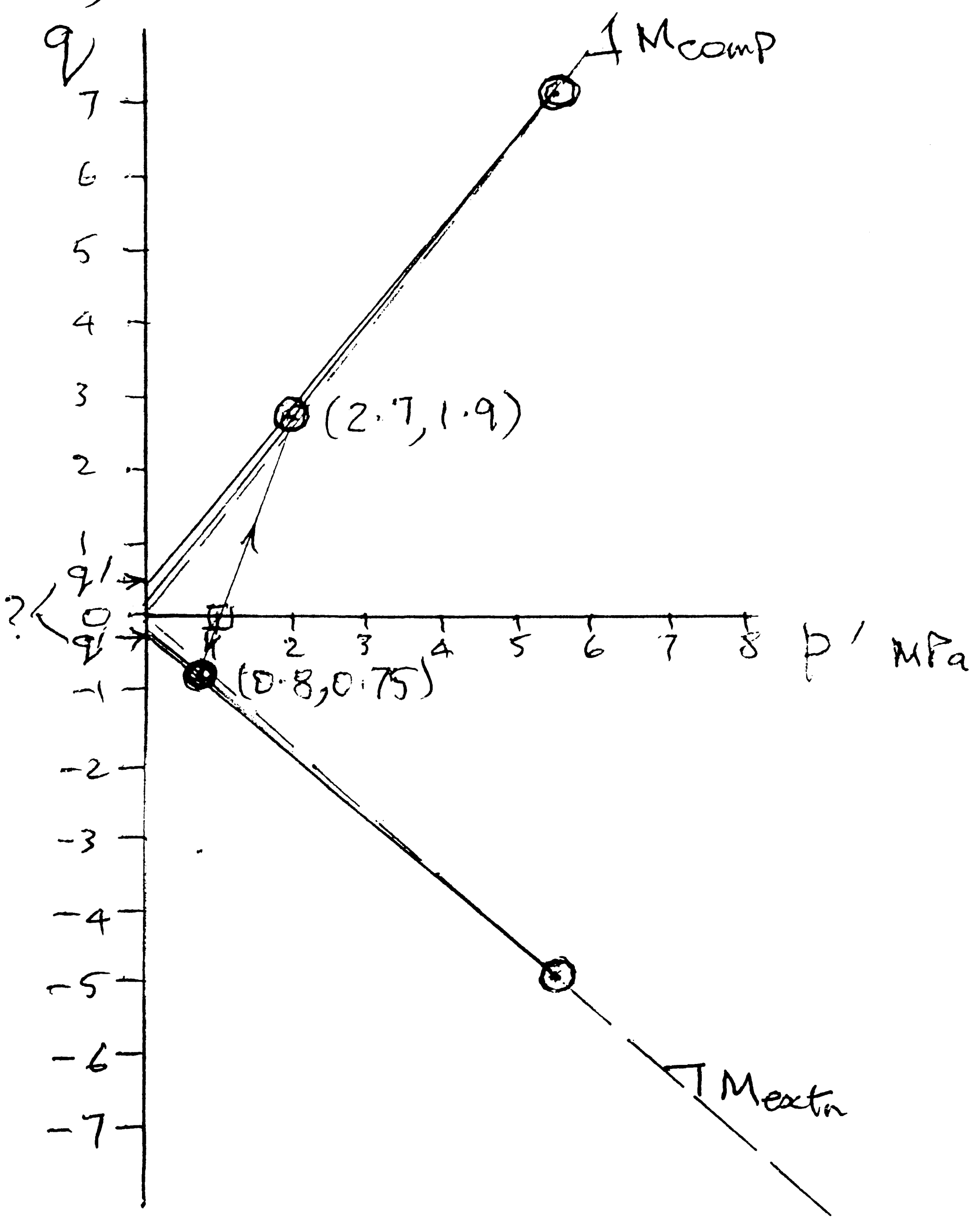
~~Generalising~~ using (b) $M_{comp} = 1.29$

$\therefore q_{comp,\,crit} = \underline{7.08} \text{ MPa}$

$M_{extn} = 0.90$

$\therefore q_{extn,\,crit} = \underline{-4.95} \text{ MPa}$ 4

4 e)



The difficulty arises at $p' \ll 1$ MPa where the real data of sands (as expressed through I_R etc) curves in to the origin with no "true cohesion" intercept at $p' = 0$. Although the distinction may look small, a q' intercept of 150 kPa would imply that dry sand could stand in 10 m vertical cliffs! 2